EVALUATION OF GENETIC ALGORITHM AND DIFFERENTIAL EVOLUTION APPLIED TO PARAMETRIC OPTIMIZATION OF A COMPONENT USED IN REACTIVE ACOUSTIC FILTERS

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Abstract. This work evaluate the velocity of convergence of the solutions obtained with the parametric optimization of the cylindrical surface tube used in acoustic filters with two methods: Genetic Algorithms (GA) abs Differential Evolution (DE). In this application, the Finite Element Method (FEM) plus the Improved Four-Pole Parameter Method are employed for calculating the value of the objective function. The Objective function is defined from the sound transmission loss (TL), which is the one of the parameters used for evaluating the acoustic filter efficiency in frequency domain. Some results illustrated in this work indicate that the versions implemented for GA e DE are very appealing methods for acoustic problems for situations where the number of design variables is relatively large.

Keywords: acoustic filter, GA, DE, optimization.

1. INTRODUCTION

The Genetic Algorithms (GA) is based on Darwin’s concept of the selection and natural evolution applied to mathematical programming. It was originally proposed by Holland in 1975 and it has been widely used in the last years for obtaining optimum solutions for many engineering efforts. The optimization performance through the GA involves the task of making the population with the best fitness evolve and survive through many generations. During optimization, a set of solutions is chosen so it “evolves” towards the optimum solution for the problem. The GA has the ability of searching for a global optimum in discontinuous and multimodal spaces without requiring a proper initial point (Goldberg, 1989). GA efficiency has been widely verified in many fields, and besides that, it has been greatly used for acoustic problems (Chang et al., 2005; Balauc et al., 2008; and Chiu, 2010).

The Differential Evolution (DE) is a direct search method that was developed for minimizing the problems represented by non-linear and non-differentiable continuous spatial functions. This method was created in 1995 by Storm and Price when attempting to solve a polynomial fitting problem by Chebychev associated with electronic filter designs. In addition to that, the method is fairly simple and presents fast convergence features for problems that involve a small number of particles (individuals) and a large number of design variables. The main idea of the differential evolution is to create vectors composed by the design variables by adding the weighted difference between two randomly chosen vectors of the population and a third particle. This method was widely tested by Storm and Price in 1997 when searching for a minimal global in the study of the functions: De Jong Functions, Corana’s parabola, Griewangk’s function, Zimmermann’s problem, Hyper-Ellipsoid function, Katsuura’s Function, Rastrigin’s Function and Ackley’s function, being very efficient when compared to the results obtained with other search methods: Adaptive Simulated Annealing (ASA), the Annealed Nelder and Mead approach (ANM), the Breeder Genetic Algorithm (BGA), the EASY Evolution Strategy and the method of Stochastic Differential Equations (SDE). Since then, the DE has been used in the solution of problems in several engineering fields: identification of hysteretic systems (Kyprianonou et al. 2001; non-linear modeling problems of laser diodes under microwave intensity (Akdagli and Yuksel, 2006); flow shop scheduling (Onwubolu and Davendra, 2006) and hydro generation scheduling (Yuan et al., 2008).

In this work, the parametric optimization of the surface form of a cylindrical tube used in acoustic filters is carried out using two direct search methods: Genetic Algorithms (GA) and Differential Evolution (DE). The tube acoustic efficiency in relation to the Transmission loss (TL) is evaluated in the optimizations. The transmission loss calculation will be carried out throughout the finite element method in the frequency domain, along with the Improved Four-Pole Parameter Method (Kim and Soedel, 1989-90).

2. THEORY

The GA allows to reach the maximum of a function $f(x(i,j))$ subject to the following design constraints:

$$x_L(i) \leq x_i \leq x_U(i) \Rightarrow i = l, 2, ... , m$$

(1)
where $x_i$ is the particles or individuals set of design variables $x(i,j)$ which lower limit is $x_L(i)$ and the upper limit is $x_U(i)$ and $m$ is the number of design variables to be defined.

The GA represents the design variables as sets of binary numbers of $nb$ bits that are called chromosomes. This way, the viable interval for each variable $x_i$ is divided into $N_i$ intervals:

$$ N = 2^{nb} - 1 $$

So, each variable $x_i$ can be represented by any discreet representation, for e.g., through a binary number: 011011. This number can be decoded as:

$$ x_i = x_L(i) + (0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0) \cdot s_i = x_L(i) + 27s_i $$

where $s_i$ is the variable interval $x_i$ defined by:

$$ s_i = (x_U(i) - x_L(i))/27. $$

This procedure defines the codification and decodification processes of the GA variables. The first step in the development of the GA is the creation of an initial population. Each particle or individual $i$ of the population is a set with $m \times nb$ bits. The set of variables, $x_i$, in the binary form, creates the genetic code for each individual. An initial population with a $z$ size is created by a random process. The second step is to decode the genetic code of each individual using the Eqn. (3) and verifying the value correspondent to its respective fitness, i.e., of its objective functions, given by $f_1, f_2, \ldots, f_z$. The most fit individuals, with higher $f$ values are considered most “optimal”. Their fitness value must reflect the design and the restrictions imposed on the problem being studied, Tab. 1.

