



ANALYSIS OF BISTABILITY PHENOMENON OF THE FLOW ON TWO CIRCULAR CYLINDERS SIDE-BY-SIDE WITH HILBERT-HUANG TRANSFORM METHOD

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Abstract. *The bistable phenomenon consists on the deviation of the flow of two cylinders placed side-by-side, where a stable characteristic is acquired for a certain period of time. Usually, this phenomenon is investigated using with Fourier Spectral Analysis and Wavelets. This paper presents an experimental study alternative tool to treat the bistability phenomenon with the Hilbert-Huang Transform, which is composed by Empirical Mode Decomposition (EMD) and the Hilbert Spectral Analysis (HSA). The flow of two rigid and flexible circular cylinder placed side-by-side was studied, where two different pitch-to-diameter ratios were studied ($p/d = 1.26$ and 1.6). The experiment was performed with arrangements of two rigid cylinders of diameters 25 mm, with an blockage ratio of the arrangements is 25.9%. The hot-wire anemometry technique was used as experimental technique, which consists of measuring velocity fluctuations in aerodynamic channel. The results show that The Hilbert-Huang Transform is a valuable additional tool to complement the Wavelet analysis in a study of the features of this phenomenon.*

Keywords: *turbulent flow, hot-wire anemometry, biestability, Hilbert-Huang Transform, wavelets.*

1. INTRODUCTION

The bistable phenomenon was a typical phenomenon that occurring in a flow on tube banks, with a large application on engineering. The manifestation of this phenomenon was present also in arrangements of two cylinders side-by-side subject to a flow.

The traditional methods to analyze this phenomenon and others in turbulence consist in a mathematical tools like Fourier spectral analysis and wavelets. The use of each method is limited by a condition of the flow, stationary for Fourier, and the choice of the wavelet adequate to treat the transient data.

This study suggests a new auxiliary method that enables the analysis of turbulent signals that are non-stationary and non-linear named Hilbert-Huang Transform (HHT) Huang et al. (1998), which is composed by Empirical Mode Decomposition (EMD), which is a direct and intuitive method, from the decomposition of the signal, and the Hilbert Spectral Analysis (HSA).

According to Huang and Shen (2005), the decomposition is based on the simple assumption that all the data consist of different simple intrinsic modes of oscillations, each of these modes of oscillation is called Intrinsic Mode Functions (IMF) and are obtained from the envelopment of the maximum and minimum values and the average signal from this envelopment. Repeating this procedure iteratively the decomposition of the original signal is obtained. The Hilbert transform is used to obtain a continuous spectral distribution of signal energy in the time frequency domain. This spectral analysis is made possible by applying the Hilbert transform to each of the intrinsic mode functions of the original signal.

A comparison between the wavelet transform and Hilbert-Huang transform to signals from earthquakes is made in Shi and Luo (2003). The authors show that the Hilbert-Huang transform performs a direct decomposition of the original signal and can show more clearly the intrinsic properties of the original signal.

In this paper, the Hilbert-Huang Transform was used like an auxiliary method to analyze the bistable phenomenon, which presents the characteristic signal to apply this method, on the flow on two cylinders side-by-side, where the signal was treated with wavelets. A comparison of the results of each method was discussed.

2. BISTABLE PHENOMENON

According to Sumner et al. (1999), the cross steady flow through circular cylinders of same diameter (d) placed side-by-side presents a wake with different modes depending on distance between the centers of the cylinders (p). In intermediate spacing ratios ($1.2 < p/d < 2.2$) identifies the flow to form two wakes behind the cylinders, a large wake behind a cylinder and a narrow belt after another, Figure. 1. The presence of these wakes make two dominant frequency vortex shedding are derived: one related to the higher narrow wake, and another is associated with lower wide wake.

The flow passing through the slit is deviated toward the wake narrower. The bistable phenomenon called, according to the technical literature, the flow pattern undergoes a change that deviated intermittent, sometimes oriented toward a cylinder, sometimes in the other direction. This phenomenon is considered an intrinsic property of the flow and is independent of Reynolds number and is not related to misalignments between the cylinders or any other external influence.

According to Kim and Durbim (1988), the transition between two asymmetric states is random, being the time scale between transitions about 10^3 times larger than the period vortex shedding, while for Peschard and Le Gal (1996), the behavior is not intrinsic to the flow, but disturbances associated with turbulent flow at the entrance.

Guillaume and LaRue (1999) define terms that describe each of the three types of bistable behavior, the *quasi-stable behavior* where the flow does not vary with time and large-scale disturbances can cause changes in the average values of the wakes, but the new values remain the same until another major disturbance is applied; the *spontaneous flopping*, where the average flow observed over time alternate between a high value and one featuring the two modes flow, even if no disturbance is applied; and the *forced flopping*, exchanges that are derived a large disturbance applied.

Sumner et al. (1999) conducted a study of flow around two and three cylinders arranged side-by-side across the flow, for pitch ratios between 1 and 6 and Reynolds number in the range between 500 and 3000. In the experiment for two cylinders, was not identified the bistable phenomenon. The non-appearance of bistability was attributed to the combined effects of the small degree of misalignment of the cylinders and experimental effects such as aspect and blockage ratios, the latter being 13%.

Zhou et al. (2002) studied about the turbulent on wake two cylinders placed side-by-side in terms of velocity fields and temperature, for the pitch ratios p/d between 1.5 and 3.0. The results were compared with the wake of a single cylinder. The authors attributed the extreme narrowing of the gap, the fact that only one frequency, not two, be measured. A gap between the cylinders very close can inhibit the generation of vortices behind the cylinders, and start to act as one body with only one wake vortex generated.

