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THE INFLUENCE OF SUBGRID MODEL IN TRANSITIONAL SCENARIO

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Abstract. *The turbulence phenomena has been widely investigated by the use of the Large-Eddy Simulation (LES) methodology. However, its success is associated with the appropriate choice of subgrid scale model and the initial conditions for the problem. An acceptable initial condition can be generated through the investigation of the transition phenomena. The propagation and interactions of Tollmien-Schlichting waves in a boundary layer flow can be a transitional way from laminar to turbulent state. Thus, this paper aims the study of the influence of Smagorinsky subgrid scale model in the evolution of Tollmien-Schlichting waves. For this the vorticity-velocity formulation is used and it is assumed periodicity in the spanwise direction. High order compact finite difference schemes are applied to discretize the spatial derivatives in the streamwise and wall normal directions. A spectral method is used to discretize the spatial derivative in the spanwise direction and the time derivative is integrated by a classical fourth order Runge-Kutta method. The Poisson equation is solved by a multigrid technique. The code parallelization is provided by the Message Passing Interface library.*

Keywords: *Large-Eddy Simulation, Smagorinsky model, laminar-turbulent transition, Tollmien-Schlichting waves*

1. INTRODUCTION

In recent years, many resources have been invested in research related to the understanding and control of turbulent flows. Thus many efforts have been focused on the development of numerical tools for the simulation of turbulent flows, which increase the demand for mathematical models and simulation methods that best estimate such effects. The main methodologies used to study numerically turbulent flows are DNS (Direct Numerical Simulation), RANS (Reynolds Averaged Navier-Stokes) and LES (Large Eddy Simulation).

DNS captures all scales of the turbulent flow then it requires a extremely refined grid and time step. In the most industrial applications, where Reynolds numbers are very high, this methodology exceeds the capacity of the most powerful computers currently available (Lesieur and Métais, 1996).

In RANS the dependent variables are decomposed into mean and fluctuating parts by applying time average in Navier-Stokes equations. As results, only the averaged motion is computed and the effect of fluctuations is modeled. The great advantage of this methodology is the use of coarser grids which implies in lower computational cost. On the other hand, this model does not capture detailed information of the flow, since these are lost by applying the time average.

In the LES approach, the turbulent scales are split into two groups: the group of large scales and small scales. The large scales are resolved directly, while small scales are modeled using so-called subgrid scale models (Wilcox, 1994; Lesieur and Métais, 1996; Rodi *et al.*, 1997). According to Lesieur and Métais (1996), usually the most relevant information is contained in the larger scales of flow which explains the idea behind this methodology. Due to the separation process, LES has become an alternative with a smaller computational effort than DNS simulations, then it can be applied to solve flows with higher Reynolds numbers justifying its growth use in recent years.

The success of simulations using LES is associated with the appropriate choice of subgrid scale model and the initial conditions for the problem. An acceptable initial condition can be generated through the investigation of the laminar-turbulent transition phenomena. The propagation and interactions of Tollmien-Schlichting (TS) waves in a boundary layer flow can be a transitional way from laminar to turbulent state (Schlichting, 1979). These waves arise when some disturbance interacts with flow, ie. wall roughness, sound waves and vibrations. Depending on the Reynolds number these TS waves can have an exponential growth or decay in the downstream direction. If they grow, depending on the initial amplitude, they can amplify until reach large enough amplitudes that nonlinearities take over and the flow can go from the laminar to turbulent state. In experimental studies of Tollmien-Schlichting waves the disturbances are introduced in the flow through a vibrating ribbon located shortly downstream of the leading edge (Medeiros and Gaster, 1999). In numerical studies this disturbances may be introduced at the wall, through a periodic blowing and suction strip. According to Fasel *et al.* (1990) the second method has proved to be a very efficient method to introduce this kind of disturbance. In this paper the second method was adopted.

In this sense, the objective of this work is to investigate the influence of Smagorinsky subgrid scale model in the evolution of TS waves in a laminar boundary layer flow. Results of amplitude development and growth rate of Tollmien-Schlichting waves in the downstream direction with a DNS code and LES code with different values of Smagorinsky

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constant are compared with Linear Stability Theory (LST) (Mendonça, 2000).

2. LARGE-EDDY SIMULATION

The earliest works involving LES were motivated by meteorological applications. In 1963, the meteorologist Smagorinsky using the ideas of Reynolds decomposition proposed this new simulation methodology and performed the 3D first attempts to simulate the climate. Other works such as Lilly (1967) and Deardorff (1974) also had great importance for the dissemination of this new simulation technique. In recent years, along with the improvement of computers and simulation techniques, LES has became one of the most used and promising simulation methodologies for the solution of turbulent flows.

