



## DESIGN OF ROBUST CONTROLLERS FOR FLIGHT ENVELOPE PROTECTION FOR LIGHT SPORT AVIATION

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**Abstract.** *The continuous growth of ultralight aviation over the world has encouraged companies to develop advanced avionic systems for small aircraft taking into account low costs. Normally some systems such as autopilots must be carefully customized for each airplane since their functionality and efficiency are directly linked to the individual aircraft's dynamic. However ultralight airplanes have small geometrical and performance difference between each other, which allows a study of robustness to develop controllers that could be effective for a wide range of ultralight aircraft. The purpose of this paper is to present a methodology to design robust controllers considering parametric variations of some aerodynamics characteristics as well as flexible mode and disturbances caused by wind gust. Still the gains of the controllers are defined considering military requirements for automatic flight systems based on phase and gain margin and quality of temporal response. After defined the controller structure and their respective gains, a protection system is designed intending to protect high attitude angle to avoid stall.*

**Keywords:** *robust controller, general aviation, autopilot, ultralight aircraft.*

### 1. INTRODUCTION

The first step of this study is to choose an airplane which offers several information in respect to its aerodynamics, performance and geometrical data. All these information are essential to estimate stability and control derivatives required to model and simulate the airplane's behavior. Open-loop stability is guaranteed by analyzing the eigenvalues position of the longitudinal mode.

Before defining the controllers structure, it is necessary to determine the robustness bounds considering parametric variations, first flexible mode and disturbances. The parametric variations are estimated based on the airplane operation weight,  $CL_\alpha$  and  $CL_q$ . The first flexible mode refers only to the structural mode and disturbance model is given by wind shear described in the later sections.

The gains of the controller are determined when the function GK remains inside the robustness bounds and also presents a satisfactory temporal response for a step command. The design requirements are determined by military norms for automatic flight control system (MIL-DTL-9490E) which is based on minimum gain and phase margin and also on temporal response quality. Having all controllers working in a satisfactory way, we define the operation logic of the protection system and its respective control laws.

### 2. STABILITY DERIVATIVES ESTIMATION

Basically two software are used to estimate stability and control derivatives, the XFLR5 and the Advanced Aircraft Analysis (AAA). The XFLR5 is able to generate many aerodynamics curves and also to calculate moments of inertia and CG position that are later required by the AAA. First of all it is necessary the model construction in three dimensions and the distribution of all points of mass along the fuselage so that the program estimates the moments of inertia. The referred points of mass consists in mass of engine, instruments, pilot, fuel, baggage, wing and horizontal and vertical stabilizers. The figure 7a shows the modeled airplane.

Although the moments of inertia estimation do not seem to be highly reliable, three different cases were considered: airplane operation weight at his maximum, minimum and normal. As we going to see in the later sections this is one of the criteria used to define the robustness bounds to guarantee controllers efficiency against parametric errors caused by model uncertainties.



$$u = \delta_e \quad (10)$$

The linearization of these equations can be carried out as follows. In the steady-state straight, level, and constant speed flight condition,  $\dot{U} = \dot{V} = \dot{W} = \dot{p} = \dot{q} = \dot{r} = 0$ . Furthermore, there is no turning in any axis so that  $p = q = r = 0$ , and the wings will be level (FRANKLIN; POWELL; EMAMI-NAEINI, 2002). The linearization method used was the Taylor Series which gives a state-space model in the form