For Alam et al. (2003) the flow around two circular cylinders of equal diameter, arranged side-by-side in the transverse direction of the flow shows that the wake vortices have different modes of flow. These studies were developed using the Reynolds number in the subcritical regime, 5.5×10^4 and, according to the authors, the forces exerted on the body are insensitive against variations of Reynolds number in this regime.

Alam e Zhou (2007) analyze the flow around two cylinders placed side-by-side on the cross-flow, for small pitch ratios ($1.1 < p/d < 1.2$), with a Reynolds number of 4.7×10^4 . The authors identified four distinct modes flow.

Olinto et al. (2009) conducted a study of the bistable phenomenon in flow in aerodynamic channel on two cylinders arranged side-by-side, with $Re = 3 \times 10^4$ and blockage ratio 33%. The author found the strong presence of bistability in measurements near to the cylinders (until $x/d = 0,93$), where "x" is the distance of the probe to the center of the cylinders. For a greater ratio distance did not identify the bistable standard.

De Paula (2008) studied the presence of the bistable phenomenon for two tubes, for pitch ratios $p/d = 1.26$ and 1.6 , and Reynolds number range of 1.85×10^4 and 2.98×10^4 . Several changes of velocity were observed during the entire period of data acquisition.

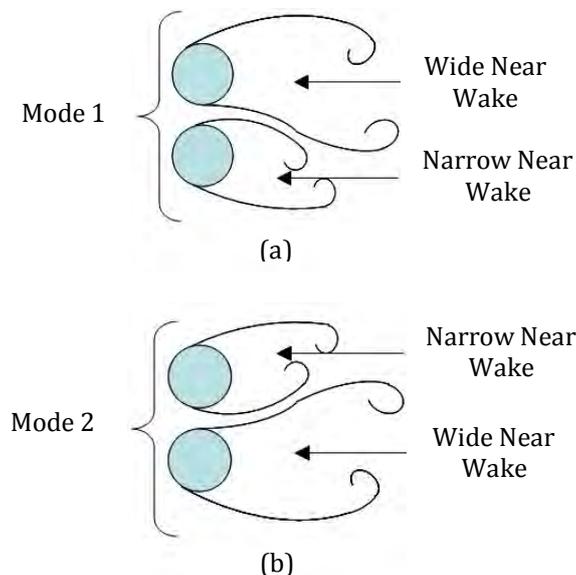


Figure 1. Bistability scheme for (a) mode 1 and (b) mode 2.

3. MATHEMATICAL TOOLS

3.1 Fourier and Wavelet Transforms

The statistical (or time domain) analysis consists on determining the first four moments of the probability density function: mean (average), standard deviation, skewness and kurtosis. The spectral (or frequency domain) analysis can be done through the power spectral density function (PSD). The joint time-frequency domain analysis was made through wavelet transform. The wavelet analysis can be applied to time varying signals, where the stationary hypothesis cannot be maintained, to allow the detection of non permanent flow structures.

The Fourier transform of a discrete time series gives the energy distribution of the signal in the frequency domain evaluated over the entire time interval. The Fourier spectrum is defined as

$$P_{xx}(f) = |\hat{x}(f)|^2 \quad (1)$$

The first attempt to deal with nonstationary processes was the Windowed Fourier Transform. However, due to the aliasing of high and low frequency components that do not fall within the frequency range of the window, the windowed Fourier transform is inaccurate for time-frequency location of transient features.

While the Fourier transform uses trigonometric functions as basis, the bases of wavelet transforms are functions named wavelets, with finite energy and zero average that generates a set of wavelet basis.

The continuous wavelet transform of a function $x(t)$ is given by:

$$\tilde{X}(a, b) = \int_{-\infty}^{\infty} x(t) \psi_{a,b}(t) dt \quad (2)$$

where ψ is the wavelet function and the parameters a and b are respectively scale and position coefficients ($a, b \in \mathfrak{R}$ and $a > 0$).

The respective wavelet spectrum is defined as:

$$P_{xx}(a, b) = |\tilde{X}(a, b)|^2 \quad (3)$$

In the wavelet spectrum, Equation (2), the energy is related to each time and scale (or frequency), Daubechies (1992). This characteristic allows the representation of the distribution of the energy of the signal over time and frequency domains, called spectrogram.

The velocity signals were analyzed using wavelet transforms to obtain the energy distribution of the turbulent flow over time-frequency domain. The continuous wavelet spectrum was obtained through continuous wavelet transform. The discrete wavelet transform was used to decompose the measured signal in wavelet approximations divided in frequency bands, Indrusiak et al. (2011).

In this work, Daubechies "db20" functions were used as bases of discrete wavelet transforms.

3.2 Hilbert-Huang Transform

The Hilbert-Huang Transform (HHT) is applied in the treatment of non-linear and non-stationary signals and it is made up of Huang Transform and Hilbert spectral analysis. The HHT is ruled by the empirical mode decomposition (EMD), known as the Huang Transform. The EMD assumes that any data set consists of different, simple, intrinsic modes of oscillation that need not be sinusoidal. Based on this, each mode of oscillation from high frequency to low frequency is derived in an objective manner from the recorded complex data. We call each of these oscillatory modes an intrinsic mode function (IMF). As discussed by Huang et al. (1998), the EMD method is necessary to deal with data from non-stationary and non-linear processes. This new method is intuitive, direct, and adaptive, with an *a posteriori*-defined basis, from the decomposition method, based on and derived from the data. The decomposition is made from the identification of all the local maxima. Connect all the local maxima by a cubic spline to produce the upper envelop of data, i.e., $x(t)$, and repeat the procedures for the local minima to produce the lower envelop of $x(t)$. All the data should be encompassed by the upper and lower envelopes. Their mean of these envelopes is designated by $m_1(t)$, and the difference between the data $x(t)$ and $m_1(t)$ provide us the first component $h_1(t)$, i.e.,

$$h_1(t) = x(t) - m_1(t) \quad (4)$$

Ideally, $h_1(t)$ should be an IMF, but all the conditions of an IMF should be achieved; the conditions are, (Huang and Shen, 2005):

