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THEORETICAL SYNTHESIZING ERRORS IN COORDINATES MEASURING MACHINES

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Abstract. *The application of sophisticated equipment used to dimensional and geometric inspection in manufactured products is evidenced by the requirement of a continuous progress in mechanical and manufacturing industries. The CMM's (Coordinate Measuring Machines) have attributes of speed during operation and ability to provide results with accuracy and repeatability in measurements, so they are considered equipment with potential for application in industrial environments, specifically in inspection processes. However, knowledge of the errors in CMM is needed and allows applying techniques of error compensation. This study aimed to develop a mathematical model of the kinematic errors of a bridge type CMM in "X", "Y" and "Z" directions. The modeling of the errors was accomplished using coordinate transformations applied to the rigid body kinematics, the method of the homogeneous transformation was used for the development of the model. The position and angular errors for the three axes of CMM, in addition to errors related to the absence of orthogonality between them were equated. This study allowed to conclude that modeling of errors applied to CMM allied to calibration is able to evaluate the metrological performance of equipment with displacement on guides, thus is possible to use this technique as error budget analysis in machines.*

Keywords: *Error Budget, modeling of errors, coordinates measuring machines*

1. INTRODUCTION

The competition between companies boosts and accelerates technological development of the industrial sector, reduce costs and increased of quality manufacturing are the large objectives , the result is the evolution of production process.

Continuous improvements in production of mechanical industry caused by the large number of companies in the sector and also by the requirement of high quality manufactured products on the market, shows the need to application of sophisticated instruments for quality control and inspection products. The use of CMM (Coordinate Measuring Machine) provides a quick and pratice dimensional inspection with accurate results, however, error are inherent in its operations and require a study of error budget.

The use of CMM is advantageous, although, as any machine, it is susceptible to geometric errors due to its kinematics and also the errors from external sources, which introduce erroneous values in the dimensions informed. Errors in CMM reduce accuracy in dimensional checking of manufactured parts and thus is essential to know it to a dimensional control appropriate.

Geometric errors in a workspace of a CMM are classified as dimensional errors caused by undesirable moves, like: movements rotation (roll, pitch and yaw) and linear movements (straightness errors and position errors) in addition to errors caused by the lack of orthogonality between the three axes of the machine (DI GIACOMO, 1986).

In recent decades, in order to facilitate the identification of the magnitude of the error in CMM, several authors have proposed techniques for theoretical synthesizing errors, applying mathematical methods and structural analysis models of the machine. In this way, the equationing of errors of coordinate measuring machines are studied.

The authors Denavit and Hartenberg (1955) used matrix model of homogeneous transformations applied to the study kinematic rigid body. Using matrix operations, the authors could know the end position of a body after to be subjected to displacement. PAUL (1981) used the homogeneous transformations in rigid bodies when he applied this technique in a robotic system. The author made possible to know the end position of a rigid body through multiplication of translatory matrix and rotation matrix of its successive movements. This study facilitated the author understand the spatial behavior of mechanical components in moving, as the CMM.

Di Giacomo (1986); Lim and Burdekin (2002) used the geometric method for synthesizing errors in CMM. They analyzed the machine structurally to define the errors present during your operation. Thus, authors proposed a mathematical model equations with the 21 geometric errors grouped in preferential directions, "x", "y", "z" of the coordinates measuring machine.

Was studied in 2008 using the method of homogeneous transformations for calibration of an articulated measuring arm, allowing to define the geometrical errors caused during its operation in directions "x", "y" and "z" (SANTOLARIA et al., 2008).

ORREGO (1999) and CARDOZA (1995) made analysis of volumetric error in movable bridge-type CMM applying the matrix method of homogeneous transformations. The authors presented equations similar of errors found with the geometric analysis.

2. OBJECTIVIES

The purpose of this investigation was to present and to compare the geometric analysis method and the homogeneous transformations method for mathematical modeling of errors in the preferential directions in a movable bridge-type CMM.

3. MATHEMATICAL MODELING

The mathematical model of errors was applied in a movable bridge-type CMM Brown & Sharpe. The geometric analysis method and homogeneous transformations method for equating the error of the CMM were elaborated with the aid of a diagram, drawn in figure (1), it represents the coordinate system of reference and the structure of the machine. The diagram ilustrated in Figure (1) also shows the sliding linear guides in the "x", "y", and "z". The coordinate system used is the same coordinate system set by the machine.