$$\begin{bmatrix} \dot{V}_T(t) \\ \vdots \\ \dot{H}(t) \end{bmatrix} = \begin{bmatrix} f_{V_T}(V_T, \alpha, \theta, q, H, \delta_e) \\ \vdots \\ f_H(V_T, \alpha, \theta, q, H, \delta_e) \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} \Delta \dot{V}_T(t) \\ \vdots \\ \Delta \dot{H}(t) \end{bmatrix} = \begin{bmatrix} \frac{\delta f_{V_T}}{\delta V_T} & \dots & \frac{\delta f_{V_T}}{\delta H} \\ \vdots & \ddots & \vdots \\ \frac{\delta f_H}{\delta V_T} & \dots & \frac{\delta f_H}{\delta H} \end{bmatrix} \begin{bmatrix} \Delta V_T(t) \\ \vdots \\ \Delta H(t) \end{bmatrix} + \begin{bmatrix} \frac{\delta f_{V_T}}{\delta \delta_e} \\ \vdots \\ \frac{\delta f_H}{\delta \delta_e} \end{bmatrix} [\Delta \delta_e] \quad (12)$$

The flight conditions to calculate the airplane's equilibrium point are defined considering  $V = 58,6$  m/s and  $H = 2286$  m as given by the Pilot Operation Handbook (VANS AIRCRAFT). For this operation point we have the follow eigenvalues for the longitudinal mode

Modes	Eigenvalues	Damping	Natural Frequency
Short Period	$-4.16 \pm 3.98$	0,723	5.75
Phugoid	$-3.47e-02 \pm 2.07e-01$	0,1.65	2.10e-01

Table 1: Eigenvalues of longitudinal dynamic.

The periods of these modes are separated by more than one order of magnitude, so they are easily identifiable as the short period and phugoid modes. The phugoid mode is very slightly damped ( $\zeta = 0,165$ ) but its period is so long that a pilot would have no difficulty in damping out a phugoid oscillation. The short-period mode is reasonably well damped ( $\zeta = 0,723$ ) in this particular flight condition, and the aircraft response to elevator commands would be acceptable to the pilot. The open-loop stability can also be proved by exciting the longitudinal short period with  $3^\circ$  in  $\alpha$ .

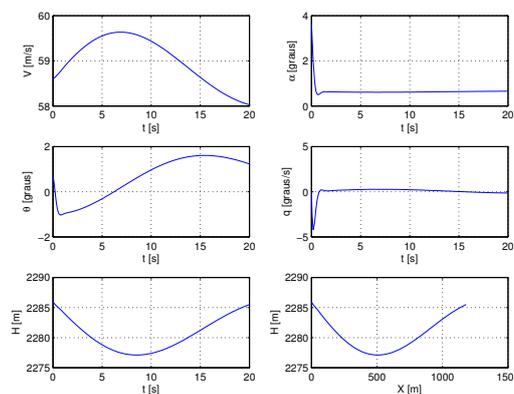


Figure 2: Longitudinal response after  $\alpha$  perturbation.

## 4. ROBUSTNESS CRITERIA

The robust control offers many techniques to guarantee the functionality of controllers against parametric uncertainties, disturbances and noise. The uncertainties are included in this project due to the possible existing differences between the theoretical and the real model. Disturbances are also considered since the controller must be robust when the airplane flies in turbulent atmosphere (SKOGESTAD; POSTLETHWAITE, 2005). The flexible mode is an extra consideration to evaluate its real influence in small aircraft. For this case noise are neglected.

### 4.1 Flexible Mode

Normally it is assumed a rigid-body aircraft model for purpose of controls design, and in so doing are neglecting flexible mode and vibrational modes at high frequencies. Thus although the design may guarantee closed-loop stability

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for the assumed mathematical model  $G(s)$ , stability is not assured for the actual plant  $G'(s)$  with flexible modes. The transfer function that represents the first flexible mode of an aircraft is given by (BLAKELOCK, 1965).

$$M(s) = \frac{-s(s + 2\zeta\omega_n)}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{13}$$

where

$\omega_n$ : natural frequency of the first flexible mode;  
 $\zeta$ : damping factor.