- (1) In the whole dataset, the number of maxima and minima as well as the number of zero-crossings must either equal or differ at most by one, and
- (2) At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

Not satisfied the condition of an IMF, the process is repeated. In the subsequent sifting process, $h_1(t)$ is treated as the data, then

$$h_{11}(t) = h_1(t) - m_{11}(t) \quad (5)$$

where $m_{11}(t)$ is the mean of the upper and lower envelopes of $h_1(t)$. Repeating k times until $h_{1k}(t)$ to satisfy the conditions of an IMF, we have

$$h_{1k}(t) = h_{1(k-1)}(t) - m_{1(k-1)}(t). \quad (6)$$

It is named the first IMF component $c_1(t)$ from the data, because $c_1(t) = h_{1k}(t)$. The component $c_1(t)$ will contain the finest-scale of the highest frequency component of the signal. The residue $r_1(t)$, given by

$$r_1(t) = x(t) - c_1(t) \quad (7)$$

contains longer-period components, is treated as new data and subjected to the same sifting process as described above. This procedure can be repeated to obtain all the subsequent $r_j(t)$'s

$$r_j(t) = r_{j-1}(t) - c_j(t) \quad (8)$$

The sifting process can be ended on any of the following predetermined criteria: a) either the component $c_n(t)$ or the residue $r_n(t)$ becomes so small that it is less than a predetermined value of consequence, or b) the residue $r_n(t)$ becomes a monotonic function from which no more IMFs can be extracted, (Huang and Shen, 2005). The original data can be expressed by the sum of the IMF components plus the final residue, thus we obtain

$$x(t) = \sum_{j=1}^n c_j(t) + r_n(t) \quad (9)$$

The components of EMD are usually physical meaningful, for the characteristic scale are defined by the physical data.

The HHT is completed by the Hilbert spectral analysis (HSA) which consists in the application of the Hilbert transform on each IMF components obtained. For one IMF $c_i(t)$ in Eq. (14), we can express the Hilbert transform as

$$H[c_i(t)] = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{c_i(t')}{t - t'} dt' \quad (10)$$

From this definition, an analytic signal may be given by

$$z_i(t) = c_i(t) + jH[c_i(t)] = a_i(t)e^{i\Phi_i(t)} \quad (11)$$

where

$$a_i(t) = \sqrt{c_i^2(t) + H^2[c_i(t)]} \quad (12)$$

$$\Phi_i(t) = \arctan \frac{H[c_i(t)]}{c_i(t)} \quad (13)$$

The instantaneous frequency is obtained from Eq. (18) as

$$\omega_i(t) = \frac{d\Phi_i(t)}{dt} \quad (14)$$

After applying the Hilbert transform to each IMF component, the original signal can be expressed as the real part (RP) in the following form (Cheng et al., 2008):

$$x(t) = RP \sum_{i=1}^n a_i(t) e^{i\Phi_i(t)} = RP \sum_{i=1}^n a_i(t) e^{i \int \omega_i(t) dt} \quad (15)$$

At this moment, the residue $r_n(t)$ is left out on purpose, for it is either a monotonic function or a constant. Eq. (16) gives both amplitude and frequency of each component as functions of time. This frequency-time distribution of the amplitude is designated as the Hilbert spectrum $H(\omega, t)$:

$$H(\omega, t) = RP \sum_{i=1}^n a_i(t) e^{i \int \omega_i(t) dt} \quad (16)$$

All mathematical analysis was made with MATLAB © software and its specific toolboxes for statistical, spectral and wavelet analysis.

4. EXPERIMENTAL TECHNIQUE

Velocity measurements were made with DANTEC *StreamLine* hot wire anemometer in an aerodynamic channel to investigate the flow around a circular cylinder and the shedding process.

The test apparatus, shown on Fig. 2, with 146 mm height and width of 193 mm. Air, at room temperature, is the working fluid, driven by a centrifugal fan of 0,75 kW, passed by a diffuser and a set of honeycombs and screens, which reduce the turbulence intensity in the channel to about 1%. A frequency inverter controls the fan speed, where the flow velocity in the aerodynamic channel can be varied from 0 to 15 m/s. To measure the velocity reference a Pitot tube fixed before to the test section is used. The measures for a single cylinders was made for free stream velocity 15 m/s for two cylinders side-by-side.

The cylinders arrangement side-by-side was positioned 220 mm from the end of the channel. The incidence angle of the flow on the cylinder is 90°.

For the measurement of velocity and velocity fluctuations, two single hot wire probes are positioned as shown in Fig. 2(b) and 2(c), where the distance "x" of the probes to the cylinders is variable according to diameter of the tube examined.

Data acquisition was performed with a 16-bit A/D-board (NATIONAL INSTRUMENTS 9215-A) with USB interface, with a sampling frequency of 3000 Hz and a low pass filter at 1000 Hz.

The data set was acquired in steady state flows at same velocity value.

Computations of the wavelet transform were performed using the MatLab © software. The experimental data were analyzed by statistical, spectral, wavelet tools.

The error of the determination of the velocity fluctuations with a hot wire is between 3 and 6 %.

