OPEN LOOP RESPONSE IDENTIFICATION DURING NON-STATIONARY OPERATION OF ACTIVELY CONTROLLED ROTATING SYSTEMS

Patrick Felscher
Felix Dornbusch
Richard Markert
Technische Universität Darmstadt, Fachgebiet Strukturdynamik, Petersenstrasse 30, 64287 Darmstadt, Germany
felscher@sdy-tu-darmstadt.de, dornbusch@sdy-tu-darmstadt.de, markert@sdy.tu-darmstadt.de

Rodrigo Nicoletti
University of São Paulo, São Carlos School of Engineering, Trabalhador São-Carlense 400, 13566-590 São Carlos, Brazil
rnicolet@sc.usp.br

Abstract. One of the challenges in designing actively controlled rotating systems is finding the appropriated controller gains of the feedback system. Some strategies of finding the controller gains require that the open loop response of the system be known or identified experimentally. This can be done by exciting the system in a given rotating speed (stationary condition). However, the most critical situations that justify controlling occur during run-up and run-down (non-stationary conditions). In these non-stationary conditions, unbalance strongly affects the results and must be considered in the analysis. In this work, one presents a methodology for identifying the open loop response of an actively controlled rotating system during non-stationary conditions. One takes advantage of the fact that, in these conditions, frequency changes with time and can be used for exciting the system. Unbalance can be disregarded in signal processing by measuring the unbalance response during run-up and forced + unbalance response during run-down. Experimental results show the feasibility of finding the open loop response of the system after a single run-up and run-down operation. Phase identification is critical and some precautions during the response measurement procedure are suggested. At the end, the obtained open loop response functions can be further used to find the controller gains of the system for non-stationary operation.

Keywords: rotor dynamics, identification, non-stationary condition, control system, frequency response

1. INTRODUCTION

Rotating machines are vital elements in industry and, for this reason, they must present not only high performance, but also high availability to avoid interruptions in the production flow. One way of increasing the efficiency of rotating machines is the attenuation/control of the vibration levels, specially the vibration that occurs in the shaft. Unbalance, misalignment, or external forces due to operational conditions are the most common contributing factors for high lateral vibration in rotors, and they can reduce performance and cause energy losses, fatigue, or even failure (Adams, 2010). However hard to eliminate, controlling vibration levels within acceptable margins is essential for the safe and reliable operation of rotating machines.

In this context, actuators and sensors have been incorporated into rotating machines, and control systems have been developed (Keogh et al., 1995; Sun and Krondkiewski, 2000; Santos et al., 2004; Pinte et al., 2010). In literature, one can find different control techniques applied to vibration control of rotating systems, most of them based on traditional control strategies: PID, optimum and robust control. A thorough review of control system design for rotating systems is presented in Schweitzer and Maslen (2009). In the experimental implementation of such control systems, it is usually necessary to have a mathematical model of the system to design the controller. In these cases, the successful control of vibration depends on the quality of the adopted model, including the model of sensors and actuators. Considering that imprecise models can jeopardize the performance of the controller, model free design of controllers began to be investigated.

The most common strategy for designing the control system not depending on mathematical models is based on the experimental identification of open loop frequency responses. In this case, the actuators can be used as exciters and open loop frequency response is obtained for the global system composed of the actuator system + plant + sensor system (Keel and Bhattacharyya, 2008). Hence, all dynamic information is embedded in the global open loop response function, and the controller can be designed with no further data. Regarding stationary operation (constant rotating speed), successful results were achieved for a rotating system with electromagnetic actuators whose flexible shaft supported a single disk (Buttini et al., 2011), and two disks (Buttini and Nicoletti, 2012).

The most critical situations that justify vibration control in rotating systems occur during non-stationary operation of the machine (run-up and run-down). In this non-stationary condition, unbalance has a strong impact in the dynamic behavior of the system, and must not be overlooked. In this work, one presents a methodology for identifying the open loop response of an actively controlled rotating system during non-stationary conditions. One takes advantage of the
fact that, in these conditions, frequency changes with time and can be used for exciting the system. Unbalance can be disregarded in signal processing by measuring the unbalance response during run-up, and forced + unbalance response during run-down (or a second run-up). Experimental results show the feasibility of finding the open loop response of the system after a single run-up and run-down operation (or two successive run-ups). Phase identification is critical and some precautions during the response measurement procedure are suggested. At the end, the obtained open loop response functions can be further used to find the controller gains of the system for non-stationary operation. In this case, due to noise in the identified open loop response, an analysis considering confidence intervals is performed to find the best PD gains for the system.