Since there is no information about aeroelasticity of small airplanes we assume  $\omega_n = 25rad/s$  and  $\zeta = 0,025$  to represent the structural mode. To guarantee stability robustness in the face of plant parameter uncertainties, one may proceed as follows (STEVEN; LEWIS, 2003).

$$\bar{\sigma}GK < \frac{1}{M(j\omega)} \tag{14}$$

where

$G$ : transfer function of the nominal plant;  
 $K$ : transfer function of the controller;

### 4.2 Parametric Variations

We may also consider that the aircraft is nonlinear, but for controller we use linearized models obtained at some operating point. In practice, it is important for the control gains to stabilize the aircraft at all points near the design operating point for this gain scheduling procedure to be effective. In passing from operating point to operating point, the parameters of the state-variable mode vary (STEVEN; LEWIS, 2003).

The parametric variations were estimated varying the airplane's operation weight (maximun and minimun) and the stability derivatives in  $\pm 20\%CL_\alpha$ ,  $\pm 20\%CL_q$ . The bounds for each case are obtained by the following equation

$$M = (G' - G)G^{-1} \tag{15}$$

where:

$G$ : transfer function of the nominal plant;  
 $G'$ : transfer function of the plant considering parametric variation.

Using the equation 14, we may design controllers that guarantee robust stability despite plant parameter variations (STEVEN; LEWIS, 2003). All estimated bounds were compared and the most restrictive/conservative was picked up to represent the limit for high frequencies as shown in the figures bellow.

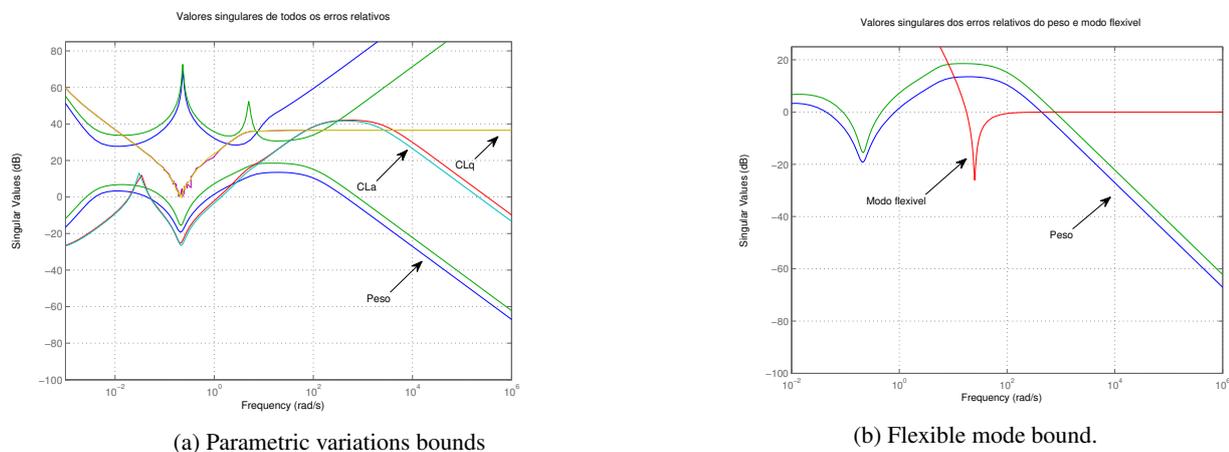


Figure 3: Robustness bounds.

Since the function GK decays -40 dB/dec at high frequencies we will use the flexible mode bound to guarantee robustness even against parametric variations.

### 4.3 Disturbance

According to MIL-F-8785C, the longitudinal ( $H_u$ ), and vertical ( $H_w$ ) wind gust noise has a spectral density given in Dryden form as

$$H_u(s) = \sigma_u \sqrt{\frac{2L_u}{\pi V}} \frac{1}{1 + \frac{L_u}{V}s} \quad (16)$$

$$H_w(s) = \sigma_v \sqrt{\frac{L_v}{\pi V}} \frac{1 + \frac{\sqrt{3}L_v}{V}s}{(1 + \frac{L_v}{V}s)^2} \quad (17)$$