The fundamental principle of this methodology is the separation of large and small scales of turbulence through a spatial filtering process (Wilcox, 1994; Lesieur and Métais, 1996; Sagaut, 2006). The filtering process is applied on primitive governing equations and the separation of scales is controlled by characteristic filter width (Δ_c), where Δ_c determines the the cutoff frequency (Lesieur and Métais, 1996; Piomelli, 1999; Meneveau and Katz, 2000; Sagaut, 2006). So, the scales that have a greater size than the cutoff frequency are called a large scale (filtered) while the other one are called small scales (subgrid scales). As result, the filtered variables are resolved directly from the filtered equations and the smaller structures are modeled.

It should be noted that after the separation process appears an additional tensor that leads to the Turbulence Closure Problem. Thus, this additional tensor is replaced by a subgrid scale model that introduces the effect of small scales on the filtered equations (Germano *et al.*, 1991; Lilly, 1992; Pope, 2000).

3. FORMULATION

The filtered Navier-Stokes and continuity equations for incompressible, isothermal Newtonian fluid flow are given by:

$$\frac{\partial u_i}{\partial t}+\frac{\partial (u_i u_j)}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial P}{\partial x_i}+\frac{\partial}{\partial x_j}\left[\nu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)+\tau_{ij}\right], \qquad i=1,2,3 \tag{1}$$

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{2}$$

where u_i are the velocity vector components in the x_i coordinate direction and P is the pressure. ρ_0 and ν are the density and kinematic viscosity of the fluid, respectively. The variable t denotes the time. The subgrid scale tensor τ_{ij} results from the unresolved subgrid scale and must be modeled by a subgrid scale model.

According to Lesieur and Métais (1996), the most subgrid scale models assumes the Boussinesq's hypothesis to model τ_{ij}:

$$\tau_{ij} = +2\nu_t S_{ij} + \frac{1}{3}\tau_{kk}\delta_{ij}, \tag{3}$$

where δ_{ij} is Kronecker delta and ν_t is subgrid scale eddy viscosity. The deformation tensor of the filtered field is defined as:

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right). \tag{4}$$

By replacing Eq. (3) and Eq. (4) in Eq. (1), one gets

$$\frac{\partial u_i}{\partial t}+\frac{\partial (u_j u_i)}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial P_{mod}}{\partial x_i}+2\frac{\partial}{\partial x_j}[(\nu+\nu_t)S_{ij}], \tag{5}$$

where $P_{mod} = P - \frac{1}{3}\rho_0\tau_{kk}$ is a modified pressure.

Now, the question is how ν_t should be modeled in order to better estimate the effects of subgrid scales in the filtered solution. In this paper, the Smagorinsky subgrid scale model (Smagorinsky, 1963) was adopted.

Using nondimensional variables, the filtered Navier-Stokes and continuity equations can be rearranged as:

$$\frac{\partial u_i^*}{\partial t^*}+\frac{\partial (u_j^* u_i^*)}{\partial x_j^*} = -\frac{\partial P_{mod}^*}{\partial x_i^*}+\frac{1}{Re}\nabla^2 u_i^* + F_{\nu x_i}, \tag{6}$$

$$\frac{\partial u_i^*}{\partial x_i^*} = 0, \tag{7}$$

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with

$$F_{\nu x_i} = \nu_t^* \nabla^2 u_i^* + 2S_{ij}^* \frac{\partial \nu_t^*}{\partial x_j^*}. \tag{8}$$

The nondimensional variables $(*)$ can be written as

$$t = \frac{t^* L}{U_\infty}; \quad x_j = x_j^* L; \quad u_j = u_j^* U_\infty; \quad P = \rho_0 U_\infty^2 P_{mod}^*; \quad \nu_t = L U_\infty \nu_t^*; \tag{9}$$

$$\tau_{kk} = \frac{\nu U_\infty}{L} \tau_{kk}^*; \quad P_{mod}^* = P^* - \frac{1}{3Re} \tau_{kk}^*. \tag{10}$$

Reynolds number is defined as $Re = \frac{U_\infty L}{\nu}$, where L is characteristic plate length and U_∞ the reference velocity. ∇^2 denotes the Laplacian operator. In the next equations, the symbol $(*)$ will be omitted to facilitate the symbolism.