2. OPEN LOOP IDENTIFICATION

During non-stationary operation, the rotating system is subjected to the unbalance excitation, whose excitation frequency changes with the increasing (run-up), or decreasing (run-down), rotating speed. By measuring the displacements presented by the shaft, one obtains the unbalance response of the system (Fig. 1(a)), whose input (unbalance force) is usually unknown. When the rotating system has actuators, the control system tries to compensate the effects of the unbalance, as shown in Fig. 1(b). However, for finding the appropriated control gains for the feedback, one must know the open loop response function of the system, which is the response of the system to the control inputs without any other disturbance (Fig. 1(c)). The problem is that unbalance is always present and, again, it is an unknown parameter.

![Figure 1. Block diagram of the system: (a) unbalance response; (b) control + unbalance response (operating condition); (c) open loop response for control system design.](image)

Hence, the challenge is finding the open loop response of the system during non-stationary conditions, where an unknown unbalance disturbance is present. For that, one can calculate:

$$H_{\text{OLFR}}(\omega) = \frac{X_{uc}(\omega) - X_u(\omega)}{U_c(\omega)}$$  \hspace{1cm} (1)$$

where the open loop frequency response (OLFR) of the system is obtained by subtracting the displacement signal due to unbalance from the displacement signal due to control + unbalance, and dividing by the control signal sent to the actuators in the control + unbalance response measurement.

In this case, one critical assumption must be valid: the unbalance disturbance during unbalance response measurement ($X_u$) shall be the same as that during the control + unbalance response measurement ($X_{uc}$), otherwise there will be a difference of phase between the responses. To assure such operating conditions, the following procedure is adopted:

**Run-up:** the system is accelerated with constant rate from the minimum up to the maximum speed. The actuating system is turned off, and no control signal is sent to the actuators. During acceleration, the system displacement response $X_u(t)$ is acquired (unbalance response). After reaching the maximum rotating speed, acquisition stops;

**Synchronization of excitation:** at the high rotating speed (above the first critical speed), the phase between the displacement and the unbalance force is almost 180°. Hence, the excitation (signal to the actuators) is synchronized in phase with the measured displacement (sinusoidal signal). As a consequence, the excitation will be in opposition of phase with the unbalance (a constant phase);

**Run-down:** after synchronization, the rotating system is then decelerated under the same rate used in the run-up process. An encoder is used to measure the actual rotating speed of the system. From this information, the frequency of the control signal (excitation) is updated and control + unbalance forces act on the system. The system displacement response $X_{uc}(t)$ is then acquired, together with the control signal $U_c(t)$, which is sinusoidal with changing frequency. After reaching the minimum rotating speed, acquisition stops.
Synchronization could also be done at low rotating speeds, and one could measure the unbalance + control response during run-up and the unbalance response during run-down (inverse procedure). However, at low rotating speeds, the static deflection of the shaft and geometric imperfections (shaft bowing) can be detected by the displacement sensors and strongly affect the results. For this reason, in this work, the synchronization of the excitation is done at the high rotating speed (above the first critical speed), where such effects are minimum.

Synchronization between excitation and unbalance is the key to the successful identification of the open loop response function. Both displacement signals, $X_u(t)$ and $X_{uc}(t)$, must start and finish at the same rotating speeds (excitation frequencies) and the phase between the excitation and the unbalance must be known. By following the procedure above, these assumptions are guaranteed. The unbalance effect is strongly present in the measurement signals, but it is not necessary to identify it (unbalance remains unknown all over the process).

In many practical situations, the rotating speed during run-down cannot be controlled, and this could be a significant problem for the proposed procedure considering that run-up and run-down must be performed at the same rate. However, this procedure can also be applied in two successive run-ups: a first run-up for measuring the unbalance response; run-down with no measurements; and a second run-up with synchronized excitation for measuring the unbalance + control response. In this case, synchronization shall be done at the low rotating speed.

