ASYNCRONOUS PARTICLE SWARM ALGORITHM FOR SIZE TRUSS OPTIMIZATION WITH STRENGTH AND NATURAL FREQUENCIES CONSTRAINTS

Fábio Henrique de Souza, henrique.souza@ufrgs.br
Herbert Martins Gomes, herbert@mecanica.ufrgs.br
UFRGS, Federal University of Rio Grande do Sul, Porto Alegre, Rio Grande do Sul, Brazil. Tel.: (+55 51)3308-3681

Abstract. *The present work has as objective the optimization of truss structures using a heuristic algorithm known as Particle Swarm Optimization. The algorithm is based on behaviors found in populations of organisms in nature when are in group that try to find solutions to survive. These behaviors are encoded to solve engineering problems. There are great advantages indicated in literature to use this algorithm in the problems considered here, because they are related to problems with high complexity and with some strong nonlinear functions to be optimized. The problem arises when a structure of less weight is wished. Therefore, to solve this problem it is necessary an objective function minimization, which means that the total mass of the structure should be reduced. Nevertheless, this optimization process faces the problem constraints that can be excessive stress, related to violation of the value of yield stress, buckling load or dynamic behaviors not allowed, related to natural frequencies. To this end, design variables such as positions of nodes of the truss structure and cross sections areas of the bars are optimized in structural analysis iterations. Finally, an optimized point is obtained for the structure while the problem constraints are satisfied. The methodology used is the encoding of the algorithms and execution of several comparative performance tests with examples found in literature and solved by other methods to prove the efficiency using this type of algorithm. The expected results are structures with minimized weight in relation to the original structures that satisfy previously required constraints.*

Keywords*: structural optimization, truss, Particle Swarm algorithm, stress, natural frequencies.*

1. INTRODUCTION

The optimization process can be described as a successful way to find the best solution of many available ones. Likewise, structural optimization tries to meet the best structural configuration following some constraints. Therefore, this work focuses in the optimization of truss structures. The challenge of working with structural optimization involves since the design requirements until the structural analysis design, considering all conditions that constrains these calculus, in other words, this is a subject that encompasses an extended area of knowledge. So, the motivation to develop this subject is the challenge of solving high complexity and non-linear problems.

The present work has as objective the optimization of truss structures using a heuristic algorithm known as Particle Swarm Optimization (PSO). This way, this algorithm is developed, tested and compared with other ones developed for the same objective, based in successful models and examples, so it is possible to have the PSO performance evaluation. The objective function of each case is the total mass of the truss structure, which is the function that is chosen to be optimized to minimize the total weight and cost of the structure. However, to find the optimal solution, it is necessary meet all problem constraints, like bars stresses (yield stress under tension and buckling under compression) or natural frequencies of the structure.

At the same time that values of mass and stiffness are changing, vibration modes (and respective natural frequencies) of the truss structure can be easily changed (for example, flexural mode to torsional mode). It changes significantly the natural frequencies and hampers the optimization method convergence. These changes are of a non-linear behavior as indicated in Grandhi (1993). This way, the choice of heuristic algorithm for implementation and comparison is based on its robustness, as indicated by the literature, with complex and non-linear optimization problems, like spatial truss optimization of weight and displacement with bars stress and natural frequencies constraints.

2. BRIEF BIBLIOGRAPHICAL REVIEW

Optimization can be explained as a success way to meet the best parameters to find a better solution than the present one, from many parameters available, considering the problem constraints. According to Bergh (2001), the task of optimization is very important in several professional areas. For example, physicists, chemists and engineers are interested in design optimization when designing a chemical plant to maximize production, respecting certain constraints, like cost and pollution. Likewise, economists and operation researches have to consider the optimal allocation of resources in industrial and social settings. Nowadays, in Mechanical Engineering area, optimization algorithms are being studied increasingly because commercial software needs to improve their processing time. It is done trough modification of the algorithm code of the software with an algorithm optimization implementation.

There are several optimization methods that are widely used in engineering. The optimization methods can be classified basically as deterministic and probabilistic methods. The deterministic algorithms can generate a deterministic sequence of possible solutions requiring, in the majority of the time, the use of, at least, the first derivative of the objective function in relation of the design variables. Some popular known deterministic methods are: Simplex Method, Sequential Linear Programming (SLP), Sequential Quadratic Programming (SQP), Method of Feasible Directions and Reduced Gradient Method. On the other hand, optimization methods based in probabilistic methods use only the evaluation of the objective function and introduce stochastic parameters. They are considered zero order method because they do not use the derivative of the function. The best known probabilistic methods are: Genetic Algorithms (GA), Evolutionary Strategy and Programming, Particle Swarm Optimization (PSO), Ant Collony Search (ACS), Simulated Annealing (SA) and others. According to Bastos (2004), the main probabilistic methods advantages if compared with deterministic methods are:

- The objective function and constraints do not necessarily have a mathematical representation;
- They do not need the objective function is continuous or differentiable;
- These methods work properly with continuous or discrete parameters, or with a combination of them;
- They do not need complex formulations or reformulations of the problem;
- There is no restriction to the starting point within the search space of the solution;
- They perform simultaneous searches in the space of possible solutions through a population of individuals, therefore they are candidates for the use of parallelism in computers;
- They optimize a large number of variables, on condition that the evaluation of the objective function does not have a high computational cost.

The biggest drawback of probabilistic methods in relation to the deterministic methods is the computational cost. Therefore, probabilistic methods, in general, demand a high processing time when used in sequential processing computers, such as in an ordinary computer.