### Table 1. Set of individuals and their genetic codes.

<table>
<thead>
<tr>
<th>$i$</th>
<th>Individual</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$f_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>101101</td>
<td>010001</td>
<td>...</td>
<td>101101</td>
<td>$f_1$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>101010</td>
<td>100001</td>
<td>...</td>
<td>111001</td>
<td>$f_2$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>110011</td>
<td>001101</td>
<td>...</td>
<td>111011</td>
<td>$f_3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
<td></td>
</tr>
<tr>
<td>$z$</td>
<td>101001</td>
<td>11110</td>
<td>...</td>
<td>101001</td>
<td>$f_z$</td>
<td></td>
</tr>
</tbody>
</table>

The third step includes the selection and crossover phases. Here, the weaker individuals are replaced by the stronger ones. The selection enables the best individuals to survive and serve as parents for the next project generation, through the fitness calculation. In this phase, the individuals are shuffled and have their order changed. After shuffling, two individuals are chosen. These are called parents and the one with the best fitness will originate from a child and will have some of its chromosomes randomly changed, Tab. 2. This procedure is carried out $z$ times, generating the number of $z$ children. The crossover transference makes parts of the parents’ chromosomes to create the next generation of the project by combining features in such a way as to create better individuals in average, but not always.

### Table 2. Selection and crossover.

<table>
<thead>
<tr>
<th>$i$</th>
<th>Individual</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$f_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>101101</td>
<td>010001</td>
<td>...</td>
<td>101101</td>
<td>$f_1$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>110011</td>
<td>001101</td>
<td>...</td>
<td>111011</td>
<td>$f_3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
<td></td>
</tr>
<tr>
<td>Child 1</td>
<td>1 1 0 0 1</td>
<td>1 0 0 1 0 1</td>
<td>...</td>
<td>1 0 1 0 1 0</td>
<td>$f_3$</td>
<td></td>
</tr>
</tbody>
</table>

The fourth step may switch a chromosome bit of an individual to its opposite value (e.g.; 0 to 1). This step is called mutation and aims to introduce a new beneficial feature that does not exist in the current population. The mutation makes the fitness value of a child to suffer a change. This procedure is performed to avoid local optimum in search spaces induced by the random fitness change of the individual. The probability of a member bit to suffer mutation must vary between 0.005 and 0.1, which demonstrates that this will rarely occur in the nature (Carrol, 1996). If the feature
introduced is not beneficial to the individual that suffered such mutation, probably it will not survive a future transformation step.

The fifth and last step verifies if the genetic code of the individual with the best fitness was replicated in any of the created children. If this does not happen, a child is randomly removed and the individual with the best fitness is added. This step improves the optimization accuracy. This operation is called elitist reproduction. In order to conclude this generation, the highest \( f \) value and its respective set of chromosomes \( x \) are stored. If the required number of generations in this project is reached, the evolution will be concluded; otherwise, a new decodification step will begin, and continue successively. All the described process transforms an initial randomly chosen population into a population that is more adapted to its environment, making them more optimal.

2.2 Differential Evolution (DE)

The Differential Evolution method consists basically of three steps: (i) create a population with \( z \) individuals in a multidimensional space, randomly distributed over all the objective function domain, and consequently, quantify its respective fitness, \( f \); (ii) substitute the current population for one with better fitness; (iii) repeat this substitution until a population with best fitness, that meets the pre-established convergence criteria, is found.

The vector for the initial population \( x(i,j)_{g} \) with \( j \) particles is randomly created and must be contained inside the solution space for the problem.

\[
x(i,j)_{g} \text{ with } i = 1, 2, ... m; \quad j = 1, 2, ..., z \quad e \quad g = 1, 2, ..., G
\]

where \( i \) is the number of design variables and \( g \) is the number of generations to be evaluated.

The DE creates a new parameters vector by adding the weighted difference between two particles of the population into a third. This operation is called mutation. The individual that comes from the mutation is mixed with the population with best fitness, that meets the pre-established convergence criteria, is found.