2.1 Test Apparatus

The experimental procedure is tested in the test rig shown in Fig. 2. The rotating system is composed of a flexible shaft (Ø20 mm) with a single disk of 6 kg in the mid span between the ball bearings. The shaft is driven by an electric motor whose speed is controlled by a frequency inverter. The actual speed of the shaft is measured by an encoder and, shaft lateral displacements are measured by proximity probes near the disk. The electromagnetic actuators are located near the disk, as well.

![Test Apparatus Diagram](image)

Figure 2. Flexible shaft rotating system with electromagnetic actuators: (1) electric motor; (2) encoder; (3) flexible shaft; (4) electromagnetic actuators; (5) disk; (6) displacement sensors; (7) self-aligning ball bearings.

3. EXPERIMENTAL RESULTS

The experimental procedure is implemented in the test rig. Initially, the rotating speed is kept at 660 rpm (11 Hz) and run-up is performed up to the rotating speed of 2400 rpm (40 Hz), above the first critical speed, at an acceleration rate of 174 rpm/s (2.9 Hz/s). During run-up, the unbalance response of the system is measured. After reaching the maximum speed, the excitation signal is synchronized with the response phase and a sinusoidal signal, 10 V in amplitude, is sent to
the actuators at the same frequency of the shaft, which is measured by the encoder. Hence, run-down is performed at the deceleration rate of -174 rpm/s, and the unbalance + control response is measured. Signal acquisition is done at 1 kHz acquisition rate during run-up and run-down, whereas the encoder works with 10kHz acquisition rate for more precise measurement in time.

After measuring the signals in time domain, the signals are filtered by a fifth order pass-band Butterworth filter, with cut-off frequencies of 12 Hz and 40 Hz, and Fourier transformed to frequency domain for applying Eq.(1). The obtained results are shown in Fig. 3 for the two possible procedures: (a) unbalance response during run-up and excitation during run-down; (b) unbalance response during run-up and excitation during second run-up.

Figure 3. Open loop response functions identified experimentally in the test rig with the proposed procedure: (a) run-up and run-down; (b) two successive run-ups.

As one can see in Fig. 3, the open loop frequency response of the system can be identified and both procedures result nearly in the same response, as expected. The rotating system, when excited by the actuators, present a resonance frequency near 25 Hz with maximum amplitude of ~0.053 mm/V, and the phase of the shaft response due to the excitation signal starts at -90°, which is typical to inductive systems such as electromagnetic actuators. It is important to emphasize that, in this case, the open loop response function is a global response function that includes the dynamic characteristics not only of the rotating system but also of the actuation system (shaft displacement per unit control voltage). The response is obtained for different rotating speeds and the excitation frequency is actually a synchronous excitation frequency, as shown in Fig. 4.

Figure 4. Open loop response function of the rotating system as a function of the excitation frequency and rotating speed.

The results present a high level of noise which are not caused by the measurement system but by the signal processing. Slight differences between the rotating speeds during run-up and run-down (or two run-ups) result in data scattering after applying Eq.(1). In order to reduce scattering in the PD controller synthesis (determination of the PD gains from the open loop frequency response), a curve fitting procedure is implemented.
3.1 Curve Fitting and Confidence Intervals

The real and imaginary parts of the open loop response function (OLRF) are plotted in Fig. 5. The data is smoothed by a moving average filter of the form:

\[
\begin{align*}
\tilde{d}(1) &= d(1) \\
\tilde{d}(2) &= \frac{1}{3}d(1) + d(2) + d(3) \\
\tilde{d}(k) &= \frac{1}{5} \sum_{i=k-2}^{k+2} d(i), \quad k = 3, \ldots, N - 2 \\
\tilde{d}(N - 1) &= \frac{1}{3}d(N - 2) + d(N - 1) + d(N) \\
\tilde{d}(N) &= d(N)
\end{align*}
\]  

where the algorithm assigns lower weight to outliers in the regression. The method assigns zero weight to data outside six mean absolute deviations of the samples in analysis. The 80% confidence interval is obtained by calculating the standard deviation of the error between the rough data and the smoothed averaged data (red line). In this case, the green lines are plotted for $1.281552\sigma$. The resultant curve fitting of the open loop response function is shown in Fig. 6.