3.1 Vorticity-velocity formulation

In this work, the vorticity-velocity formulation is used as an alternative of the primitive variables formulation in order to eliminate the pressure terms from governing equations. Assuming, the vorticity ω of a flow as

$$\omega = -\nabla \times \mathbf{u}, \tag{11}$$

the vorticity vector components become

$$\omega_x = \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}, \tag{12}$$

$$\omega_y = \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}, \tag{13}$$

$$\omega_z = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}, \tag{14}$$

where u, v and w are the velocity vector components of vector $\mathbf{u}$ in the x, y and z directions, respectively.

Thus, the nondimensional filtered Navier-Stokes equations (Eq. (6)) result in a transport equations for the components of vorticity in x-, y-, and z- directions:

$$\frac{\partial \omega_x}{\partial t} + \frac{\partial a}{\partial y} - \frac{\partial b}{\partial z} = \frac{1}{Re} \nabla^2 \omega_x + \frac{\partial F_{\nu y}}{\partial z} - \frac{\partial F_{\nu z}}{\partial y}, \tag{15}$$

$$\frac{\partial \omega_y}{\partial t} + \frac{\partial c}{\partial z} - \frac{\partial a}{\partial x} = \frac{1}{Re} \nabla^2 \omega_y + \frac{\partial F_{\nu z}}{\partial x} - \frac{\partial F_{\nu x}}{\partial z}, \tag{16}$$

$$\frac{\partial \omega_z}{\partial t} + \frac{\partial b}{\partial x} - \frac{\partial c}{\partial y} = \frac{1}{Re} \nabla^2 \omega_z + \frac{\partial F_{\nu x}}{\partial y} - \frac{\partial F_{\nu y}}{\partial x}, \tag{17}$$

where $a = \omega_x v - \omega_y u$, $b = \omega_z u - \omega_x w$, and $c = \omega_y w - \omega_z v$ are the nonlinear terms.

Taking into account the filtered continuity equation (Eq. (7)) and the vorticity definition above, the Poisson equations for u, v and w are as follows:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} = -\frac{\partial \omega_y}{\partial z} - \frac{\partial^2 v}{\partial x \partial y}, \tag{18}$$

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = -\frac{\partial \omega_z}{\partial x} + \frac{\partial \omega_x}{\partial z}, \tag{19}$$

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} = \frac{\partial \omega_y}{\partial x} - \frac{\partial^2 v}{\partial y \partial z}. \tag{20}$$

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3.2 Subgrid scale model

To compute the subgrid scale eddy viscosity ν_t it is used the Smagorinsky subgrid scale model (Smagorinsky, 1963):

$$\nu_t = (C_s\Delta_c)^2|S|, \tag{21}$$

where C_s is Smagorinsky constant and $|S| = \sqrt{2S_{ij}S_{ij}}$ is the characteristic filtered rate of strain, and $\Delta_c = (\Delta_x\Delta_y\Delta_z)^{\frac{1}{3}}$ is the characteristic filter width.

Despite the growing demand for more sophisticated models of turbulence, the Smagorinsky model has been successfully applied in different situations. However, this model has some limitations - it is dissipative, especially near walls. One possible solution is to decrease the value of ν_t close to rigid boundaries. For this, a wall-damping function is commonly adopted then the Eq. (21) becomes

$$\nu_t = (C_s\Delta_c)^2 f_w|S|, \tag{22}$$

where the wall-damping function is given by

$$f_w = \left[1 - \exp^{\left(-\frac{y^+}{25}\right)^3}\right], \qquad 0 < f_w < 1. \tag{23}$$

$y^+ = \frac{yu_\tau}{\mu}$ is the distance from the wall in viscous wall units and $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$ is the shear velocity. τ_w is the shear stress at the wall.

4. BOUNDARY CONDITIONS

Figure 1 shows the computational domain where the equations Eq. (15) - (20) are solved. The fluid enters the at $x = x_0$ and exits at x_{max}. In the region between x_3 and x_4, a buffer zone is adopted, where the fluctuations are damped in order to prevent numerical reflections at outflow boundary (Kloker *et al.*, 1993). The basic idea behind the use of this technique is to multiply the vorticity by a ramp function after each step of the integration method. Also, disturbances are inserted in the domain by a mass suction/blowing technique at the wall in the region between x_1 e x_2.