5. RESULTS

In this study the flow for two cylinders side-by-side, pitch-to-diameter ratio $p/d = 1.26$ and $p/d = 1.6$, were used. "p" is the distance between the centers of two cylinders and "d" is the diameter. The circular cylinder of diameter 25 mm was chosen. In this configuration, the blockage ratio on the channel is 25.90%. The values of Reynolds Number of each experiment are based on the average velocity of undisturbed flow (characteristic velocity) and the diameter of the cylinder (characteristic length).

5.1. Results for $p/d = 1.26$

For this pitch-to-diameter ratio, the bistable phenomenon was identified in the cylinders arrangement. Figure 3 shows the original data signal, where the bistability is most clearing seen in the signal of probe 1 and many changes are observed. The fact in the probe 2 did not collect the changes characteristics of the phenomenon can be explained by the sensibility of this probe position. The energy distribution of the velocity signals is displayed in spectrograms made from the continuous wavelet transform, Fig. 4. The signal analyzed showed the mode change, characterizing the bistable phenomenon. Reconstructions of the signals processed by discrete wavelet analysis were grouped together with spectrograms. The Wavelet function Db20 level 9 was used, with frequencies between 0 and 2.93 Hz. The Fig. 4 shows also in the spectrograms with the reconstruction of the velocity signals for cylinders, regions that concentrate more energy are related to higher velocities. Consequently, lower velocities are associated with regions whose energy is lower.

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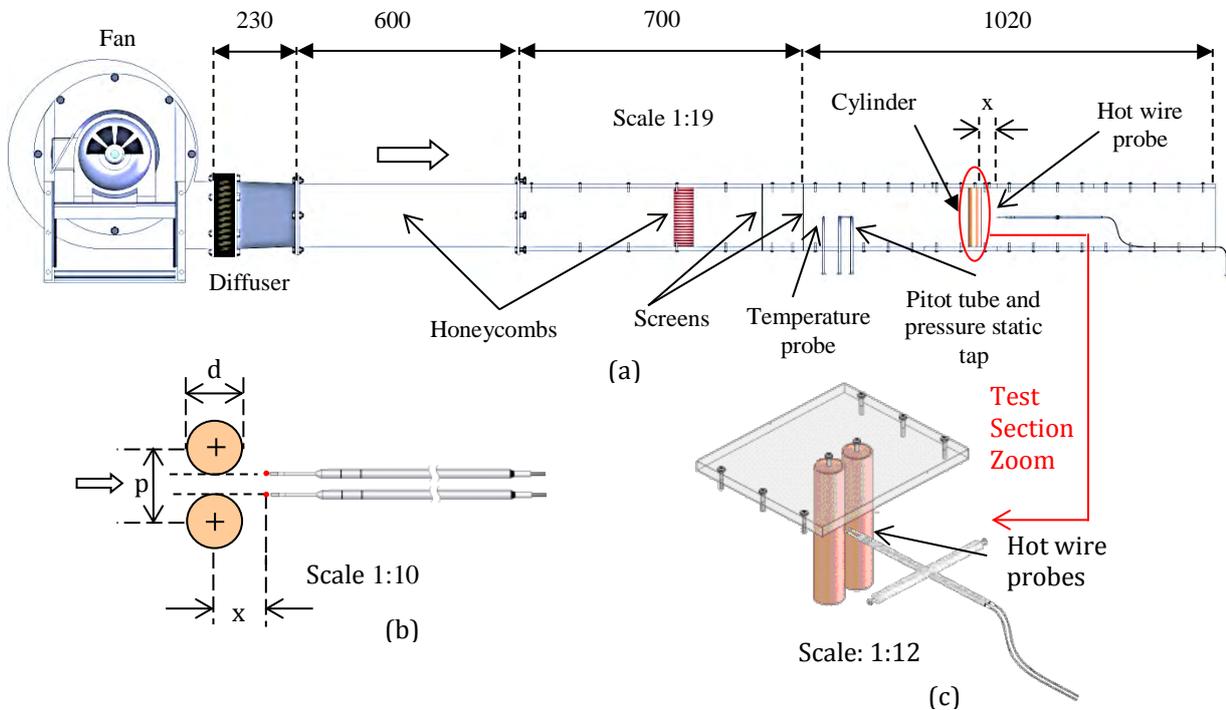


Figure 2. Schematic view of (a) the aerodynamic channel, (b) probes position for two cylinders and (c) test section (measures in mm).

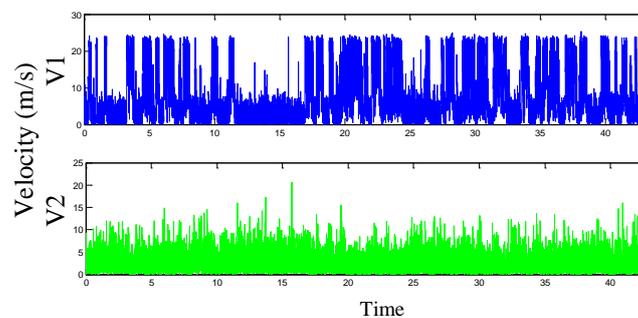


Figure 3. Signal Velocities for cylinders $x/d = 0.338$ and $Re = 2.28 \times 10^4$. Probe 1 – V1, Probe 2 – V2.

The Hilbert-Huang transform is applied on the data signal to obtain the Intrinsic Mode Functions (IMF's) with the Empirical Mode Decomposition (EMD). Figure 5 show us the first four IMF's components of the signal, whose decomposition on EMD generate 17 IMF's, besides the residue, in each signal, but we choice the first four that are the most important contents, with the highest frequency.