The Particle Swarm Optimization method is based in behavior of population from nature. It is important to know that algorithms based in behavior of nature individuals had an evolutionary process. Reynolds (1987) developed a model of collective intelligence called ‘Boid’ with simple rules of programming, and generated a complex collective behavior trough graphic animation. Boyd and Richerson (1985) checked the human decision process and created individual learning and knowledge transmission concept. According to their researches, human beings make decisions from their experiences and from other people experiences. A new optimization technique using a collective behavior analogy was developed in the beginning of 90’s. Colorni; Dorigo and Maniezzo (1991) developed the Ant Colony Optimization (ACO) based mainly in insects’ social behavior, in special ants. Each individual changes information with other one trough emission of pheromones. Eberhart and Kennedy (1995) developed the Particle Swarm Optimization (PSO) based in analogy of behavior of bird flocks and fish shcooling. According with Fukuyama et al. (2004), one of the main trends in development of collective intelligence is to analyze how nature creatures behavior in groups and try to model this behavior in a computer model.

There were several cases in which the optimization algorithms were applied to structures. Deb and Gulati (2001) applied genetic algorithms optimization to several configurations of truss structures to reduce their weights. Park and Ryu (2004) used Simulated Annealing modified aiming at minimization of the cross section areas of bars from truss structures. Cristodolou (2005) used Ant Colony Optimization for topology optimization to truss structure. Li, Huang and Liu (2009) applied Particle Swarm Optimization to truss structures in a case in which the variables were discrete. Most of these cases is recent and this evidences that the study of structure optimization has been an increasingly researched topic and that development in this area has been increasingly expanding over the years.

3. THEORETICAL FOUNDATIONS

3.1. Asynchronous Particle Swarm Optimization (APSO)

Eberhart and Kennedy (1995) created the swarm intelligence concept (collective intelligence) to optimize continuous non-linear functions, based on modeling the behavior of social-simplified groups. Nevertheless, their goal was to obtain a description of the movement of groups of birds or fishes, which, even in great quantity, used to have synchronism when they were moving. In reference to shoals of fish, the socio-biologist Wilson (apud EBERHART; KENNEDY, 1995) says that “In theory at least, individual members of the school can profit from the discoveries and previous experience of all other members of the school during the search for food. This advantage can become decisive, outweighing the disadvantages of competition for food items, whenever the resource is unpredictably distributed in patches”. The hypothesis that this exchange of information between individuals in a group is an evolutionary advantage was essential for the creation of the Particle Swarm Optimization method.

The PSO algorithm keeps a population of individuals where each individual is called "particle" because it does not have characteristics such as mass and volume. Another important fact in PSO is that particles do not behave like fishes or birds, because if it happened, it would be the same that particles would have some individual cognitive capacity, which contradicts the schema of particle, which are only able to act if they are in a group.

Some definitions for the PSO are as follows:

- Design variables: they are variables that change during the optimization process and may be continuous (real), integer or discrete;
- Constraints: they are functions of equality or inequality on the design variables that describe design situations considered undesirable. Often referred to as boundary conditions;
- Search Space: is the whole space or region that includes the possible or viable solutions on the design variables of the problem to be optimized, and they are delimited by the restriction functions;
- Cost Function: it is the function of one or more design variables that are needed to be optimized, through their minimization or maximization. Also known as objective function;
- Best Point: it is the point formed by the design variables that extremize the objective function and meet the constraints. Also known as optimum point;
- Best Value: it is the value of the objective function at the optimum point.

In the PSO, it is simulated a swarm of particles that "fly" in an n-dimensional space (the search space), attracted by regions of high fitness value. The "position" of the particle characterizes it as a candidate solution, while the topology of the search space is represented by the objective function of the problem. Each particle has a "speed", which has information such as direction, rate of position change by the "time" and the attribute of fitness (or performance of the particle), the latter obtained by evaluating the objective function in the position of the particle. The position and velocity variation of the particle is given by his own experience (the historical information of good and bad regions in which the particle has passed), and by observing their succeed neighbors.

Considering $\vec{X}_i(t) = \{x_{i,1}(t),...,x_{i,j}(t)\}$ and $\vec{V}_i(t) = \{v_{i,1}(t),...,v_{i,j}(t)\}$, respectively, the position (the vector which is solution candidate) and speed (rate of change) of the particle *i* at time *t*, in a search space of *n*-dimensional. Considering also $\vec{p}Best_i(t) = \{pBest_{i,1}(t),..., pBest_{i,j}(t)\}$ the best position ever found by particle *i* until time *t*, and $\vec{g}Best(t) = \{gBest_1(t),..., gBest_j(t)\}$ the best position ever found by the swarm by time *t*. The PSO algorithm follows the following rules to update their velocities and positions:

$$v_{i,j}(t+1) = \chi.[w.v_{i,j}(t) + c_1.r_1.(pBest_{i,j}(t) - x_{i,j}(t)) + c_2.r_2.(gBest_j(t) - x_{i,j}(t))] \tag{1}$$

$$x_{i,j}(t+1) = x_{i,j}(t) + v_{i,j}(t+1) \tag{2}$$

$$\chi = \frac{1.6}{\left|2 - (c_1 + c_2) - \sqrt{(c_1 + c_2)^2 - 4(c_1 + c_2)}\right|} \tag{3}$$

where:

- $v_{i,j}$(t+1): it is the velocity to update of the particle *i* corresponding to the design variable *j*;
- $v_{i,j}$(t): it is the current velocity of the particle *i* corresponding to the design variable *j*;
- $x_{i,j}$(t+1): it is the position to update of the particle *i* corresponding to the design variable *j*;
- $x_{i,j}$(t): it is the current position of the particle *i* corresponding to the design variable *j*;
- $pBest_{i,j}$(t): it is the best position ever found by particle *i* corresponding design variable *j*;
- $gBest_j$(t): it is the best position ever found by the swarm of particles corresponding to design variable *j*;
- *w*: weight of inertia for particles velocity, controlling their ability to explore and exploit information gathered by the experience of other particles;
- c_1: cognitive constant acceleration (individual), referring to $pBest_{i,j}$(t);
- c_2: social acceleration constant (swarm), referring to $gBest_j$(t);
- r_1 and r_2: random numbers that can be between zero and one, having sole purpose of keeping the stochastic characteristic of the function;
- χ: parameter used to avoid the divergent behavior of the algorithm.