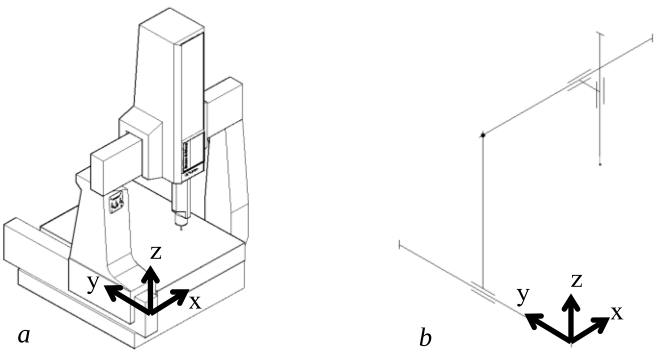

Figure 1. Movable bridge-type CMM (a) and diagram of the CMM (b)

Symbols adopted for errors analyzed in this mathematical modeling and the nomenclatures of the CMM regions with the plans adopted are shown in table (1) and figure (2) respectively.

Table 1. Symbols adopted for errors analyzed

Preliminary definitions of the nomenclature used for the error analysis	
$\delta u(v)$	Position error in the direction u due to movement in the direction v
$\delta\theta u(v)$	Angular motion around the direction u due to movement in the direction v
$\delta\theta wo$	Deviation of orthogonality between the axes perpendicular to the direction w
$sen\ \delta\theta = \delta\theta$ $\cos \delta\theta = 1$	Approximations made for infinitesimal angular displacements

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3.1 Geometric analysis method for errors synthesizing

Angular errors in geometrical analysis were obtained using trigonometric relations of the rotations (roll, pitch and yaw) and also of the errors due lack of orthogonality between the axes of CMM, these errors result in undesirable displacements on the tip of measuring probe. The budget of angular errors is made for the three planes of analysis (xz, xy and yz). It is considered in this study that rotation center of the machine components and the center of the set of aerostatic bearings are coincident, allowing to know the linear components, of the angular errors, in preferential directions of the machine, "x", "y" and "z". The sum of components of errors found in a particular preferential direction is considered the total error in the direction analysed.