In additions, the Fast-Fourier Transform (FFT) was applied on the each IMF showed, in the Fig. 6. This procedure is useful to give us the most clearly visualization of the reduction of the frequency with the increase of the IMF's components obtained, characterized by a frequency peak value, which are, 500, 250 and 150 Hz. These values of the frequency can be seen in the Fig. 7 in the Hilbert-Huang Spectrum, but due the high number of points of the data signal, the visualization becomes obfuscated. In this case, we cannot observe any relation between the spectrogram with the continuous wavelet transform, from the Fig. 4, and the Hilbert-Huang spectrum, from the Fig. 7.

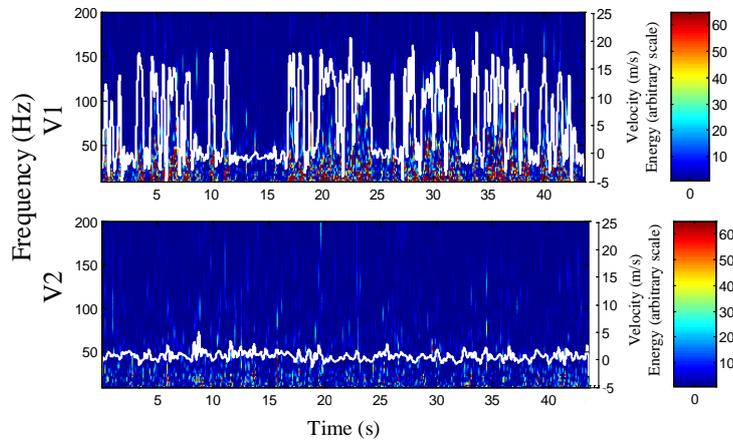


Figure 4. Spectrograms and reconstruction for signals of cylinders, for $p/d = 1.26$.

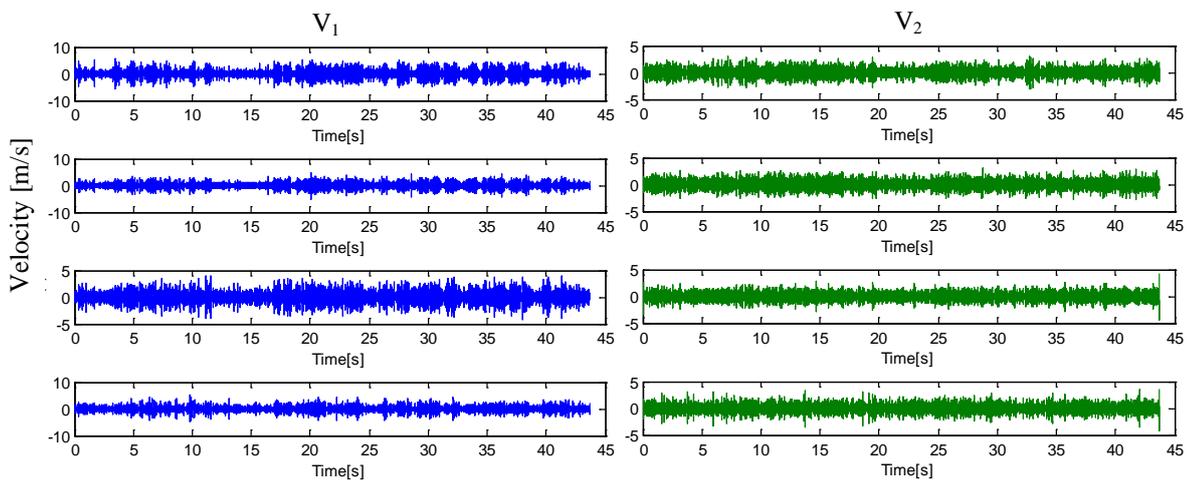


Figure 5. First four IMF's components of the signal.

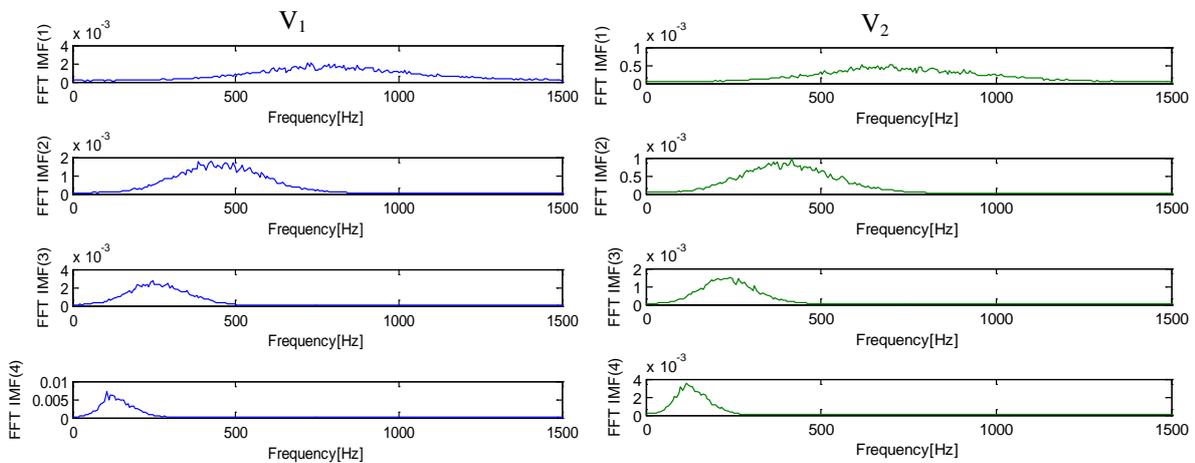


Figure 6. FFT of the first four IMF's components of the signal.