In the optimization algorithm PSO, the swarm of particles is initialized randomly, so that positions and velocities of the particles have random values. Thus, while the stop criterion is not met (in this case the criterion is the ratio of standard deviation and mean for the value of the cost function for all particles, which must be less than or equal to a stipulated amount for the iterative process is interrupted), a loop is executed with the following steps:

- The particles are evaluated according to the cost function and the fitness values of each particle are determined;
- Values of $pBest_{i,j}$(t) and $gBest_j$(t) are updated;
- The particles move according to the updating equations for the velocity and the position (equations (1) and (2), respectively).

In Fig. 3.1 can be viewed the pseudocode of the PSO.

(a)
```
Algoritmo PSO
begin
    for i=1 to n_particles do begin
        randomize(Xi); randomize(Vi);
    end;
    for iter=1 to itermax do begin
        for i=1 to n_particles do evaluate (Xi);
        for i=1 to n_particles do update(pBesti,gBest);
        for i=1 to n_particles do begin
            Vi = χ*(w*Vi+c1*r1*(pBesti-Xi)+c2*r2*(gBest-Xi));
            Xi = Xi + Vi;
        end;
    end;
end.
```

(b)
```
Algoritmo APSO
begin
    for i=1 to n_particles do begin
        randomize(Xi); randomize(Vi);
    end;
    for iter=1 to itermax do begin
        for i=1 to n_particles do begin
            evaluate (Xi);
            update(pBesti,gBest);
            Vi = χ*(w*Vi+c1*r1*(pBesti-Xi)+c2*r2*(gBest-Xi));
            Xi = Xi + Vi;
        end;
    end;
end.
```

Figure 3.1. (a)Pseudocode of PSO algorithm and (b) Pseudocode for Asyncronous PSO Algorithm.

The routine shown above in the pseudocode illustrates the steps previously mentioned. After reaching the stopping criterion, in case of satisfactory convergence, the iterations are stopped and the optimized result is achieved. Evidences of convergence of the method indicating that it is an algorithm of global convergence, *i.e.*, after a certain number of iterations it will reach the global optimum, can be found in the literature (MEI; LIU and Xiao, 2010; Bergh, 2001) and is not object of this work.

As stated by Groenwold and Schutte (2003), differently of the synchronous PS, where the average swarm values are updated once the whole swarm has been evaluated in an iteration, in the asynchronous approach, the average swarm values are updated upon evaluation of each individual in the swarm. In general, the asynchronous implementation is superior to the synchronous implementation.

3.2. Algorithm and Structural Analysis

In structural projects the main criterion for no failure is that the structure, given its boundary conditions, resists the stresses imposed in its design. Similarly, in this work, it is optimized structures having design criteria directly linked to possible failure. Therefore, the PSO algorithm is applied to the objective function (or cost function) of the problems (total mass of the structures) and its optimization is satisfactory when it has satisfied the design constraints and when the objective function is minimized. The cost function of truss structures is given by the following equation:

$$Mass = \sum_{i=1}^{n} \rho . A_i . L_i \tag{4}$$

Where Mass is the total mass of the structure, n is the number of bars, ρ is the material density of the bars, A_i is the cross-sectional area of each bar and L_i is the length of each bar. In the optimization process, the algorithm follows the routine given by the flowchart in Fig. 3.2, below.

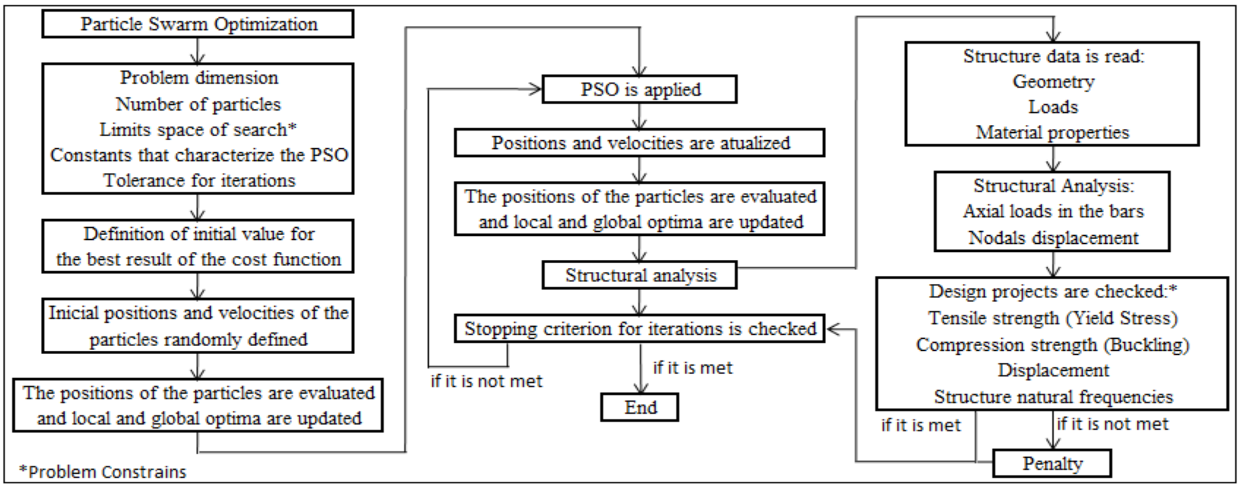


Figure 3.2. Flow chart of PSO algorithm with structural analysis of trusses.