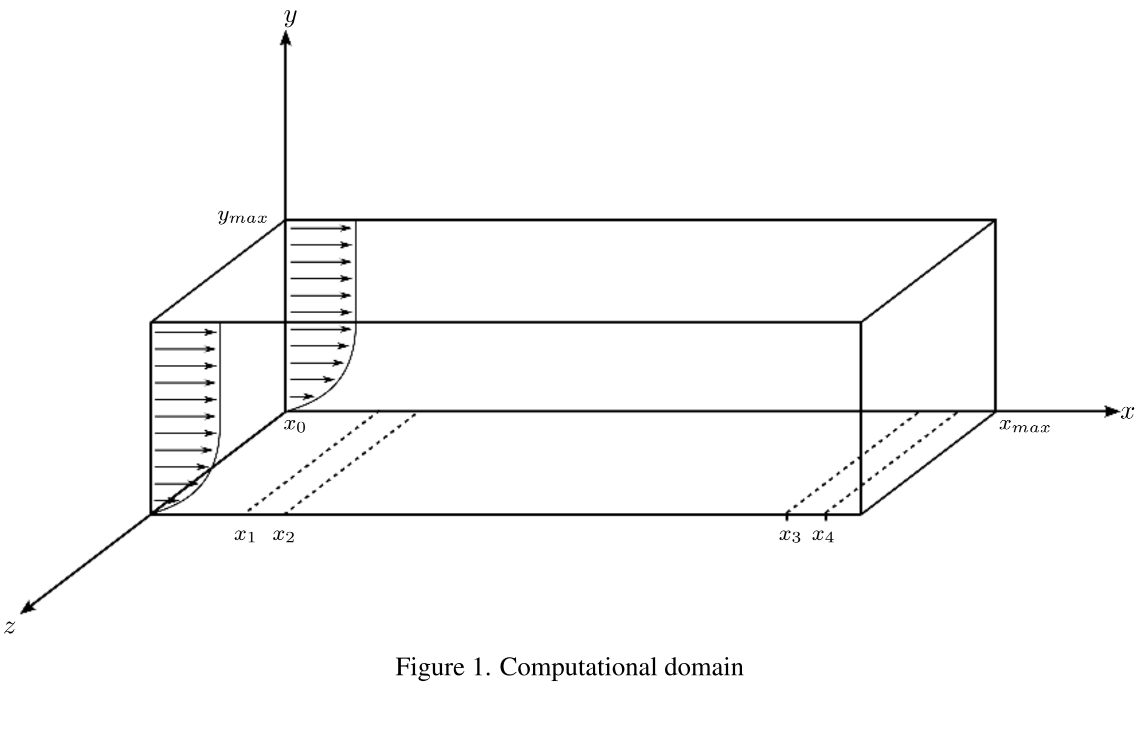

Figure 1. Computational domain

The boundary conditions used at the upper boundary is $\omega_x = \omega_y = \omega_z = 0$ (the flow is supposed to be irrotational). At the wall, the no-slip condition is assumed then u, v, and w are set zero. At the outflow boundary, second derivative of the velocity and vorticity components in the x-direction are set zero. At the inflow is specified a boundary condition of

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the Dirichlet type for v and ω_z based on Falkner-Skan boundary layer solutions. After that, u is calculated by z- vorticity definition (Eq. (14)).

The function used to introduce disturbance via a slot at the wall for the normal velocity v is:

$$v(i,0,t) = f(x)_i A\sqrt{Re}\sin(\omega_t t), \qquad i_1 \leq i \leq i_2,$$
$$v(x,0,t) = 0, \qquad i_2 \leq i \quad \text{and} \quad i \leq i_1, \tag{24}$$

where i_1 and i_2 are the first (x_1) and the last (x_2) point of the disturbance strip, respectively. The value of A is a real constants that can be chosen to adjust the amplitude of the disturbances. The constant ω_t is the dimensionless frequency. The function $f(x)_i$ adopted here is a fifth order function was proposed by Zhang and Fasel (Zhang and Fasel (1999)):

$$\begin{aligned} f(x)_i &= \frac{1}{48}(729\epsilon^5 - 1701\epsilon^4 + 972\epsilon^3) \qquad \text{if} \qquad i_1 \leq i \leq \frac{1}{2}(i_1+i_2),\\ \text{where} \qquad & \epsilon = 2\frac{i-i_1}{i_2-i_1}\\ f(x)_i &= \frac{-1}{48}(729\epsilon^5 - 1701\epsilon^4 + 972\epsilon^3) \qquad \text{if} \qquad \frac{1}{2}(i_1+i_2) \leq i \leq i_2,\\ \text{where} \qquad & \epsilon = 2\frac{i_2-i}{i_2-i_1}. \end{aligned} \tag{25}$$

5. NUMERICAL METHOD

This work assumes periodicity in z-direction therefore it was adopted a spectral method in this direction. Using the conditions of periodicity, all variables can be written as combinations of K Fourier modes

$$f(x,y,z,t) = \sum_{k=0}^{K} F_k(x,y,t)e^{-i\beta_k z}, \tag{26}$$

where f is a generic variable, $i = \sqrt{-1}$ is the imaginary unit and β_k is the wave number in the spanwise direction, given by

$$\beta_k = \frac{2\pi K}{\lambda_z}, \tag{27}$$

where λ_z is the spanwise wavelength of the fundamental Fourier mode. Also, the variables of physical space are represented by lowercase letters (f) and the variables of the Fourier space by capital letters (F).

