HYPERFIT – CURVE FITTING SOFTWARE FOR INCOMPRESSIBLE HYPERELASTIC MATERIAL MODELS

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Abstract. *Constitutive models for hyperelastic materials, such as rubber, foams and certain biological tissues, are usually defined in terms of several constants. By means of a non-linear curve fitting procedure, stress-strain test data can be used to evaluate those constants for a given model. Routines devoted to this task exist on all major finite element software packages. Nevertheless, in most cases the user has little or no control over the curve fitting process and cannot define the optimization parameters used by the software. Furthermore, those functionalities are often restricted to few pre-selected hyperelastic models and there is no direct way of comparing the results yielded by different modeling choices. Selecting the model that best fits a particular test curve can be a challenge. HyperFit is a research code developed to help minimizing these difficulties. Non-linear least squares methods are applied to match theoretical curves with user provided experimental data, resulting in optimized material constants. The code allows multi-criteria optimization, so that the analyst can optimize the constants against more than one type of experimental data. In addition, an interface was developed to simplify the calibration and comparison of several different hyperelastic models, easing the task of selecting the best model for a given set of experimental data. An example of application and usage is presented and discussed.*

Keywords*: hyperelasticity, constitutive models, curve fitting*

1. INTRODUCTION

The analysis of rubber components is a difficult task due to the material's intrinsic non-linear elastic behavior. Finite element simulations are widely used to help designing such components. In this context, the choice of a hyperelastic constitutive model defines the material behavior and is of capital importance.

Hyperelasticity is used to model the mechanical behavior of rubber, foams and many biological tissues. Traditionally, the constitutive models are expressed in terms of a strain energy function that depends on the principal stretches or invariants of the strain tensor. This function is directly linked to the material's stress-strain relationship and depends on a series of parameters (*material constants*). In order to determine those constants, experimental data obtained from simple tests are fitted to a given model's theoretical behavior.

On all major commercial finite element software, there are optimization routines that can be used to find hyperelastic material constants. Regardless, in most cases the user cannot change some optimization parameters and there are no direct means of comparing different models' performances. Since there exist dozens of proposed hyperelastic models in the literature, each one having their advantages and limitations, finding out which one best matches some experimental data is not a straightforward task.

HyperFit is a software package developed under Matlab® whose objective is simplifying the choice of an hyperelastic model and the determination of its constants for a particular material. It consists on a number of routines that apply non-linear least squares methods on experimental data to find optimized constants for a given model. Besides the standard curve fitting procedure, where the results of a single test are used to determine constants, HyperFit offers the possibility of multi-criteria optimization, meaning that different test results can be simultaneously used for optimization. Stumpf (2009) has shown that this approach gives better curve fittings in certain situations. HyperFit also counts with post-processing tools that aid the user to objectively determine the fit quality of regressions, allowing direct comparison between models. Those functionalities make it a research platform that is a useful tool for the analysis of hyperelastic materials.

2. INCOMPRESSIBLE HYPERELASTICITY FUNDAMENTALS

2.1. Strain energy function

The stress-strain behavior of a hyperelastic material can be expressed using a strain energy function W (Marczak *et al.*, 2006). Typically, W is defined in terms of three strain invariants I_1, I_2 and I_3. For incompressible materials, the I_3 dependency is eliminated, so that I_1 and I_2 are given by:

$$I_1 = \lambda_1^2 + \lambda_2^2 + \frac{1}{\lambda_1^2 \lambda_2^2} \qquad I_2 = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} + \lambda_1^2 \lambda_2^2 \tag{1}$$

where λ_1 and λ_2 are the principal stretches. The incompressibility assumption is acceptable for a wide range of rubbery materials and is currently the only possibility offered by HyperFit.

A hyperelastic model consists on a particular form of strain energy function depending on arbitrary constants. To describe a particular material behavior, these constants must be determined by means of curve fitting to experimental data. In most hyperelastic models the constants have no direct physical meaning, relating only to the curve fitting process. Examples of traditional models and their corresponding strain energy functions can be seen in the list of models implemented in HyperFit, given in Section 3.1.