As it can be seen in the flowchart above, the algorithm begins with the definition of dimension of the problem, which is the number of design variables to be optimized. The number of particles is chosen according to the problem, checking which value is appropriated to get a satisfactory convergence. The boundaries for the search space are highlighted with an asterisk because they can be restrictions of the problem, and in this work, they are called as primary constraints, because they delimit the search space of the particles. The constants that characterize the PSO are the constants discussed in the previous item (item 3.1). The tolerance for the iterations is a value chosen for the standard deviation between one iteration and another one, which is the stopping criterion to the program.

Then, it is set an initial value for the best result of the cost function, which is an initial value for the global optimum of the objective function of the problem. Then, the initial position and velocity of the particles have their values set randomly. It would be the same that the particles are scattered within of search space to start looking for their local optima. So, once the local optima are defined the process can be initialized in order to find the swarm global optimum. Then, the PSO algorithm is applied (equations (1), (2) and (3)). The positions and velocities of the particles are updated. The local optima and the global optimum are evaluated. Therefore, structural analysis is done.

The first step of the structural analysis is the reading of the data structure, that are about geometry, loading and material properties. The structural analysis is then performed using the Finite Element Method (FEM). The linear system generated by the stiffness matrix and the vectors of force and displacement of the structure is solved. When the displacements are found, the stresses in each bar are calculated. For the examples with natural frequency to be considered, an eigenvalues and eigenvectors problem is created with the matrices of mass and stiffness and then solved, obtaining the first *n* natural frequencies as well as their corresponding vibration modes.

Then, there are the design conditions, also highlighted with an asterisk (Figure 3.2), because they are part of the restrictions of the problem. However, in this case, they are called secondary constraints, because they are part of the failure criterion in the structural analysis. As the structures are trusses, the only type of stress is tensile or compressive stresses. Consequently, the only possibilities for failure are for yield stress (by tension) or buckling (compression) of the component bars of the truss structure. Another condition that can also be evaluated refers to the natural frequencies of the structure. If any design condition is violated, a penalty factor is applied to the cost function. The cost function with penalty factor is given by:

$$Mass^* = \left(\sum_{i=1}^{n} \rho . A_i . L_i\right) . (1 + PF) \tag{5}$$

Where Mass* is the cost function (total mass of the structure) changed by a penalty factor *PF*, which varies according to the design condition of the constraint violation. Generally, the penalty factor is represented by:

$$PF = \left| \frac{K_i}{K_{máx}} - 1 \right| \tag{6}$$

Where K_i is the value found that violates the design condition for each bar, and it can be strain or natural frequency, and K_{max} is the maximum allowable value for the violated condition.

Coming back to the structural analysis (with the design specifications met), the stopping criterion to the iterations is applied. As discussed earlier, this criterion is defined by the ratio of standard deviation and average values of the $gBest_j(t)$ in the iterations over the individuals of the swarm, a value that is set at the beginning of the algorithm routine. This criterion represents the diversity of the solution in the swarm. If the criterion is not satisfied, it is reapplied the PSO and the subsequent processes described above are done again until the convergence is reached. Once the stopping criterion is satisfied, the algorithm is finalized and the value of the cost function and their respective optimized variables are obtained.

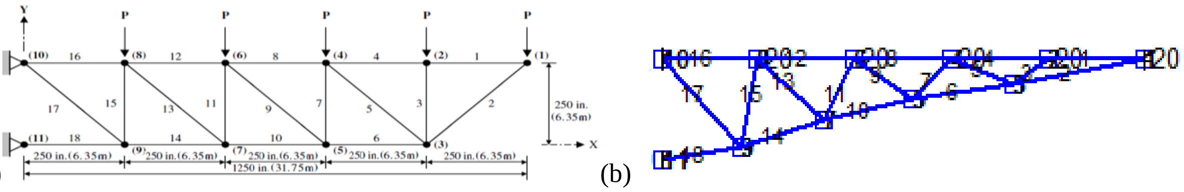
4. PROBLEMS

4.1. Methodology

It will be optimized four cases of truss structures, which are problems already solved by other authors using others optimization algorithms. Thus, a performance comparison can be done between the PSO algorithm and the others algorithms used for these problems considered benchmarks, because they are reference cases in the literature. All the problems have in common the fact that they are optimizations of truss structures. However, each one has its peculiarities as regards its configuration and its structural design constraints. The PSO algorithm and the structural analysis of these trusses are programmed in MATLAB® software, a mathematical tool that stands out for its quality and wide use in literature. The four cases to be solved are truss structures of 18, 120, 72 and 37 bars with, respectively, 12, 7, 16 and 37 design variables to be optimized. These problems and their solutions will be presented minutely hereafter.

4.2. 18 bars plane truss – 12 design variables to be optimized

The plane cantilevered truss structure shown in Fig. 4.1 is one of the most classic design optimization of structures. As a reference, it will be used the work done by the authors Lee and Geem (2005), who used the algorithm of Harmony Search (HS) for the optimization of this structure and they compared it with other optimization algorithms.



(a) (b)

Figure 4.1. (a)Initial structure to be optimized of the problem 4.2 (Source: LEE; GEEM, 2005) and (b)obtained optimized structure.

The problem is related to a truss structure of 18 bars and 11 nodes. The material density is 0.1 lb/in³ (2768 kg/m³) and the modulus of elasticity is 10000 ksi (68,95 GPa), which correspond roughly to the properties of aluminum. The initial area of cross section of all the bars adopted in this work is 10,25 in² (0,00661 m²), which corresponds to the mean of the maximum and the minimum allowable cross-sectional area of the bars for this problem. The loads that are concentrated at nodes 1, 2, 4, 6 and 8 have a value of 20 kips (88964 N).