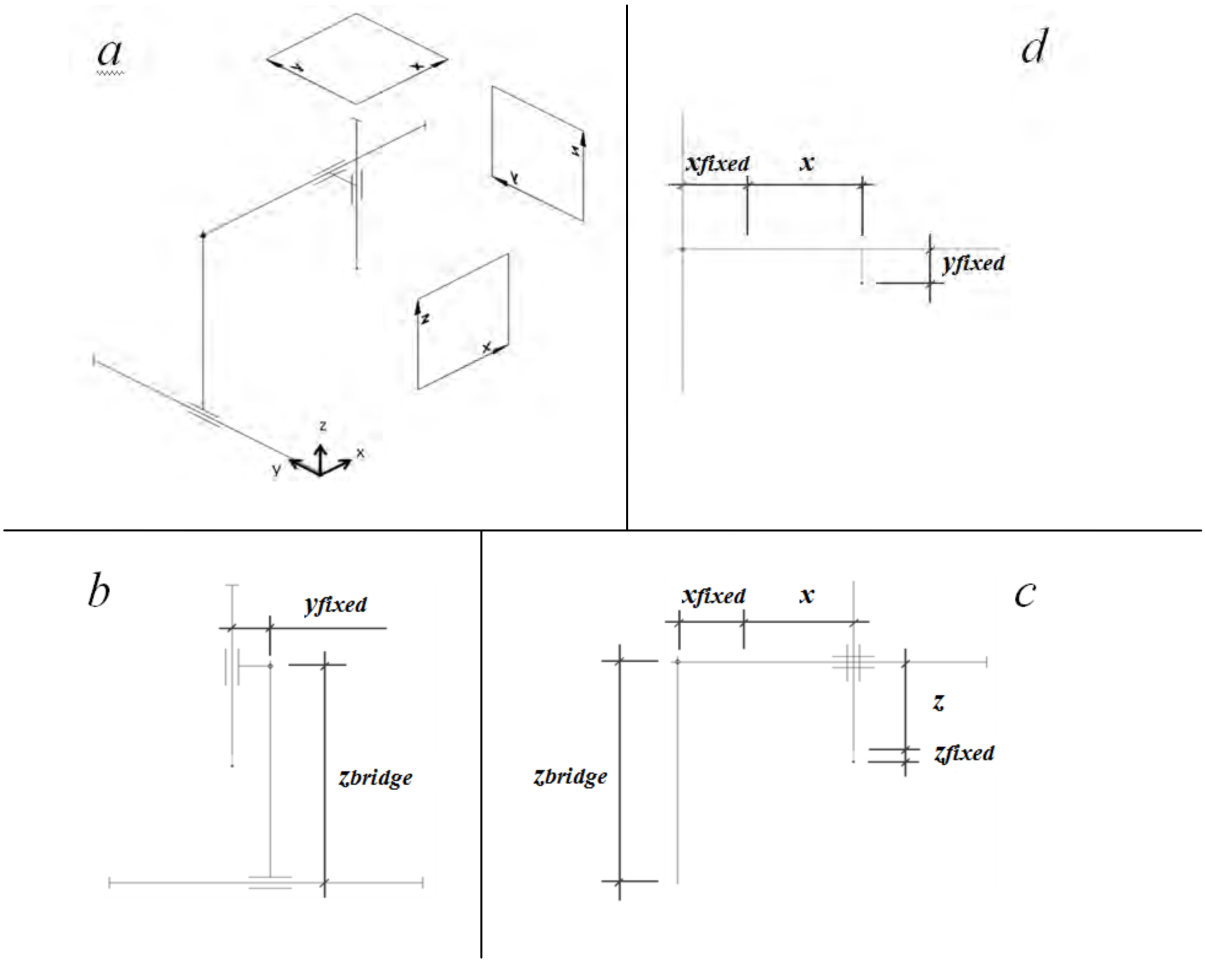

Figure 2. Definition of planes of CMM (a), side view (right) of the diagram proposed, yz plane (b); Front view of the diagram, xz plane (c), top view of the diagram, xy plane (d).

3.2 Homogeneous Transformations method for errors synthesizing

The errors obtained in preferential directions, "x", "y" and "z" using the method of homogeneous transformations, are obtained with a vector analysis. Error vector $\overrightarrow{Ev}$ is the difference between the vector connecting to the origin system of the machine to the tip of the probe (path without error, represented by the vector $\overrightarrow{Maq}$) and the vector that joining the origin system of the machine to the probe tip after undesirable displacements (path with error, represented by the vector $\overrightarrow{VetMaq}$). Coordinates of error vector correspond to errors grouped in the directions "x", "y" and "z", which are the error equations in the directions "x", "y" and "z".

Matrix method of homogeneous transformations is applied with the product of the matrices found to the movements of rotations and translations of the CMM. In this method are applied coordinate systems in the centers of the set of aerostatic bearings and others in the structure of the machine, thus allowing to analyze and assemble the matrix of homogeneous transformations in sections.

With the structural and kinematic analysis of CMM are elaborated matrices of homogeneous transformations to know the coordinates of the probe with undesirable displacements in relation to previous coordinate system. To find the

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undesirable displacements total in the preferential directions is made the product of all matrices of homogeneous transformations.

Coordinate system, the vector analysis and nomenclature used for synthesizing errors with homogeneous transformations are illustrated in figure (3).

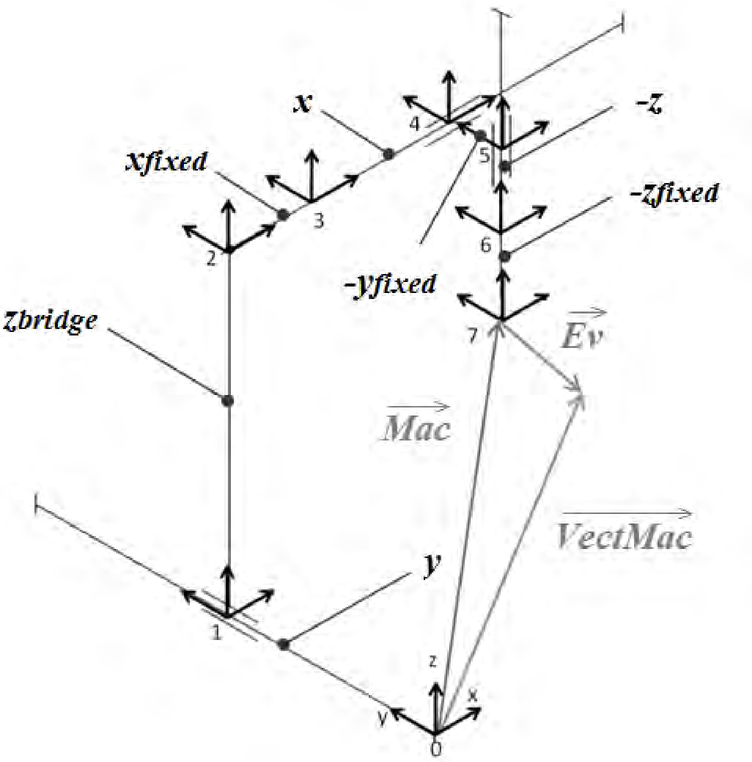

Figure 3. Diagram with nomenclature adopted, positioning of coordinate systems and vector analysis used.