Substituting Eq. (26) in the vorticity transport equations Eq. (15) - (17) and in the Poisson equations Eq. (18) - (20), these equations can be rewritten, for each K Fourier mode as

$$\frac{\partial \Omega_{x_k}}{\partial t} + \frac{\partial A_k}{\partial y} + i\beta_k B_k = \frac{1}{Re}\nabla_k^2\Omega_{x_k} - i\beta_k F_{y_k} - \frac{\partial F_{z_k}}{\partial y}, \tag{28}$$

$$\frac{\partial \Omega_{y_k}}{\partial t} - i\beta_k C_k - \frac{\partial A_k}{\partial x} = \frac{1}{Re}\nabla_k^2\Omega_{y_k} + \frac{\partial F_{z_k}}{\partial x} + i\beta_k F_{x_k}, \tag{29}$$

$$\frac{\partial \Omega_{z_k}}{\partial t} + \frac{\partial B_k}{\partial x} - \frac{\partial C_k}{\partial y} = \frac{1}{Re}\nabla_k^2\Omega_{z_k} + \frac{\partial F_{x_k}}{\partial y} - \frac{\partial F_{y_k}}{\partial x}, \tag{30}$$

$$\frac{\partial^2 U_k}{\partial x^2} - \beta_k^2 U_k = i\beta_k\Omega_{y_k} - \frac{\partial^2 V_k}{\partial x \partial y}, \tag{31}$$

$$\frac{\partial^2 V_k}{\partial x^2} + \frac{\partial^2 V_k}{\partial y^2} - \beta_k^2 V_k = -\frac{\partial \Omega_{z_k}}{\partial x} - i\beta_k\Omega_{x_k}, \tag{32}$$

$$\frac{\partial^2 W_k}{\partial x^2} - \beta_k^2 W_k = \frac{\partial \Omega_{y_k}}{\partial x} + i\beta_k\frac{\partial V_k}{\partial y}, \tag{33}$$

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with modified Laplacian operator

$$\nabla_k^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \beta_k^2 \right). \tag{34}$$

The domain discretization is performed using high order compact finite difference schemes in the streamwise and wall normal directions. The temporal integration of the transport of vorticity equations are carried out by a 4^{th} order Runge-Kutta method. The solution of Poisson's equation is done via a multigrid method called FAS (Full Approximation Scheme). In order to reduce the computational cost and improve numerical resolution near to the wall, we adopted a technique known as stretching in the wall-normal direction. In the present paper, the stretching factor stf is constant then the spacing between the computational points in the wall-normal direction is obtained by a geometric progression. In addition, the code is parallelized by domain decomposition strategy in the streamwise direction. The Message Passing Interface library is used for the communications in the parallelization process.

6. RESULTS

In the present results the Reynolds number is $Re = 322326$ and four values to C_s (0.01, 0.02, 0.032, 0.05) are used. A DNS code was also used in order to compare with the LES results. The domain is discretized using 473 points in the streamwise direction and 137 points in the wall-normal direction. The distance between two consecutive points in the streamwise direction is 0.008. The number of Fourier modes is $K = 2$ with 4 points in the physical space and the time step is $dt = 0.022$. Also, the disturbances at the wall are inserted between $1.12 < x < 1.4$ and the buffer zone is situated in $3.9 < x < 4.3$. The stretching value adopted was $stf = 1\%$ and the distance from the wall to the first point in wall-normal direction was 0.00018. The fundamental wavenumber in the spanwise direction is $\lambda_z = 0.4$. The parameter to adjust the disturbance amplitude was $A = 1 \times 10^{-6}$, and $\omega_t = 22.27$, corresponding to 550 Hz.

Figure 2 shows the growth rate of a TS wave simulated using DNS, LST and LES strategies. The DNS and LST results are in good agreement. After the receptivity region, the TS wave shows an unstable behavior up to the streamwise position $x = 2.8$. After this region, disturbances tend to be suppressed. Regarding LES cases on DNS mesh parameters, the growth rate of the TS wave starts with values that gives a more stable behavior. For these cases the Smagorinsky model acts damping the disturbances, and the damping effect is higher with higher values of the Smagorinsky constant. This results agrees with literature observations.