With regard to the cost function, what is intended to optimize is the total mass of the structure. Therefore, it is necessary to optimize the design variables involved in the problem, which corresponds to cross-sectional areas of the bars and locations of some nodes. However, it should be taken into account the primary and secondary constraints involved in the problem (as explained in section 3.2).

With regard to the primary constraints of the problem, the structure is two-dimensional, so it does not have movement in the z-axis (perpendicular to the x and y axes). Left end nodes are fixed and labeled, so do not move. In this optimization problem, the only nodes that can change their positions in order to minimize the structure mass are the nodes 3, 5, 7 and 9. The minimum cross sectional area of the bars is 3,5 in ² (0,00226 m²) and the maximum is 18 in² (0,01161 m²). The areas are classified into groups that depend on structure design, as follows: Group 1 - A1 = A4 = A8 = A12 = A16, Group 2 - A2 = A6 = A10 = A14 = A18, Group 3 - A3 = A7 = A11 = A15 Group 4 - A5 = A9 = A13 = A17, where the numbering of the areas corresponds to the numbering of the bars.

The secondary constraints are related only to strains in the bars. For tensile strains, the maximum allowable stress is 20 ksi (137,90 MPa). For compression, considering the buckling phenomena, the permissible maximum stress is given by:

$$\sigma_i = \frac{-K.E.A_i}{L_i^{\,2}} \tag{7}$$

Where σ_i is the Euler buckling stress, K is a constant determined by the geometry of the cross section of the bars (K = 4), E is the material modulus of elasticity, A_i is the cross-sectional area of the bars and L_i is the length of the bar.

The twelve variables to be optimized correspond to coordinates of the four nodes that can have changed their positions and areas of cross sections of four groups of bars. The results of the PSO algorithm compared with the other cases can be viewed in Tab. 4.1.

Table 4.1. Results of problem 4.2 compared with results of other authors.

Optimization Algorithm Used		HS	Multi-Level Approach	AG	AG	AG	AG	HS	PSO
		1) Imai and Schmit	2) Felix	3) Yang	4) Soh and Yang	5) Rajeev and Krishn	6) Yang and Soh	7) Lee and Geem	8) Present study
Cross-section areas	G1 (in²)	11,24	11,34	12,61	12,59	12,50	12,33	12,65	12,24
	G2 (in²)	15,68	19,28	18,10	17,91	16,25	17,97	17,22	18,00
	G3 (in²)	7,93	10,97	5,47	5,50	8,00	5,60	6,17	4,97
	G4 (in²)	6,49	5,30	3,54	3,55	4,00	3,66	3,55	3,74
Coordinates of the nodes	X3 (in)	891,10	994,60	914,50	909,80	891,90	907,20	903,10	914,94
	Y3 (in)	143,60	162,30	183,00	184,50	145,30	184,20	174,30	189,11
	X5 (in)	608,20	747,40	647,00	640,30	610,60	643,30	630,30	648,91
	Y5 (in)	105,40	102,90	147,40	147,80	118,20	149,20	136,30	151,31
	X7 (in)	381,70	482,90	414,20	410,00	385,40	413,90	402,10	418,46
	Y7 (in)	57,10	33,00	100,40	97,00	72,50	102,00	90,50	101,21
	X9 (in)	181,00	221,70	200,00	200,90	184,40	202,10	195,30	205,04
	Y9 (in)	-3,20	17,10	31,90	32,00	23,40	30,90	30,60	30,90
Structure Mass	M (lb)	4667,90	5713,00	4552,80	4531,90	4616,80	4520,00	4515,60	4491,98
	M (kg)	2117,30	2591,40	2065,10	2055,60	2094,10	2050,20	2048,20	2037,48

The Particle Swarm algorithm had a satisfactory performance in the optimization of this structure, reaching a value of 4491,98lb (2037,48kg). This result was better than all the other authors, as shown in Tab. 4.1. There was no constraint violation in the final result using PSO. Figure 4.2 shows the obtained geometry for the optmized structure.

4.3. 120 bars space truss – 7 design variables to be optimized

The second case study is a dome, a three-dimensional structure formed by trusses, as shown in Figure 4.2. Kaveh and Talatahari (2009) applied Heuristic Particle Swarm Ant Colony Optimization (HPSACO) in the optimization of this structure, work that will be used as reference for the application of PSO.

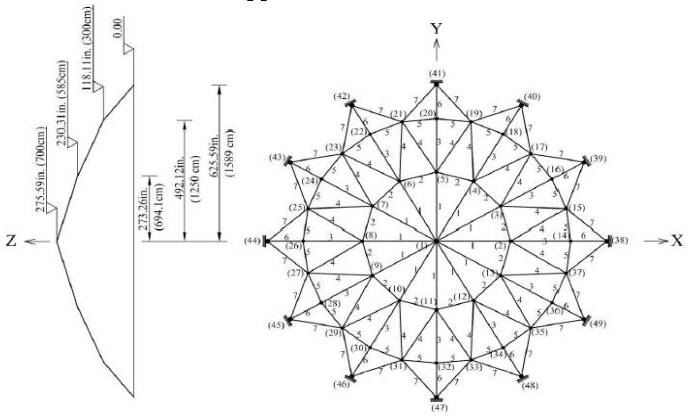


Figure 4.2. Structure to be optimized of the problem 4.3 (Source: KAVEH; TALATAHARI, 2009).