![Figure 5. Curve fitting of the (a) real and (b) imaginary parts of the open loop response function identified experimentally (mean value and 80% confidence intervals).](image)

![Figure 6. Resultant fitting of the open loop response function from the real and imaginary part curve fitting (mean value and 80% confidence intervals).](image)

4. PD CONTROLLER SYNTHESIS

From the smoothed results, one can apply the D-decomposition technique to find the region of stable gains of the system (Buttini et al., 2011). The basic idea of the D-decomposition approach consists in dividing the controller parameter...
space into root invariant regions, i.e. regions with a fixed number of stable and unstable roots. This division is obtained calculating boundaries which map the imaginary axis in the complex plane into curves in the controller parameter space. Because each region delimited by these boundaries corresponds to a root invariant region, if a set of gains in a certain region leads the system to stability, all gains inside this region will do it as well.

By considering a PD controller of the form:

$$C(j\omega) = j\omega G_D + G_P$$

where $G_P$ and $G_D$ are the proportional and derivative gains of the PD controller, the boundaries of the root invariant regions are given by:

$$G_P = -\frac{1}{H_{OLFR}(0)} \quad \text{Real Root Boundary (RRB)}$$

$$G_D = -\frac{1}{H_{OLFR}(\infty)} \quad \text{Imaginary Root Boundary (IRB)}$$

$$\begin{cases} 
G_P = -\Re[H_{OLFR}(\omega)]/|H_{OLFR}(j\omega)|^2 \\
G_D = \Im[H_{OLFR}(\omega)]/\omega|H_{OLFR}(j\omega)|^2
\end{cases} \quad \text{Complex Root Boundary (CRB)}$$

In the case $H_{OLRF}(j\omega)$ is obtained experimentally, the infinite root boundary is seldom applicable because either information at infinity is not available or system response tends to zero ($G_D \to \infty$).

The application of Eqs.(4) and (6) in the smoothed open loop response functions resulted in the curves shown in Fig. 7. As one can see, the boundaries in the $G_D \times G_P$ plane form four distinct regions (A,B,C,D) where closed loop system roots are invariant. By performing a stability analysis in regions A and B, e.g. the Nyquist stability criterion, it is found that region A is a region of unstable gains, whereas region B is a region of stable gains (Fig. 8). In region C, there is not enough information to determine if it is a region of stable or unstable roots (frequency content of the open loop frequency response is no lower than 15 Hz). In region D, due to scattering of experimental data, the invariant root region is uncertain. Hence, by applying the D-decomposition method in the open loop response function, identified experimentally and smoothed in post processing, the region of stable PD gains is the shaded area in Fig. 8.

**Figure 7.** Complex and real root boundaries of the closed loop system in the PD gain domain.

**Figure 8.** Regions of stabilizing gains of the closed loop system.

### 4.1 Performance Criteria

In order to choose appropriated gains in the stable region of the $G_D \times G_P$ plane, one can adopt the following performance criteria:

$$S(j\omega) = \frac{1}{1 + (G_P + j\omega G_D)H_{OLRF}(j\omega)}$$

$$H_{CL}(j\omega) = H_{OLRF}(j\omega) S(j\omega) = \frac{H_{OLRF}(j\omega)}{1 + (G_P + j\omega G_D)H_{OLRF}(j\omega)}$$
\[ T(j\omega) = (G_P + j\omega G_D) H_{CL}(j\omega) = \frac{(G_P + j\omega G_D)H_{OLRF}(j\omega)}{1 + (G_P + j\omega G_D)H_{OLRF}(j\omega)} \]  

(9)

where the performance criterion (8) represents the closed-loop transfer function of the system, the criterion (7) represents the sensitivity function of the system, and the criterion (9) represents the complementary sensitivity function of the system (Li et al., 2011).