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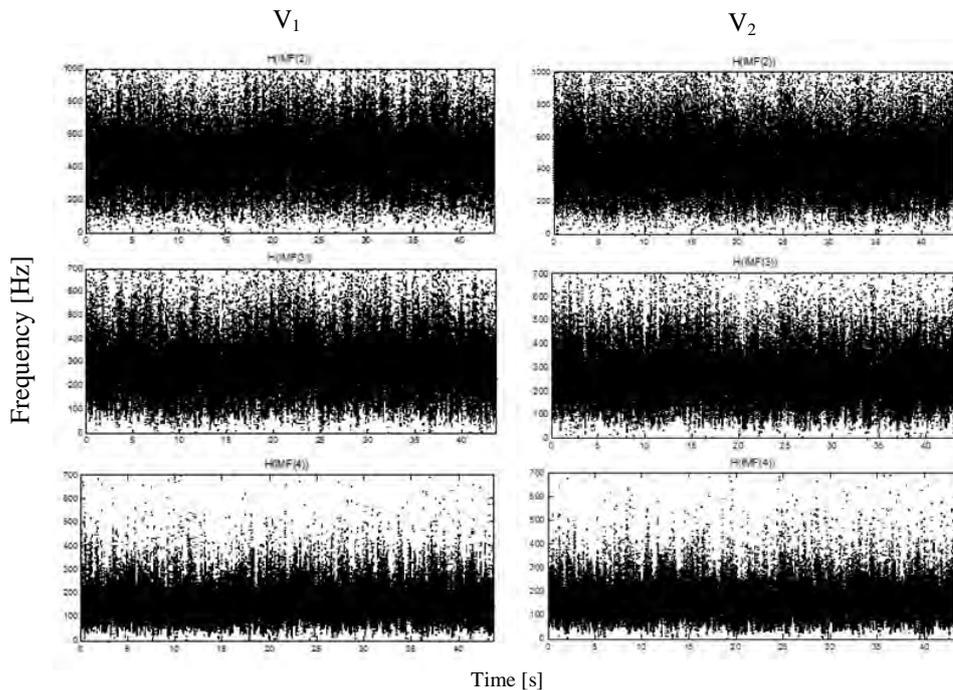


Figure 7. Hilbert-Huang Spectrum of the second, third and fourth IMF's.

5.2. Results for $p/d = 1.6$

The arrangements of cylinders presents the changes of velocity levels for this p/d ratio, as shown in Fig. 8, for the two signals obtained from the each probe. This characterizes the bistable phenomenon seen in a more complementary. The features of the signal also are analyzed from the Fig. 9 with the reconstructions of the velocity signals via discrete wavelet transform in the spectrogram with the continuous wavelet transform. The regions with more energy it is associated with the highest velocities and, analogously, the lower velocities are related to smallest concentrations of the energy signal.

We applied the Hilbert-Huang transform and we obtain 17 IMF's for the signal V1 and 17 IMF's for the signal V2, besides the residue for each signal, by the EMD method, Fig. 10. Again, the first four IMF are showed. From the same way, the FFT of the each IMF are made, Fig. 11, and the same characteristics are observed from the p/d ratio 1.26: the decrease of the main frequency of the component with the increase of the IMF, and tis visualization became better to the Hilbert-Huang Spectrum, Fig. 12, which shows, for the second, third and fourth IMF, the frequency distribution around the peak value of, 450, 300 and 200 Hz, respectively. As seen also in the p/d ratio 1.26, these peaks of frequency can be related to the shedding process around these cylinders, and no relation between the Hilbert-Huang spectrum and the spectrogram by wavelets was observed.

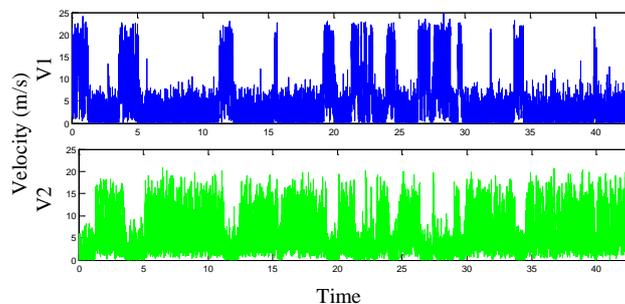


Figure 8. Signal Velocities for cylinders, $x/d = 0.28$ and $Re = 2.63 \times 10^4$.

Probe 1 – V1, Probe 2 – V2.

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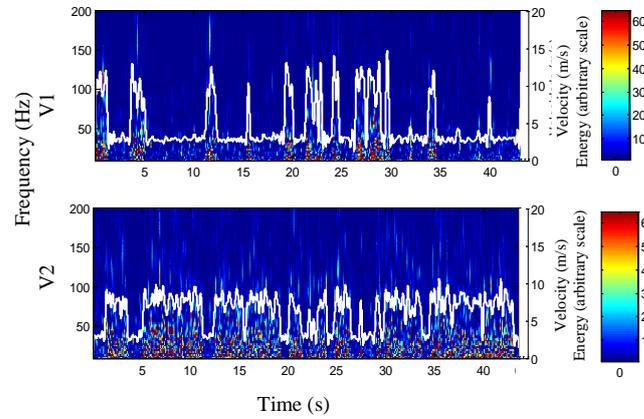


Figure 9. Spectrograms and reconstruction for signals of cylinders, for $p/d = 1.6$.