The structure consists of 120 bars and 49 nodes. The density of the material is 0,288 lb/in³ (7971,810 kg/m³) and the modulus of elasticity is 30450 ksi (210 GPa), which correspond to the properties of steel. The initial area of cross section of all the bars was 0,31 in ² (0,0002 m²). All the loads have the vertical direction and downward direction (negative direction of z axis): node 1, P_1 = -13,488,54 psi (-60000 N), node 2 to node 13, P_{2-13} = -6744,268 psi (-30000 N), the node 14 to node 37, P_{14-37} = -2248,089 psi (-10000 N).

Again, the objective function is the total mass of the structure. The design variables in this design optimization are referring only to the areas of cross sections of the bars. With regard to the primary problem constraints, the structure is three dimensional. Nodes placed at height zero (at z = 0m) were fixed with clamp labeled, so they do not move. In this problem, there are no nodes that can have changed their positions, because the only design variables are the areas of cross sections of the bars. The minimum cross sectional area of the bars is 0,775 in ² (0,0005 m²) and the maximum is 5 in ² (0,00323 m²). The areas are classified into groups depending on the position of the bars in the structure, as follows: 1 to 12 bars - A_1, bars 13 to 24 - A_2, 25 to 36 bars - A_3, 37 to 60 bars - A_4, bars 61 to 84 - A_5, 85 to 96 bars - and bars A_6 97-120 - A_7, where the numbering corresponds to numbering of the areas of design variables.

The secondary constraints are only the stresses in the bars. The maximum allowable tension stress is equal to 60% of yield stress considered to steel (58 ksi, equivalent to 400 MPa), in other words, equal to 34,8 ksi (240 MPa). For compression, considering buckling, the maximum allowable stress is given by:

$$\sigma_i = \frac{\pi^2.E}{\lambda_i^{\,2}} \qquad , if\ \lambda_i < C \tag{8}$$

$$\sigma_i = \frac{\sigma_y.\left(1 - \frac{\lambda_i^{\,2}}{2.C^2}\right)}{\frac{5}{3} + \frac{3.\lambda_i}{8.C} - \frac{\lambda_i^{\,3}}{8.C^3}} \qquad , if\ \lambda_i \geq C \tag{9}$$

$$\lambda_i = \frac{K.L_i}{r_i} \tag{10}$$

$$C = \sqrt{\frac{2.\pi^2.E}{\sigma_y}} \tag{12}$$

$$r_i = \sqrt{\frac{L_i^{\,2}}{12}} \tag{13}$$

Where σ_i is the buckling stress, K is a constant determined by the geometry of the bars and its fixing conditions (in this case K = 12/23), E is the material modulus of elasticity, λ_i is the slenderness ratio, σ_y is the yield stress of the material, C is the slenderness ratio, which divides the buckling in elastic or inelastic and ri is the radius of gyration of the bars. These equations were modified according to the literature of the dome's original case (SOH; YANG, 1996), because there were inconsistencies in the equations of reference publication.

The design variables to be optimized are seven groups of areas of the structure. In Tab. 4.2 the results are compared with the ones from other authors.

Table 4.2. Results of problem 4.3 compared with results of other authors.

Optimization algorithm used		HS	PSO	PSOPC	HPSACO	PSO
Author		1) Lee and Geem	2) Lee and Geem	3) Lee and Geem	4) Kaveh , Talatahari	5) Present Study
Cross-Section areas	A1 (in²)	3,295	3,147	3,235	3,311	4,058
	A2 (in²)	3,396	6,376	3,370	3,438	5,000
	A3 (in²)	3,874	5,957	4,116	4,147	4,282
	A4 (in²)	2,571	4,806	2,784	2,831	2,956
	A5 (in²)	1,150	0,775	0,777	0,775	0,310
	A6 (in²)	3,331	13,798	3,343	3,474	3,284
	A7 (in²)	2,784	2,452	2,454	2,551	3,042
Mass of the structure	M (lb)	19707,8	32432,9	19618,7	19491,3	21648,1
	M (kg)	8939,3	14711,3	8898,9	8841,1	9819,4

As can be seen in Tab. 4.2, the PSO algorithm used in this work reached a better result, just, than the other PSO used by the second author. The optimal value of the mass was 9819,4 kg, about 66% of the value obtained by the other author who also used PSO.

4.4. 37 bars plane truss – 19 design variables to be optimized.

A two-dimensional structure with lumped masses at its lower nodes not fixed, representing a bridge, should be optimized considering natural frequencies constraints. This optimization problem was solved by Wang, Zhang and Jiang (2004) through the Evolutionary Node Shift Method (ENSM) and by Lingyum et al (2005) by Novel Hybrid Genetic Algorithm (NHGA). This structure has 37 bars and 20 nodes. The density of the material is 7800 kg/m³ and the modulus of elasticity is 210 GPa each one of the concentrated masses has 10 pounds of weight.

Again, the cost function is the total mass of the structure. The design variables involved are the coordinates of the upper nodes and the nine cross-sectional areas of the bars (symmetric about the central vertical bar), totaling 19 variables. Figure 4.3 represents the structure to be optimized.

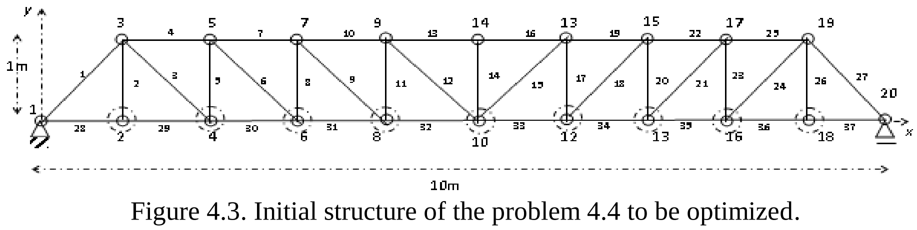


Figure 4.3. Initial structure of the problem 4.4 to be optimized.