2.2. Homogeneous deformation cases

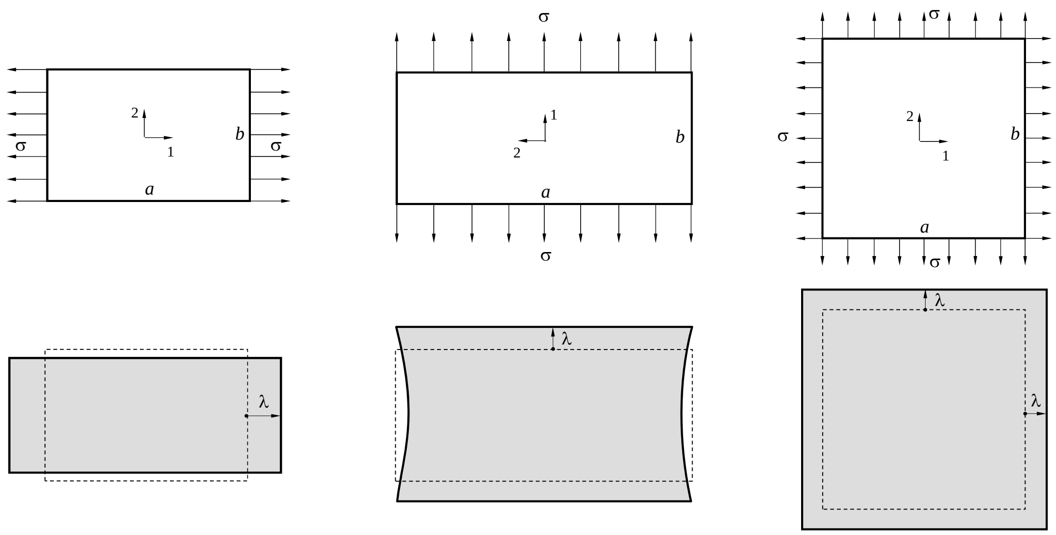
Data necessary for the aforementioned curve fitting are originated from experiments replicating homogeneous incompressible deformation cases, such as simple tension, compression, pure shear, simple shear and equibiaxial tension. For these tests, there is a simple analytical relationship between stress and strain, as shown below. For details on the derivation of those expressions, see Marczak *et al.*, 2006.

The simple tensile test of a sample is depicted in Fig. 1(a). In this configuration, stress can be related to stretch and the strain energy function in the following manner:

$$t = 2\left(\lambda - \frac{1}{\lambda^2}\right)\left(\frac{\partial W}{\partial I_1} + \frac{1}{\lambda}\frac{\partial W}{\partial I_2}\right) \tag{2}$$

where t denotes the first Piola-Kirchhoff stress (*engineering stress* – related to the initial sample cross-section area). Using $\lambda_1 = \lambda$ and $\lambda_2 = \lambda_3 = 1/\sqrt{\lambda}$ (see Fig. 1(a)), I_1 and I_2 can be calculated using Eqs. (1).

The compression test generates analogous results, resulting in exactly the same relationship between stress and strain (however, stress values are different since λ assumes negative values).



(a) (b) (c)

Figure 1. Homogeneous deformations of an incompressible material: (a) simple tensile test, (b) pure shear test, (c) equibiaxial tensile test

Figure 1(b) depicts the stresses occurring on the pure shear test of an incompressible material, along with its deformation pattern. The stress-strain relation in this case is given by:

$$t = 2\left(\lambda - \frac{1}{\lambda^3}\right)\left(\frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2}\right) \tag{3}$$

where using $\lambda_1 = \lambda$, $\lambda_2 = 1/\lambda$ and $\lambda_3 = 1$, I_1 and I_2 can be directly calculated with Eqs. (1).

It can be shown that simple shear and pure shear tests represent the same deformation mode, so their test results can be directly transposed from one form to the other.

The equibiaxial tensile test configuration illustrated in Fig. 1(c) results in the following stress-strain relationship:

$$t = 2\left(\lambda - \frac{1}{\lambda^5}\right)\left(\frac{\partial W}{\partial I_1} + \lambda^2 \frac{\partial W}{\partial I_2}\right) \tag{4}$$

where using $\lambda_1 = \lambda_2 = \lambda$ and $\lambda_3 = 1/\lambda^2$, I_1 and I_2 can be calculated with Eqs. (1).