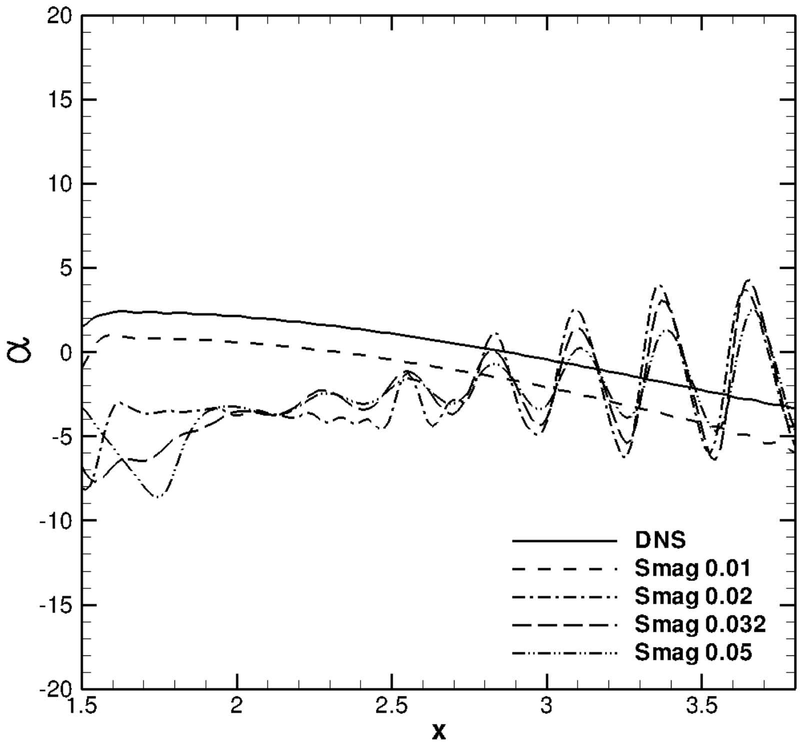

Figure 2. Growth rate of disturbances

The results for the streamwise development of the maximum streamwise velocity disturbance in the wall normal

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direction for the DNS and LES simulations are presented in Fig. 3. It can be observed that the disturbance is initially damped by the LES model, and the damping effect is higher with higher values of Smagorinsky constant value.

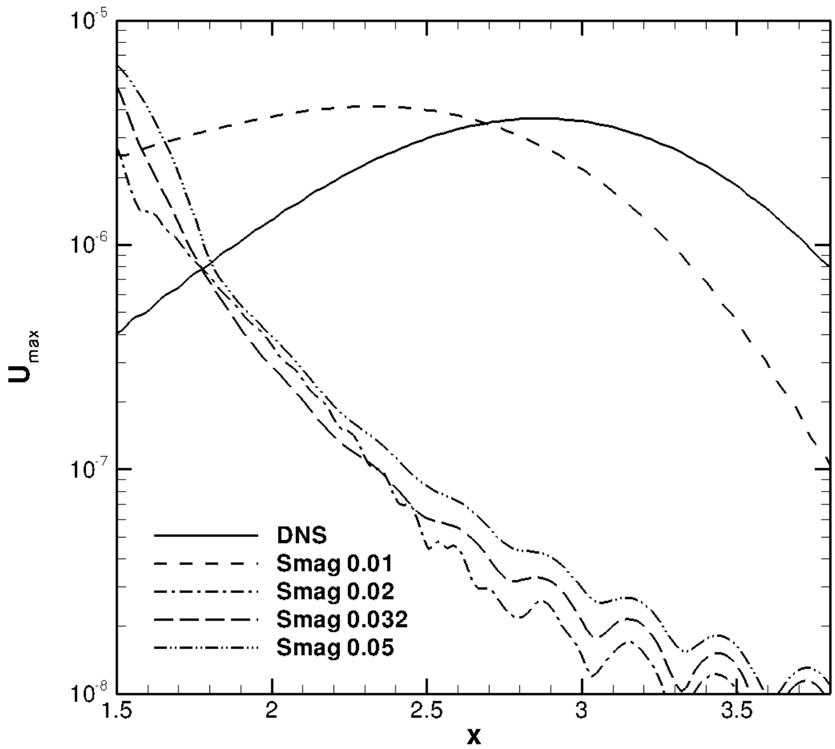

Figure 3. Downstream development of the maximum streamwise velocity disturbance in the wall-normal direction

The disturbance vorticity contours of each simulation (DNS and LES) are showed in the Fig. 4. It can be observed that the disturbance introduced by the LES model increases with the Smagorinsky constant value. The values showed with in all cases goes from -0.002 to 0.002. It can be observed that the Tollmien-Schlichting waves are damped with the LES model, and it vanishes where it should have some amplitude.