With regard to the primary problem constraints, node 1 is prevented of moving horizontally and vertically, and the node 20 only vertically. The whole structure was modeled with bar elements, bars 1-27 with cross-sectional area starting from 1x10-4 m² and bars 28 to 37 with rectangular section (initial dimensions of 50x80 mm ²), so the initial area was 4x10-3 m². The higher nodes can have their vertical positions changed during the optimization.

The secondary constraints are related to natural frequencies of the structure. The first natural frequency must be $\omega_1 \geq 20$ Hz, the second $\omega_2 \geq 40$ Hz and the third $\omega_3 \geq 60$ Hz. Therefore, the considered problem has three frequency constraints and 19 design variables.

The initial configuration of the structure (Fig. 4.4) violates the constraints of frequency, therefore, the optimization should have a mass greater than the initial setup. The results for this problem are shown in Tab. 4.3.

From Tab. 4.3 it is noted that none of the constraints is violated. Nevertheless, the mass found by PSO algorithm (377,20 kg) was above the other resulting masses of other authors, overcoming, for example, at 2.9% the mass reached by Wang, Zhang and Jiang (2004). This difference may be due to numerical approximations of the FEM.

Table 4.3. Results of problem 4.4 compared with results of other authors.

Optimization method used		-	ENSM	NHGA	PSO	Optimization method used		-	ENSM	NHGA	PSO
Author		1) Initial configuration	2) Wang, Zhang and Jiang	3) Lingyum	4) Present Study	Author		1) Initial configuration	2) Wang, Zhang and Jiang	3) Lingyum	4) Present Study
Vertical Coordinates (m)	Y_3 , Y_{19}	1,000	1,209	1,200	0,964	Cross-section Areas (cm²)	A_9 , A_{18}	1,000	1,826	1,953	2,121
	Y_5 , Y_{17}	1,000	1,579	1,655	1,398		A_{10} , A_{19}	1,000	2,302	1,971	3,860
	Y_7 , Y_{15}	1,000	1,672	1,965	1,593		A_{11} , A_{17}	1,000	1,310	1,829	2,982
	Y_9 , Y_{13}	1,000	1,770	2,074	1,881		A_{12} , A_{15}	1,000	1,407	1,236	1,202
	Y_{11}	1,000	1,850	2,305	2,086		A_{13} , A_{16}	1,000	2,190	1,405	1,256
Cross-section Areas (cm²)	A_1 , A_{27}	1,000	3,251	2,893	2,680		A_{14}	1,000	1,000	1,000	3,328
	A_2 , A_{26}	1,000	1,236	1,120	1,157	Mass	M (kg)	336,30	366,50	368,84	377,20
	A_3 , A_{24}	1,000	1,000	1,000	2,348	Natural Frequencies (Hz)	ω1	8,890	20,085	20,001	20,000
	A_4 , A_{25}	1,000	2,539	1,866	1,718		ω2	28,820	42,074	40,031	40,000
	A_5 , A_{23}	1,000	1,371	1,596	1,275		ω3	46,920	62,938	60,000	60,000
	A_6 , A_{21}	1,000	1,368	1,264	1,482		ω4	63,620	74,454	73,044	73,044
	A_7 , A_{22}	1,000	2,429	1,825	4,685		ω5	76,870	90,058	89,824	89,824
	A_8 , A_{20}	1,000	1,652	2,001	1,125	-	-	-	-	-	-

Figure 4.5 shows a comparison between the optimized structures of this problem obtained by the authors.

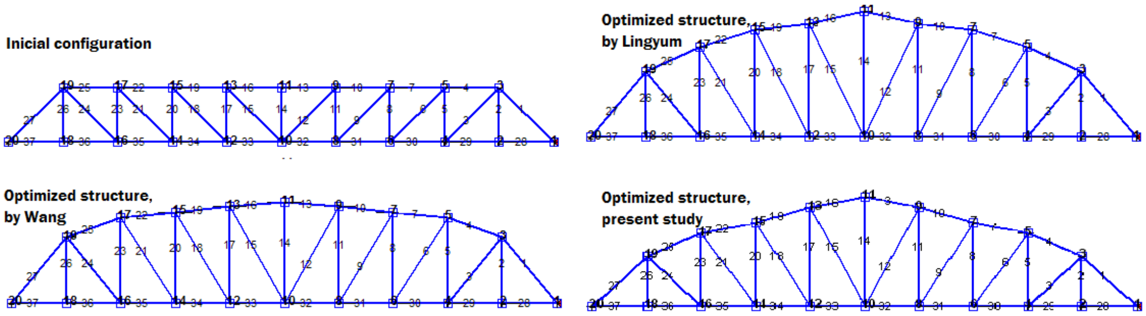


Figure 4.4. Comparison between the optimized structures.

5. CONCLUSION

The Particle Swarm algorithm performance was satisfactory for the optimization problems studied in this paper. In particular, the first problem, the truss structure of 18 bars, which design variables were the cross-sectional areas of the truss bars and the positions of the nodes, with stress constraints, the algorithm achieved excellent results of the minimization of mass, and even it was better than some authors. In the second case, the dome problem, truss structure of 120 bars, with areas of cross sections of the bars as design variables and constraints of stress, the value that was reached for the mass was better than the other author which also applied PSO algorithm optimization. In the third and case, truss structure of 37 bars, 19 design variables considering areas of cross section and position of the nodes, with frequency constraints, the results achieved for the minimization of the masses are slightly higher than the results obtained by other authors who previously worked with these problems. The reason for this difference may be associated with numerical approximations of the finite element method, observing that the result said optimal by others authors, when analyzed at this work, results in frequency constraint violation. Another possibility is a deficiency of the optimization algorithm PSO.