The general error matrix, shown by equation (1), is used to determine angular, straightness and position matrices of errors. In all transformations of coordinate systems this matrix is used , but is need fill it accordingly with the characteristic of error pointed at each position analyzed.

$$M_{erro} = \begin{pmatrix} 1 & -\delta\theta z(v) & \delta\theta y(v) & \delta x(v) \\ \delta\theta z(v) & 1 & -\delta\theta z(v) & \delta y(v) \\ -\delta\theta y(v) & \delta\theta x(v) & 1 & \delta z(v) \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

From vector analysis shown in Figure (3), the error vector is determined by equation (2).

$$\overrightarrow{Ev} = \overrightarrow{VectMac} - \overrightarrow{Mac} \tag{2}$$

In the homogeneous transformations method the matrices that determine the vector $\overrightarrow{VectMac}$ and the error vector $\overrightarrow{Ev}$ was obtain with the software Matlab R2012a.

4. RESULTS AND DISCUSSIONS

Synthesizing of errors by geometric analysis in CMM makes it possible to find the error plots showed in Table (2). The error contributions of second order were not considered according to study of Di Giacomo (1986).

Table 2. Error plots obtained from geometric analysis

Direction "x"	Direction "y"	Direction "z"
$\Delta x = \delta\theta y(y) \times (zbridge - z - zfixed)$	$\Delta y = \delta\theta z(y) \times (x + xfixed)$	$\Delta z = -\delta\theta y(y) \times (x + xfixed)$
$\Delta x = -\delta\theta y(x) \times (z + zfixed)$	$\Delta y = \delta\theta zo \times (x + xfixed)$	$\Delta z = -\delta\theta x(x) \times yfixed$
$\Delta x = -\delta\theta y(z) \times (z + zfixed)$	$\Delta y = \delta\theta x(x) \times (z + zfixed)$	$\Delta z = -\delta\theta x(y) \times yfixed$
$\Delta x = -\delta\theta yo \times (z + zfixed)$	$\Delta y = -\delta\theta x(y) \times (zbridge - z - zfixed)$	-
$\Delta x = \delta\theta z(y) \times (yfixed)$	$\Delta y = \delta\theta x(z) \times (z + zfixed)$	-
$\Delta x = \delta\theta z(x) \times yfixed$	$\Delta y = \delta\theta xo \times (z + zfixed)$	-
$\Delta x = \delta\theta zo \times (yfixed)$	-	-

By structural analysis and theory of homogeneous transformations, matrices representing the course with and without error proposed through the approach vector were expressed by equations 3 and 4, respectively.

$$Mac = \begin{pmatrix} 1 & 0 & 0 & x + xfixed \\ 0 & 1 & 0 & y + yfixed \\ 0 & 0 & 1 & zbridge - z - zfixed \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

$$VectMac = \left[{}^{0}T_1\right] \times \left[{}^{1}T_2\right] \times \left[{}^{2}T_3\right] \times \left[{}^{3}T_4\right] \times \left[{}^{4}T_5\right] \times \left[{}^{5}T_6\right] \times \left[{}^{6}T_7\right] \tag{4}$$

Structural and dynamic analysis of CMM determined that the homogeneous transformation matrices of sections analyzed were expressed by equations (5) (6) (7) (8) (9) (10) (11).

$$\left[{}^{0}T_1\right] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & -\delta\theta z(y) & \delta\theta y(y) & \delta x(y) \\ \delta\theta z(y) & 1 & -\delta\theta x(y) & \delta y(y) \\ -\delta\theta y(y) & \delta\theta x(y) & 1 & \delta z(y) \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{5}$$

$$\left[{}^{1}T_2\right] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & zbridge \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & -\delta\theta z(z) & \delta\theta y(z) & \delta x(z) \\ \delta\theta z(z) & 1 & -\delta\theta x(z) & \delta y(z) \\ -\delta\theta y(z) & \delta\theta x(z) & 1 & \delta z(z) \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{6}$$