7. CONCLUSION

This work presented a numerical method using LES to solve a three-dimensional incompressible flow in order to investigate the influence of Smagorinky subgrid scale model on evolution of TS waves. The numerical results for amplitude development and growth rate of TS waves in the downstream direction using DNS and LES codes were compared with LST. The results obtained demonstrated that the is not possible to compute transitional scenario with the Smagorinsky model.

The next step of this research is to investigate other subgrid scale models such as WALE (Wall-Adapting Local Eddy-viscosity) (Nicoud and Ducros, 1999) model and Dynamics model (Germano *et al.*, 1991) to modelate ν_t. In the end, the objective is to obtain a three-dimensional code using the LES methodology to simulate atmospheric flows.

8. ACKNOWLEDGEMENTS

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9. REFERENCES

Deardorff, J.W., 1974. “Three-dimensional numerical study of the height and mean structure of a heated planetary boundary layer.” *Boundary-Layer Meteorology*, Vol. 7, pp. 81–106.

Fasel, H.F., Rist, U. and Konzelmann, U., 1990. “Numerical investigation of the three-dimensional development in boundary-layer transition”. *AIAA*, Vol. 28, pp. 29–37.

Germano, M., Piomelli, U., Moin, P. and Cabot, W.H., 1991. “A dynamic subgridscale eddy viscosity model”. *Physics of Fluids*, Vol. 4, pp. 1760–1765.

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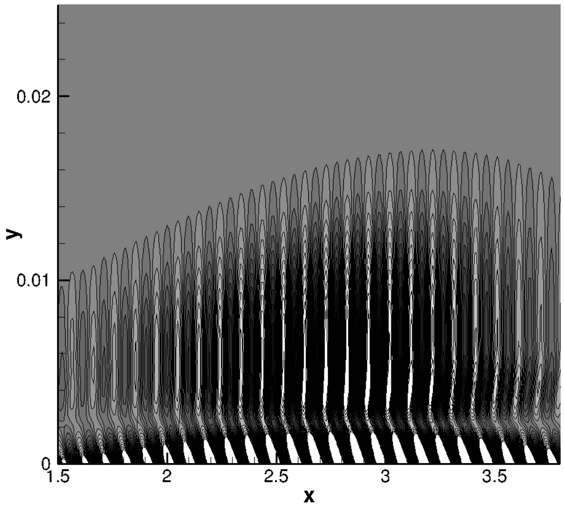
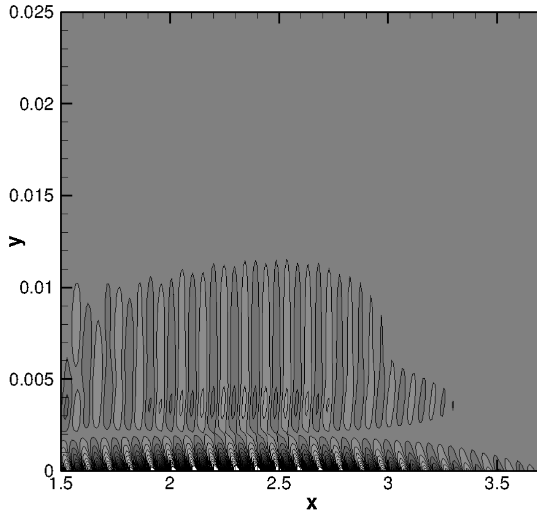
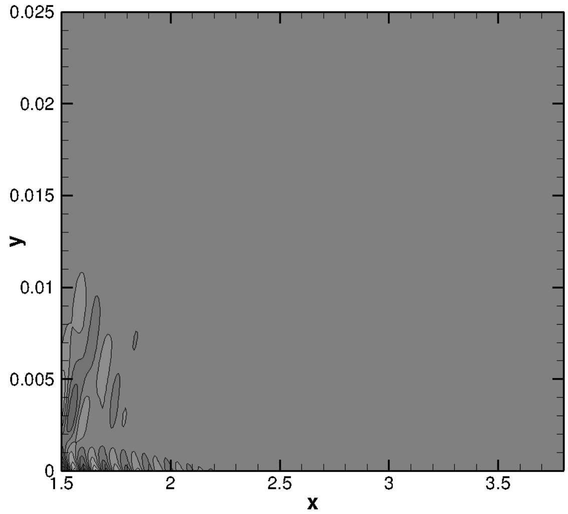