Varying the values of \( G_P \) and \( G_D \), one can calculate the expected response of the closed-loop system for each set of gains of the controller. To choose the appropriate gains, one can rely on the performance specifications below:

\[ M_H = \max_{\omega \in [\omega_1, \omega_2]} |H_{CL}(j\omega)| < \gamma_H \]  

(10)

\[ M_S = \max_{\omega \in [\omega_1, \omega_2]} |S(j\omega)| < \gamma_S \]  

(11)

\[ M_T = \max_{\omega \in [\omega_1, \omega_2]} |T(j\omega)| < \gamma_T \]  

(12)

According to Li et al. (2011), typical values of \( M_S \) are in the range of 1.2 to 2.0, and the minimum of this value (\( 1/M_S \)) represents a good evaluation of the controller robustness in face of uncertainties (the smaller \( M_S \) is, the closer the distance between the Nyquist curve and the critical point \(-1 + j0\) becomes). Typical values of \( M_T \) are in the range of 1.0 to 1.5, and this value is closely related to the peak overshoot at the plant output. The values of \( M_H \), as it was defined, depend on the system in study, and can vary from case to case. In the present study, the minimum value of \( M_H \) is, the better the controller becomes.

The set of gains to be analyzed in closed loop lies within the shaded area of Fig. 8. By calculating the performance criteria of Eqs.(7) to (9), using the mean value smoothed curve, one obtained the results shown in Fig. 9. As one can see, the maximum amplitude of the closed loop response function \( (M_H) \) decreases for positive derivative gains and negative proportional gains. Considering the range of \( 1.2 < M_S < 2.0 \) and \( 1.0 < M_T < 1.5 \), one can choose the sets of gains plotted in Fig. 9 for implementation. The value of such gains are listed in Table 1.

![Figure 9. Contour plots of the performance criteria in the area of stable gains.](image)

| Table 1. Chosen sets of gains in the stable region of the \( G_D \times G_P \) plane. |
|---|---|---|---|---|---|---|
| gains | 1 | 2 | 3 | 4 | 5 | 6 |
| \( G_P \) (V/mm) | 0 | 0.17 | 0.25 | 0.3 | 0.35 | 0.4 |
| \( G_D \) (V.s/mm) | 0 | 0 | -10 | -20 | -30 | -40 |

The performance of the closed-loop system with the chosen sets of gains can be analyzed by applying Eqs.(7) to (9) in the identified and smoothed frequency response functions. The results are shown in Fig. 10. As expected, the maximum amplitude of the closed-loop response function decreases as one decreases the proportional gain and increases the derivative gain (Fig. 10(a)). The uncertainty in the results is higher when the gain values are smaller. Considering the sensitivity function (Fig. 10(b)), one can see that the results remain within the 1.2-2.0 limits, but uncertainty increases as the gain values increase. In the case of the complementary sensitivity function (Fig. 10(c)), the results also remain within the desired limits (1.0-1.5). The uncertainty in this case is higher for smaller gain values.
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Figure 10. Values of the performance criteria for the chosen sets of gains: (a) maximum amplitude of closed loop response function; (b) sensitivity function; (c) complementary sensitivity functions.

5. CONCLUSION

In this work, one presents an experimental procedure for finding the open loop response of actively controlled rotating systems during non-stationary operation. The procedure successfully identifies the open loop response of the system by disregarding the effects of unbalance disturbance, which is an unknown parameter. For that, some precautions are needed to guarantee a constant and known phase between the excitation signal and the unbalance force, which is the correct synchronization between the excitation signal and system response.

As a result, one has an effective procedure for finding the open loop response of actively controlled rotating systems, which can be used to find the best gains for the control law and feedback. Besides, the procedure requires a single run-up and run-down operation (or two successive run-ups), after which the appropriated gains can be calculated and are already available for operation.

Considering the scattering in the identified closed loop response data, a synthesis of the controller gains is proposed based on smoothed curves and respective confidence intervals. A d-decomposition procedure is applied and the best gains of a PD controller are found, based on three performance criteria: the maximum amplitude of the closed loop response, the sensitivity function, and the complementary sensitivity function. Results show that uncertainty from the identification method does not strongly affect the performance of the closed loop system from the adopted gains.

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7. REFERENCES


8. RESPONSIBILITY NOTICE

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