with  $V$  the airspeed in m/s,  $\sigma$  the turbulence intensity in m/s, and  $L$  the turbulence scale length. It was assumed  $V = 58,6\text{m/s}$ ,  $\sigma = 2,13\text{m/s}$  and  $L = 533\text{m}$  representing light turbulence for low altitudes. These transfer functions are converted in state-space and added in the nominal plant model creating a new augmented system in the form

$$\begin{bmatrix} \dot{x} \\ \dot{x}_w \end{bmatrix} = \begin{bmatrix} A & GC_w \\ 0 & A_w \end{bmatrix} \begin{bmatrix} x \\ x_w \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} GD_w \\ B_w \end{bmatrix} n \quad (18)$$

$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_w \end{bmatrix} + v \quad (19)$$

Using the above state-space matrix we define the barrier that represents the influence of the vertical wind component in the variable  $\theta$  shown in the figure 4a.

## 5. CONTROLLERS DESIGN

The present protection system requires only one controller, the longitudinal attitude controller ( $\theta$ ). The gains are obtained by iterative method using the Matlab function *looptune* which optimizes tunable parameters to minimize the norm  $H_\infty$  (MATLAB, 2012). As requirements one can stipulates controller structure, crossover frequency, time response, target function, steady-state error, etc. The function than returns tuned parameters and a parameter called *gam* which indicates the degree of success of all restrictions previously determined.

Using the iterative method varying crossover frequency and time response we defined the gains when the robustness and temporal response requirements are reached. The suggested  $\theta$  controller is the Proportional-Integral type with gains  $K_p = -0.2720$ ;  $K_i = -0.1600$ .

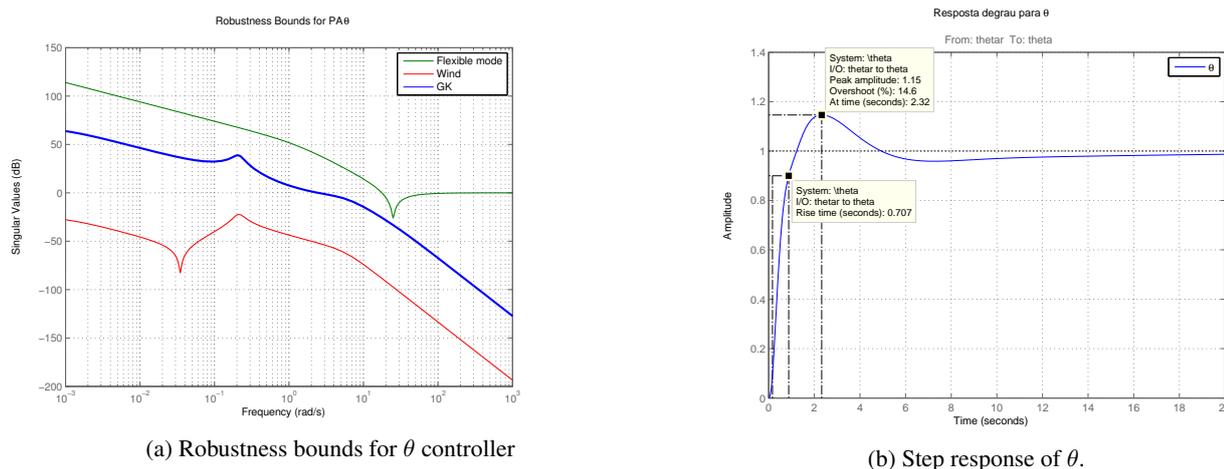


Figure 4: Controller design.

The give requirements were  $t_s = 0,7$  s, zero steady-state error and  $\omega_c = 1$  rad/s, resulting an overshoot of 14,6% and time response of 0,707 s.

## 6. FLIGHT PROTECTION SYSTEM

The current protection system works using  $\theta$  controller to avoid high attitude angles during flight cruise with constant thrust, normally 75%. Such a system would be important if we imagine an airplane flying into unintended IMC (instrument meteorological conditions) conducted by an inexperienced pilot. Intending to get out of that situation the pilot can give too much elevator command and lead the airplane to an unsafe flight condition.