The vector for the initial population \( x(i,j)_{g} \) with \( j \) particles is randomly created and must be contained inside the solution space for the problem.

\[
x(i,j)_{g} = x_{i,j}^{g} + F(x_{r2}^{g} - x_{r3}^{g})
\]

In Eqn. (6), \( x_{i,j}^{g} \) is the target vector. The indexes \( r1, r2, r3 \in \{1, 2, ..., z\} \) are integral and different among themselves. The integral ones \( r1, r2, r3 \) randomly chosen are different from the index \( j \) in such a way that \( z \) must be larger or equal to 4 in order to allow for this condition. Factor \( F \) that controls the amplification of the differential variation \( (x_{r2}^{g} - x_{r3}^{g}) \) is real and constant \( \in [0, 2] \), (Storn and Price, 1995).

In order to increase the diversity of the particles, the crossover is introduced. For this purpose, the trial vector is:

\[
\begin{align*}
u(i,j)_{g+1} &= \{ u(1,j)_{g+1}, u(2,j)_{g+1}, ..., u(m,j)_{g+1} \}
\end{align*}
\]

where

\[
\begin{align*}
u(i,j)_{g+1} &= \{ v(i,j)_{g+1} \quad \Rightarrow \quad \text{if} \quad (\text{rand}1(i) \leq \text{CR}) \quad \text{or} \quad i = \text{rand}2(j) \\
x(i,j)_{g} &= \{ u(i,j)_{g} \quad \Rightarrow \quad \text{if} \quad (\text{rand}1(i) > \text{CR}) \quad \text{and} \quad i \neq \text{rand}2(j) \}
\end{align*}
\]

In Eqn. (8) \( \text{rand1}(i) \) is a real number chosen randomly \( \in [0, 1] \). \( CR \) is a crossover constant \( \in [0, 1] \). \( \text{rand2} \) is a randomly chosen index \( \in [1, 2, ..., m] \), which guarantees that \( u(i,j)_{g+1} \) receives at least a parameter of \( v(i,j)_{g+1} \). The selection, the last step of the process, decides if the trial vector \( u(i,j)_{g+1} \) should or not become a member of the generation \( g+1 \). In this step \( u(i,j)_{g+1} \) is compared to the target vector \( x(i,j)_{g} \) using the best fitness criterion, i.e.:

\[
x(i,j)_{g+1} = \begin{cases} u(i,j)_{g+1} & \text{if} \quad f(u(i,j)_{g+1}) > f(x(i,j)_{g}) \\ x(i,j)_{g} & \text{if} \quad f(u(i,j)_{g+1}) \leq f(x(i,j)_{g}) \end{cases}
\]
3. NUMERICAL ANALYSES – OPTIMIZATION PROBLEM

The main objective of this paper is evaluating the quality of the solutions obtained with the parametric optimization of the surface of a cylindrical straight tube used in acoustic filters. The evaluated optimization methods are: GA and DE.

The straight cylindrical tube is 300 mm long and has a radius of 17.3 mm. The optimization region is defined as the central part of the tube with 100 mm in length. In the optimization region, for each particle, the boundary radius, \( r_i \) in relation to the center of the tube, is defined by the subtraction of the design variables \( x_i \) from the tube radius value, i.e.:

\[
r_i = 17.3 - x_i
\]  

where the design variable restrictions \( x_i \) of the particle \( x(i, j) \) are given by the Eqn. (1) and for this study they are: \( 0 \leq x_i \leq 5 \) mm with \( i = 1, 2, \ldots, m \) design variables.

The objective function value is calculated for each change in the tube surface boundary. All the results shown in this article are obtained by employing the objective functions written as a function of the average TL in the frequency range between 1000 and 3500 Hz. The average TL has already been used by Lima, et al., (2011) for the optimization of acoustic silencers and its mathematical expression is:

\[
f_i(x(i, j)) = \frac{1}{\omega_f - \omega_i} \int_{\omega_i}^{\omega_f} (TL(\omega_i) - TL(\omega)) d\omega
\]  

where \( \omega_i = 1000 \) hz – the beginning of the frequency interval in study and \( \omega_f = 3500 \) hz – the end of the frequency interval in study.

The value for the objective function defined in Eqn. (11) is determined using the Finite Element Method (FEM) in addition to the Improved Four-Pole Parameters Method. The numerical analyses comprised in this article use the air at 20°C as a work fluid, the specific mass being 1.21 kg/m³ and sound speed \( c \) equal to 343 m/s.