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$$\left[{}^{2}T_{3}\right]=\begin{pmatrix}1 & -\delta\theta zo & 0 & 0\\ \delta\theta zo & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\times\begin{pmatrix}1 & 0 & 0 & x\ fixed\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix} \quad (7)$$

$$\left[{}^{3}T_{4}\right]=\begin{pmatrix}1 & 0 & 0 & x\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\times\begin{pmatrix}0 & -\delta\theta z(x) & \delta\theta y(x) & \delta x(x)\\ \delta\theta z(x) & 1 & -\delta\theta x(x) & \delta y(x)\\ -\delta\theta y(x) & \delta\theta x(x) & 1 & \delta z(x)\\ 0 & 0 & 0 & 1\end{pmatrix} \quad (8)$$

$$\left[{}^{4}T_{5}\right]=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & y\ fixed\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix} \quad (9)$$

$$\left[{}^{5}T_{6}\right]=\begin{pmatrix}1 & 0 & \delta\theta yo & 0\\ 0 & 1 & -\delta\theta xo & 0\\ -\delta\theta yo & \delta\theta xo & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\times\begin{pmatrix}1 & -\delta\theta z(z) & \delta\theta y(z) & \delta x(z)\\ \delta\theta z(z) & 1 & -\delta\theta x(z) & \delta y(z)\\ -\delta\theta y(z) & \delta\theta x(z) & 1 & \delta z(z)\\ 0 & 0 & 0 & 1\end{pmatrix}\times\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & -z\\ 0 & 0 & 0 & 1\end{pmatrix} \quad (10)$$

$$\left[{}^{6}T_{7}\right]=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & -z\ fixed\\ 0 & 0 & 0 & 1\end{pmatrix} \quad (11)$$

In this study, because of error analysis be performed in relation to preferred directions of measuring machine, matrices $\overrightarrow{VectMac}$ and $\overrightarrow{Mac}$ were changed into column matrices, since only the last column of the matrices are composed to error elements in the preferential directions. On both matrices were maintained the last column and by the difference between them, it was possible to find a matrix $\overrightarrow{Ev}$, in which your cooordinates are Ex, Ey and Ez.

From the geometric analysis methods and homogeneous transformations for errors synthesizing in the CMM, not considering the errors of second order according to Di Giacomo (1986), it was possible to obtain the error equations in directions "x", "y "and" z " (12, 13 and 14, respectively). Thus, error equations in the preferential directions of CMM are described as follows:

$$\begin{aligned}Ex &= \delta\theta y(y)\times(z+zfixed)-\delta\theta y(x)\times(z+zfixed)-\delta\theta y(z)\times(z+zfixed)-\delta\theta yo\times(z+zfixed)+\\ &+\delta\theta z(y)\times(yfixed)+\delta\theta z(x)\times yfixed+\delta\theta zo\times(yfixed)+\delta x(x)+\delta x(y)+\delta x(z)\end{aligned} \quad (12)$$

$$\begin{aligned}Ey &= \delta\theta z(y)\times(x+xfixed)+\delta\theta zo\times(x+xfixed)+\delta\theta x(x)\times(z+zfixed)+\\ &-\delta\theta x(y)\times(zbridge-z-zfixed)+\delta\theta x(z)\times(z+zfixed)+\delta\theta xo\times(z+zfixed)+\delta y(x)+\delta y(y)+\delta y(z)\end{aligned} \quad (13)$$

$$Ez=-\delta\theta y(y)\times(x+xfixed)-\delta\theta x(x)\times yfixed-\delta\theta x(y)\times yfixed+\delta z(x)+\delta z(y)+\delta z(z) \quad (14)$$

It is possible to observe that the methods used showed similar results in equationing of errors, not considering second order errors.

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5. CONCLUSIONS

Considering the industrial growth are essential equipments that verify the quality of production of manufactured parts accurately and quickly. In this regard, methods have been developed for finding the effectiveness on coordinate measuring machines.

In this study, it was verified that the geometric analysis method and homogeneous transformations method enable to equate errors of CMM. The methods of equating the errors presented, quantified by calibration, make it possible to know individual contribution of error in each measuring direction analyzed. Application of homogeneous transformation is a convenient and practical way for the error budget.

6. ACKNOWLEDGMENTS

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