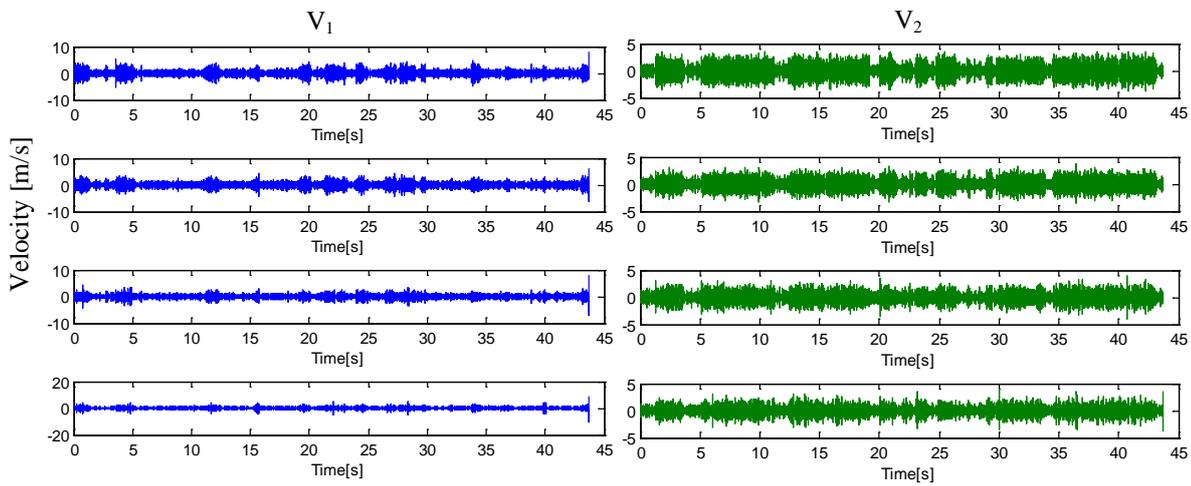


Figure 10. First four IMF's components of the signal.

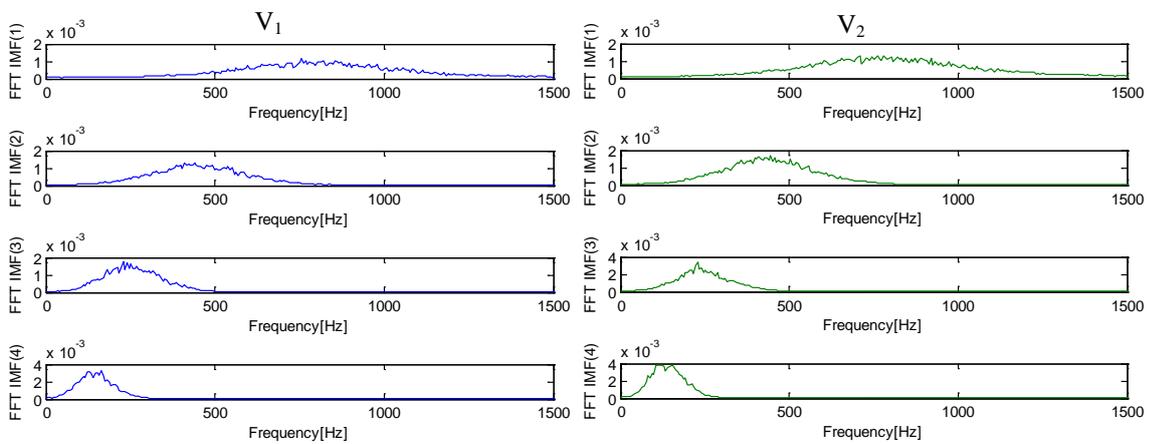


Figure 11. FFT of the first four IMF's components of the signal.

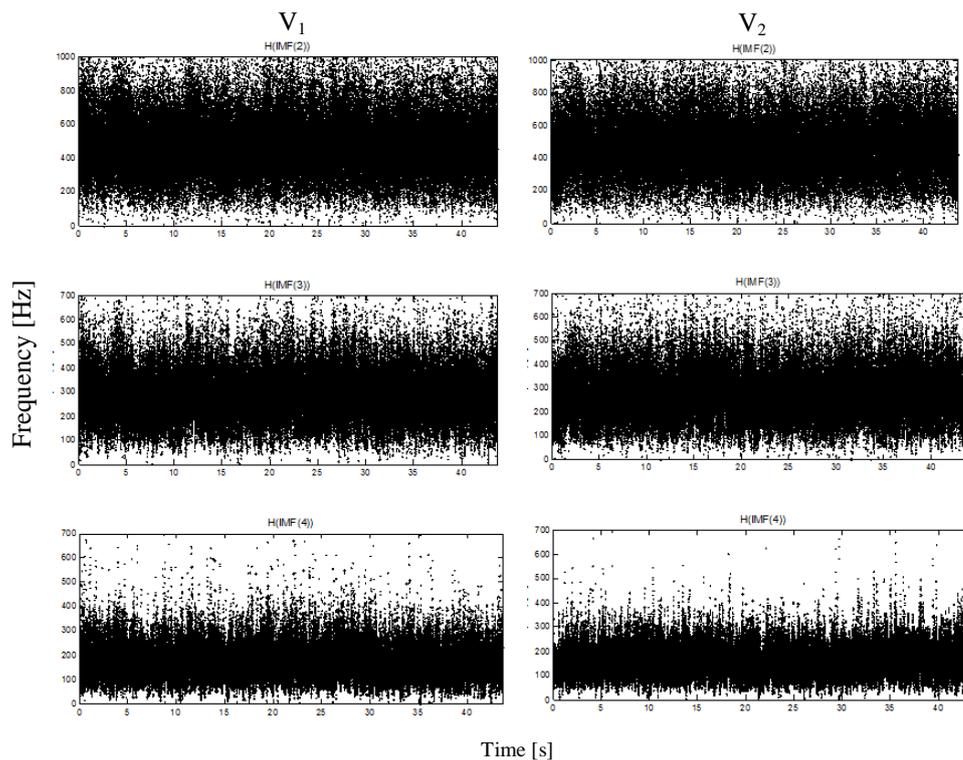


Figure 12. Hilbert-Huang Spectrum of the second, third and fourth IMF's.