In the design region, for the highest excitation frequency, \( f \) is equal to 3500 hz, the finite element mesh has a characteristic length of 1.087 mm. This provides for a sufficient number of elements by wavelength (\( \lambda \)) in order to reduce amplitude and phase errors in the evaluation of the objective function using the FEM (Ihlenbur et al., 1997). In this article, the wavelength is defined by:

\[
\lambda = \frac{c}{f} = 0.098 \text{ m}
\]

With \( \lambda = 0.098 \text{ m} \) we assure that all the numerical analyses will be carried out with at least 90 elements by wavelength. The finite element analyses were carried out by employing the linear isoparametric element with axisymmetric formulation. The design region boundary is defined by the known Hermite polynomials for each particle. This approximation aims to locally and smoothly approximate the boundary (continuity \( C^1 \)) and to reduce the number of design variables. This way, we only have the need to obtain 22 points equally distant from the optimization algorithm, i.e., \( m = 22 \). The points between two design variables are obtained by using the Hermite polynomials as shown in Fig. 1.

![Figure 1. Definition of the optimized tube profile and the design variables](image-url)
This work will be conducted in three 3 steps:
I. Definition of the DE parameters;
II. Study of the convergence speed for the two methods;
III. Analysis of the tube acoustic efficiency with the obtainment of the optimized profile.

3.1 Step I – Determination of the DE parameters

GA effectiveness in optimization analyses of acoustic filters have been widely studied by several authors (Chang et al., 2005; Chiu et al., 2008, Chiu, 2010 and Airaksinen et. al., 2011). However, there are not many works describing the efficiency of the DE. The following parameters were used in the analyses with the GA: crossover probability = 50% and mutation probability = 2%. The DE is a relatively simple method that represents only 2 parameters: CR and F. In order to determine the best set of parameters for the problem in study, 16 analyses are carried out for the DE, Tab. 3. Several parameter configurations are shown in these analyses. Besides that, the number of evaluations necessary to reach the maximum value for the objective function is shown in the tables. As a stopping criterion, 40,000 evaluations of the objective function were done for each analysis. In Tab. 3, it can be easily noticed, with the exception of the analysis 4, that the objective function reaches the same value for all the analyses carried out, i.e., 3.73 dB. However, for the next step of this work, the analysis parameters are chosen where the convergence of the objective function was faster: CR =5 and F = 0.5 (Analysis 3).

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Population</th>
<th>CR</th>
<th>F</th>
<th>Number of evaluations</th>
<th>Maximum objective function (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>0.9</td>
<td>0.5</td>
<td>19680</td>
<td>3.73</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>0.7</td>
<td>0.5</td>
<td>19080</td>
<td>3.73</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>0.5</td>
<td>0.5</td>
<td>18400</td>
<td>3.73</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>0.3</td>
<td>0.5</td>
<td>35040</td>
<td>3.72</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>0.9</td>
<td>0.7</td>
<td>21160</td>
<td>3.73</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>0.7</td>
<td>0.7</td>
<td>20960</td>
<td>3.73</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>0.5</td>
<td>0.7</td>
<td>25200</td>
<td>3.73</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
<td>0.3</td>
<td>0.7</td>
<td>23600</td>
<td>3.73</td>
</tr>
<tr>
<td>9</td>
<td>20</td>
<td>0.9</td>
<td>0.9</td>
<td>26300</td>
<td>3.73</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>0.7</td>
<td>0.9</td>
<td>23160</td>
<td>3.73</td>
</tr>
<tr>
<td>11</td>
<td>20</td>
<td>0.5</td>
<td>0.9</td>
<td>20360</td>
<td>3.73</td>
</tr>
<tr>
<td>12</td>
<td>20</td>
<td>0.3</td>
<td>0.9</td>
<td>21520</td>
<td>3.73</td>
</tr>
<tr>
<td>13</td>
<td>20</td>
<td>0.9</td>
<td>1.3</td>
<td>39840</td>
<td>3.73</td>
</tr>
<tr>
<td>14</td>
<td>20</td>
<td>0.7</td>
<td>1.3</td>
<td>29400</td>
<td>3.73</td>
</tr>
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<td>20</td>
<td>0.5</td>
<td>1.3</td>
<td>28000</td>
<td>3.73</td>
</tr>
<tr>
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<td>20</td>
<td>0.3</td>
<td>1.3</td>
<td>37840</td>
<td>3.73</td>
</tr>
</tbody>
</table>