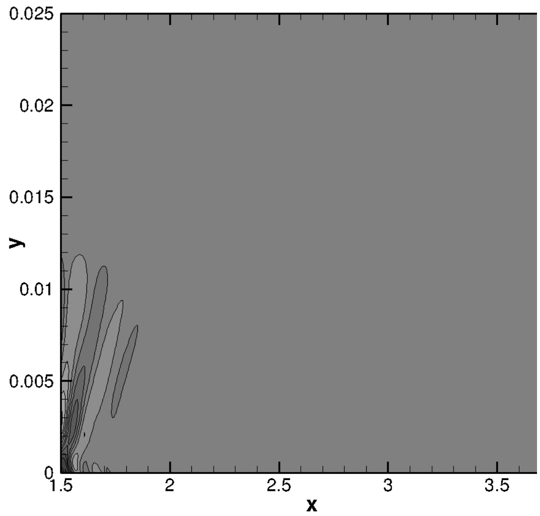
(a) DNS results



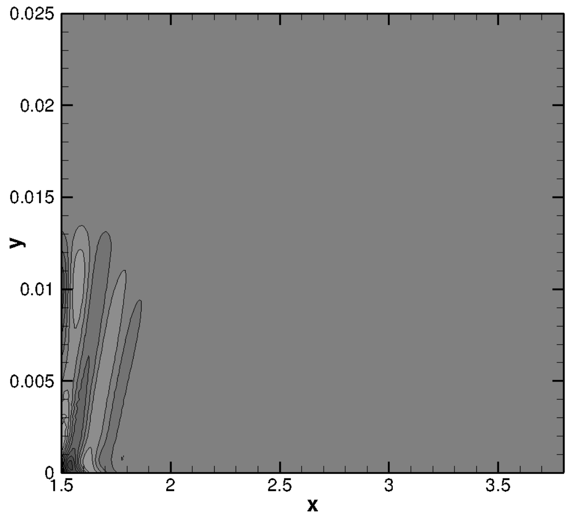
(b) $C_s = 0.01$



(c) $C_s = 0.02$



(d) $C_s = 0.032$



(e) $C_s = 0.05$

Figure 4. Isocountours of disturbance vorticity in the spanwise direction for each simulation

Kloker, M., Konzelmann, U. and Fasel, H.F., 1993. "Outflow boundary conditions for spatial Navier-Stokes simulations of transition boundary layers". *AIAA Journal*, Vol. 31, pp. 620–628.

Lesieur, M. and Métais, O., 1996. "New trends in large-eddy simulations of turbulence". *Annual Review Fluid Mechanics*, Vol. 28, pp. 45–82.

Lilly, D.K., 1967. "The representation of small-scale turbulence in numerical simulation experiments". In *IBM Scientific Computing Symposium on Environmental Sciences*.

Lilly, D.K., 1992. "A proposed modification of the germano subgridscale closure method". *Physics of Fluids*, Vol. 3, pp. 633–635.

Medeiros, M.A.F. and Gaster, M., 1999. "The production of sub-harmonic waves in the nonlinear evolution of wavepackets in boundary layers". *Journal of Fluid Mechanics*, Vol. 397, pp. 301–318.

Mendonça, M.T., 2000. "Parabolized stability equations: A review". In *National Congress of Mechanical Engineering - CONEM 2000*. Natal - RN.

Meneveau, C. and Katz, J., 2000. "Scale-invariance and turbulence models for large eddy simulation". *Annual Review Fluid Mechanics*, Vol. 32, pp. 1–32.

Nicoud, F. and Ducros, F., 1999. "Subgrid-scale modelling based on the square of the velocity gradient tensor". *Flow, Turbulence and Combustion*, Vol. 62, pp. 183–200.

Piomelli, U., 1999. "Large-eddy simulation: achievements and challenges". *Progress in Aerospace Sciences*, Vol. 35, pp. 335–362.

Pope, S., 2000. *Turbulent flows*. Cambridge University Press.

Rodi, W., Fergizer, J.H., Breuer, M. and Pourquié, M., 1997. "Status of large eddy simulation: results of a workshop". *Journal of Fluids Engineering*, Vol. 119, pp. 248–262.

Sagaut, P., 2006. *Large eddy simulation for incompressible flows*. Springer Verlag.

Schlichting, H., 1979. *Boundary-layer theory*. McGraw-Hill, IncH.

Smagorinsky, J., 1963. "General circulation experiments with the primitive equations". *Monthly Weather Review*, Vol. 91, pp. 99–164.

Wilcox, D.C., 1994. *Turbulence modeling for CFD*. DCW Industries.

Zhang, H. and Fasel, H.F., 1999. "Spatial direct numerical simulation of Görtler vortices". In *AIAA Fluid Dynamics Conference*. Norfolk, USA.

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