It is important to understand that every airplane has a different attitude angle to be protected. However it was stipulated a  $\theta$  maximum of  $20^\circ$  since this attitude avoids stall speeds for the current airplane flying with cruise thrust.

Basically the protection system works as a  $\theta$  limiter. The controllers do not closed the feedback loop in normal flight condition, but the pilot does. The protection system switches on only when  $\theta$  achieves its maximum value. In this situation the autopilot maintains the airplane at its maximum attitude angle, in case the pilot is trying to protect the airplane against some obstacle.

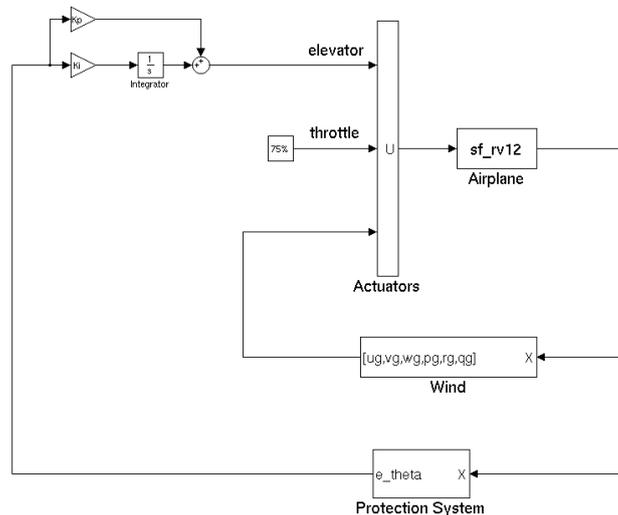
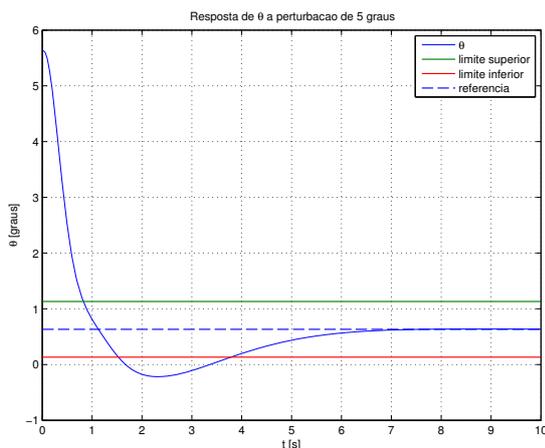


Figure 5: Block diagram of the system.

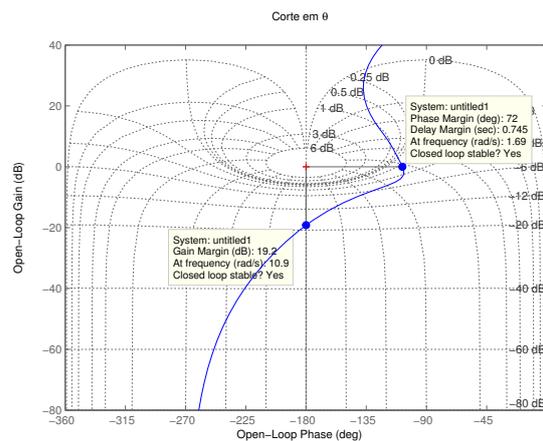
7. RESULTS

The military norm MIL-DTL-9490E says that attitudes shall be maintained in smooth air with static accuracy of  $\pm 0,5^\circ$  in pitch attitude (with wings level). Accuracy requirements shall be achieved within 5 seconds of mode engagement for a  $5^\circ$  attitude disturbance. The figure 6a shows this requirement being satisfied.