3.2 Step II - Study of the convergence speed of the GA and DE

To verify which is the most efficient optimization method in relation to the convergence speed, four evaluations per method are carried out, i.e., with the populations of 10, 20 and 40 particles. The parameters defined in Sec. 3.1 are used in this step. The results for the convergence behavior of the optimizations are shown in Figs. 2 and 3. Figure 4 presents a comparative amongst the best results for the two methods. According to Figs. 2 and 3, it can be clearly noticed that the increase in the number of particles has only increased the convergence speed for the optimization with the GA. In Fig. 4, it can be noticed that the convergence is obtained faster through the GA after 5,900 evaluations of the objective function (see continuous vertical line). However, the maximum objective function value is only reached after 29,600 (GA) and 10,820 (DE). The maximum values reached for the objective function through the two methods evaluated are: 3.70 (GA) and 3.73 (DE), Tab. 4. Table 5 shows the design variables values for the two methods when they all reached 5,900 evaluations and when their objective functions reached the maximum values. In addition, GA and the DE results are similar. In this case, the surface that was found in the tube optimization is shown in Fig. 5. The GA and DE presented results without significant differences when their objective functions reached the maximum values.
Figure 2. GA convergence behavior.

Figure 3. DE convergence behavior.

Figure 4. The best results of the convergence behavior between the GA 40 and DE 10.
Table 4. Values for the objective function obtained through the optimization methods.

<table>
<thead>
<tr>
<th>Evaluation number</th>
<th>Optimization method</th>
<th>Number of particles</th>
<th>Objective function value (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,900</td>
<td>DE</td>
<td>10</td>
<td>3.69</td>
</tr>
<tr>
<td></td>
<td>GA</td>
<td>40</td>
<td>3.68</td>
</tr>
<tr>
<td>10,820</td>
<td>DE</td>
<td>10</td>
<td>3.73</td>
</tr>
<tr>
<td>29,600</td>
<td>GA</td>
<td>40</td>
<td>3.70</td>
</tr>
</tbody>
</table>

Table 6. Values of the design variables obtained through the optimization methods.

<table>
<thead>
<tr>
<th>Evaluation number</th>
<th>Optimization method</th>
<th>Values of design variables in (mm).</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,900</td>
<td>DE</td>
<td>4.894 4.990 4.915 4.979 4.980 4.994 4.987 0.012 0.023 0.000 0.009</td>
</tr>
<tr>
<td></td>
<td>GA</td>
<td>0.006 0.008 0.001 0.001 0.491 0.004 0.084 0.006 0.063 0.009 0.024</td>
</tr>
<tr>
<td>10,820</td>
<td>DE</td>
<td>4.971 4.990 4.990 5.000 4.990 4.976 4.980 0.070 0.003 0.000 0.000</td>
</tr>
<tr>
<td></td>
<td>GA</td>
<td>0.044 0.010 0.034 0.010 0.061 0.056 0.048 0.009 0.000 0.000 0.000</td>
</tr>
<tr>
<td>29,600</td>
<td>GA</td>
<td>5.000 4.999 5.000 4.990 4.990 4.999 4.999 0.029 0.090 0.000 0.000</td>
</tr>
<tr>
<td></td>
<td>GA</td>
<td>0.005 0.000 0.015 0.005 0.496 0.495 0.495 0.495 0.495 0.495 0.495</td>
</tr>
</tbody>
</table>

Figure 7. Profile of the optimized tube after 5,900 evaluations of the objective function for the GA and DE.

### 3.3 Analysis of the tube acoustic efficiency with the obtainment of the optimized profile

The acoustic efficiency of the optimized tube is measured through the determination of the sound transmission loss in the frequency domain, $TL(\omega)$. Fig. 8 shows the $TL(\omega)$ results for the tube of Fig. 7 in a black line. For just one optimized tube, the average $TL$ reaches the average value of approximately 6 dB for a frequency range between 2100 and 3400 Hz. In order to show the optimized tube efficiency, the result of the two optimized tubes assembled in series is showed red line in the Fig. 8 with red line. The $TL(\omega)$ results reach higher values in the same frequency range for this configuration.

Figure 8. $TL(\omega)$ behavior for one and two optimized tubes.
4. CONCLUSIONS
The main observations and conclusions obtained in this work are:
a) The optimization with the GA and the DE present are fast.
b) The optimization with the DE has proven to be more efficient for this type of problem;
c) The increase in the number of particles in a population is not always beneficial to the convergence of the solution, see Figs. 2, 3 and 4;
d) The use of tubes with an optimized profile has proven to be effective for attenuating the noise in high frequencies for acoustic filter.

5. REFERENCES


6. RESPONSIBILITY NOTICE
The authors are the only responsible for the printed material included in this paper.