6. CONCLUSIONS

The Hilbert-Huang transform give us the main characteristics of the signal associated to its oscillations modes. The gain in the use of the HHT method is the analysis of the frequency instantaneous distribution over the time and your behavior obtained from the each IMF component from the EMD method. This procedure can be useful to determination of this information on the mapping of any signal. In the turbulence, more specifically, in the flow around the cylinders showed in this paper, HHT appears with an additional method to obtain other analysis parameters as instantaneous frequency and instantaneous energy signal distribution. Together with Fourier analysis and wavelet analysis, HHT may qualify as an additional and practice tool in the study of this type of data signal and the bistable phenomenon.

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8. REFERENCES

- Alam, M. M., Moriya, M. and Sakamoto, H., 2003, "Aerodynamic characteristics of two side-by-side circular cylinders and application of wavelet analysis on the switching phenomenon", *Journal of Fluids and Structures*, v. 18, pp. 325–346.
- Alam, M. M., and Zhou, Y., 2007, "Flow Around Two Side-by-Side Closely Spaced Circular Cylinders", *Journal of Fluids and Structures*, v. 23, pp. 799–805.
- Can-Yang, H.; Qing-Jun, C.; Tian-Li, H., 2008. Estimation of Local Spectral Density of Seismogram by Orthogonal Hilbert-Huang Transform and Application of the Spectral Density, *The 14th World Conference on Earthquake Engineering*, Beijing, China.
- Cheng, J.; Yu, D.; Tang, J.; Yang, Y., 2008. Application of the frequency family separation method based upon EMD and local Hilbert energy spectrum method to gear fault diagnosis, *Mechanism and Machine Theory*, v. 43, p. 712–723.
- De Paula, A.V., 2008, "Estudo sobre o Fenômeno da Biestabilidade de Escoamentos Turbulentos em Bancos de Tubos de Arranjo Triangular", *Dissertação de Mestrado, PROMEC-UFRGS, Porto Alegre*, available in: <http://hdl.handle.net/10183/16309>.

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- Guillaume, D. W. and LaRue, J. C., 1999, "Investigation of the Flopping Regime with Two-, Three- and Four-Cylinder Arrays", *Experiments in Fluids*, v. 27, pp. 145-156.
- Huang, N.E.; Shen, Z.; Long, S.R.; Wu, M.C.; Shih, H.H.; Zheng, Q.; Yen, N.C.; Tung, C.C.; Liu, H.H., 1998. The Empirical Mode Decomposition and the Hilbert Spectrum for Nonlinear and Non-stationary Time Series Analysis, *Proceedings of the Royal Society*, v. 454, p. 903-995.
- Huang, N.E.; Shen, S.S.P., 2005. *Hilbert-Huang Transform and Its Applications*, World Scientific, v. 5, Singapore.
- Indrusiak, M.L.S. and Möller, S.V., 2011, "Wavelet Analysis of Unsteady Flows: Application on the Determination of the Strouhal Number of the Transient Wake Behind a Single Cylinder", *Experimental Thermal Fluid Science*, v. 35, pp. 319-327.
- Kim, H. J. and Durbin, P. A., 1988, "Investigation of the Flow Between a Pair of Circular Cylinders in the Flopping Regime", *Journal of Fluid Mechanics*, v. 196, pp. 431-448.
- Olinto C.R., Indrusiak M.L.S., Endres L.A.M. and Möller SV, 2009, "Experimental Study of the Characteristics of the Flow in the First Rows of Tube Banks", *Nuclear Engineering and Design*, v. 239, pp. 2022-2034.
- Peschard I. and Le Gal, P., 1996, "Coupled Wake of Cylinders", *Physical Review Letters*, v. 77, pp. 3122-2125.
- Shi, C.X.; Luo, Q.F., 2003. Hilbert-Huang Transform and Wavelet Analysis of Time History Signal, *ACTA Seismologica Sinica*, v. 16, no. 4, p. 422-429.
- Sumner, D., Wong, S. S. T., Price, S. J. and Paidoussis, M. P., 1999, "Fluid Behaviour of Side-by-Side Circular Cylinders in Steady Cross-Flow", *Journal of Fluids and Structures*, v. 13, pp. 309-338.
- Yu, D., Cheng, J., Yang, Y., 2003. Hilbert Energy Spectrum and Its Application to Gear Fault Diagnosis, *Journal of Human University*, v. 30, N^o. 4, p. 47-50.
- Zdravkovich, M. M., 1997, *Flow Around Circular Cylinders*, v. 1-2, Oxford University Press Inc., New York.
- Zdravkovich, M. M., 1977, "Review of Flow Interference Between Two Circular Cylinders in Various Arrangements", *Journal of Fluids Engineering*, v. 4, pp. 618-633.
- Zhou, Y., Zhang, H. J. and Yiu, M. W., 2002, "The Turbulent Wake of Two Side-by-Side Circular Cylinders", *Journal of Fluid Mechanics*, v. 458, pp. 302-332.

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