In structural optimization, some algorithms may perform better than others when applied to specific problems. It is known, for example, that genetic algorithms are better to solve problems that have integer variables than algorithms based on gradients. So, it is evident that it would be a mistake try to justify with just a few examples that, in general, the PSO is better or worse than the other optimization methods. Some types of constraints hamper the optimization process. Natural frequencies, for example, represent strongly nonlinear functions, because, as it optimizes the structure mass, the values of stiffness and mass are changed, modifying, consequently, the natural frequencies and vibration modes of the structure, hindering the convergence of the algorithm.

For future work would be of great importance to do a study for the convergence of the PSO algorithm by varying its parameters. Another study that could be done is the implementation of parallelism in computers, this feature is allowed by the PSO algorithm and it is not possible with many other optimization algorithms. Furthermore, it could be further examined the cases where frequency constraints have been violated for the optimized structures by other authors. What could be done is to implement techniques that reduce or avoid the numerical errors in the algorithm of structural analysis, as well as the research of which process was used by these authors to do this analysis on these problems. Finally, another point where it could improve the optimization system is the implementation of other optimization algorithms on specific points on the Particle Swarm programming, for example, Kaveh and Talatahari (2009) who applied Heuristic Particle Swarm Ant Colony Optimization (HPSACO), a mixture of two algorithms exploiting specific characteristics of each in specific parts of the programming (a hybrid algorithm).

6. REFERENCES

BASTOS, E. A., 2004, “Otimização de Seções Retangulares de Concreto Armado Submetidas à Flexo-compressão Oblíqua utilizando Algoritmos Genéticos”, Dissertação de Mestrado, Universidade Federal do Rio de Janeiro, Coordenação dos Programas de Pós-graduação em Engenharia (in Portuguese).

BERGH, F. van der., 2001, An Analysis of Particle Swarm Optimizers, PhD thesis, University of Pretoria, Faculty of Natural and Agricultural Science.

BOYD, R.; RICHERSON P. J., 1985, “Culture and the Evolutionary Process”. Chicago: University of Chicago Press.

CHRISTODOULOU, S., 2005, “Optimal Truss Design using Ant Colony Optimization”. Limassol: 5th GRACM International Congress on Computational Mechanics.

COLORNI , A.; DORIGO, M.; MANIEZZO, V., 1991, “Distributed Optimization by Ant Colonies”. Cambridge, MA: Proceedings of First European Conference on artificial Life (MIT Press). p. 134–142.

DEB, K.; GULATI, S., 2001, “Design of truss-structures for minimum weight using genetic algorithms”. Finite Elements in Analysis and Design, Vol. 37;

FUKUYAMA, Y.; KITAGAWA, S.; TAMURA, T.; TSUKADA, T., 2004, “Particle Swarm Optimization for Optimal Operational Planning of a Cogeneration System”. Proceedings of the 4th IASTED International Conference on Modeling, Simulation, and Optimization (MSO2004), ACTA Press.

GRANDHI, R. V. , 1993, “Structural optimization with frequency constraints – a review”, AIAA Journal, Vol. 31, No. 12, pp.2296-2303.

KAVEH, A.; TALATAHARI, S., 2009, “Particle Optimizer, Ant Colony Strategy and Harmony Search Scheme Hybridized for Optimization of Truss Structures”. Computers and Structures 87, p. 274 a 280.

KENNEDY, J.; EBERHART, R. C., 1995 “Particle Swarm Optimization”. In: Proceedings of the IEEE Int. Conference on Neural Networks.

KONZELMAN, C. J., 1986, “Dual Methods and Approximation Concepts for Structural Optimization”, M.Sc. Thesis, Department of Mechanical Engineering, University of Toronto.

LEE, K. S.; GEEM, Z. W., 2005, “A New Meta-heuristic Algorithm for Continuous Engineering Optimization: Harmony Search Theory and Practice”. Computer methods in applied mechanics and engineering, Elsevier.

LI, L. J.; HUANG, Z. B.; LIU, F., 2009, “A Heuristic Particle Swarm Optimization Method for Truss Structures with Discrete Variables”. Computers and Structure 87.

LINGYUN, W.; MEI, Z.; GUANGMING, W.; GUANG, M., 2005, “Truss optimization on shape and sizing with frequency constraints based on genetic algorithm”, Journal of Computational Mechanics, Vol. 25, pp.361-368.

MEI, C.; LIU, G.; XIAO, X., 2010, “Improved particle swarm optimization algorithm and its global convergence analysis”, Chinese Control and Decision Conference, pp.1662-1667.

PARK, J.; RYU, M., 2004, “Optimal Design of Truss Structures by Rescaled Simulated Annealing”. KSME International Journal, Vol. 18 No. 9, p. 1512-1518.

REYNOLDS, C., 1987, “Flocks, Herds, and Schools: A Distributed Behavioral Model”. Computer Graphics, pp. 25-34.

SEDAGHATI, R.; SULEMAN, A.; TABARROK, B., 2002, “Structural optimization with frequency constraints using finite element force method”. AIAA Journal, Vol.40, No.2, pp.382-388.

SOH, C. K.; YANG, J., 1996, “Fuzzy Controlled Genetic Algorithm Search for Shape Optimization”. Journal of computing in civil engineering.

SCHUTTE, J. F., GROENWOLD, A. A., 2003, Sizing design of truss structures using particle swarms, Journal of Structural and Multidisciplinary Optimization, No.25, pp.261-269.

WANG, D.; ZHANG, W. H.; JIANG, J. S., 2004, “Truss optimization on shape and sizing with frequency constraints”, AIAA Journal, Vol.42, No.3, pp.1452-1456.

7. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.