For stability margins the required gain and phase margins about nominal must be  $\pm 6dB$  and  $\pm 45^\circ$  respectively. The figure 6b shows the obtained gain margin of 19,2 dB and phase margin of  $\pm 72^\circ$ .



(a) Performance requirement.



(b) Stability requirement.

Figure 6: Automatic control system requirements.

To illustrate the situation explained in the earlier section we consider the airplane flying in steady-state. At 5 seconds the pilot applies approximately  $-2^\circ$  elevator command trying to fly back into visual conditions. This command results in a pitch-up attitude and would lead the airplane to stall. In the figure 7a the red line represents the command that the pilot was intending to give and the blue line represents the autopilot acting. At approximately 10 seconds the protection system switches on, when  $\theta$  achieves  $20^\circ$  (figure 7b), and maintain the airplane with  $\theta$  maximum.

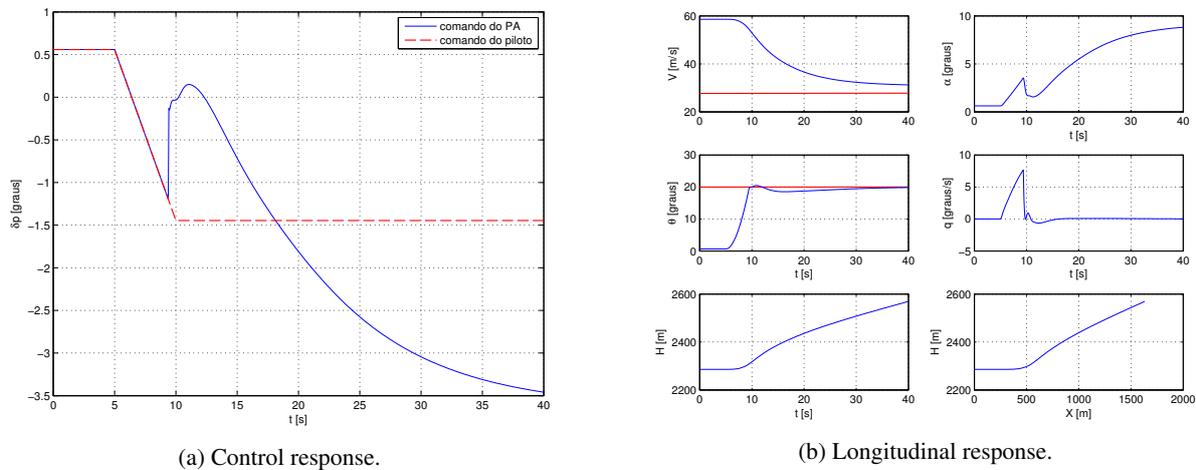


Figure 7: Aircraft response.

As explained before, the speed stall (26,8 m/s) is not achieved and the protection system maintains the maximum attitude until the pilot decides to switch off the autopilot.

## 8. CONCLUSIONS

The idea to start a deeper research for ultralight aircraft is fundamental to improve safety inside all segments of aviation. Indeed the experimental aviation does not have much information about the airplanes characteristics to build a mathematical model with a high level of reliability. That is the reason why the design of the controllers was made considering parametric variations. Even varying CG, moments of inertia, points of mass and some aerodynamics coefficients, there was no big difference on the final results. None of the six models approximated to the limit of instability or behaved themselves different comparing to the nominal plant. This is an important observation to be made because the focus of this study is to design controllers robust enough to be efficient in other airplane models of the same category.

The stability bound given by the first flexible mode seemed not to be appropriated for small airplanes, since aeroelasticity is more indicated for airplanes with considerable wingspan. This bound restricted the controllers performance and consequently the airplane's response and must be reconsidered or studied carefully. Although the autopilots responses were satisfactory there is still the chance to improve them even more.

The control laws created to switch the autopilot appeared to be quite efficient. The main focus to develop the control logic was based on the simplicity of flying small aircraft. A system using angle of attack could substitute  $\theta$  but the most airplanes of this category do not provide this sort of sensor.

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