3. HYPERFIT

HyperFit's core procedures are outlined in Fig. 2. The software has an interface where the user must choose which hyperelastic models to consider in the analysis. An optimization routine must then be selected: non-linear least squares, multi-criteria optimization or compromise optimization (explained on detail in Section 3.2). The subsequent steps are executed for each model considered in the analysis in order to obtain optimized material constants.

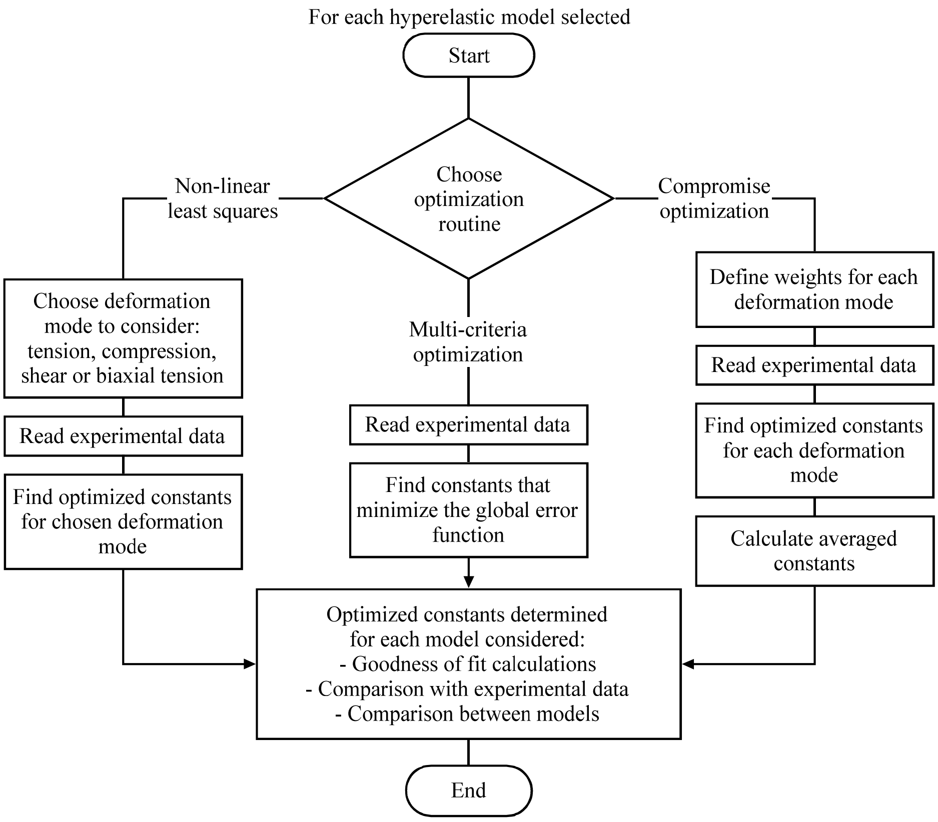


Figure 2. HyperFit's main flowchart.

With the material constants sets determined, the user has at his disposal some post-processing tools that indicate the different models' performances. The goodness of fit estimator detailed in Section 3.3 is calculated for each model, allowing the user to objectively determine which model better predicts each deformation mode. Theoretical *versus* experimental curve plots further ease the comparison between hyperelastic models for a particular curve fit.

3.1. Supported strain energy functions

HyperFit comes with 42 incompressible hyperelastic models, ranging from well-established classics (such as the Ogden models) to modern approaches. Those models are listed on Tab. 1 along with their strain energy functions. The variables in those equations other than I_1, I_2, λ_1, λ_2 and λ_3 are the material constants of the corresponding hyperelastic model. For more information and the original references, the reader is referred to Hoss (2009).

It is possible to add strain energy functions to HyperFit, should the user wish to work with other hyperelastic models. In order to do that, it is necessary to define in Matlab the derivatives $\partial W/\partial I_1$ and $\partial W/\partial I_2$, used in Eqs. (2), (3) and (4) to calculate the theoretical stresses (GMAp, 2011).

3.2. Optimization routines

In order to obtain the constants for modeling a given material, HyperFit offers three main optimization routines, described on the next subsections.

3.2.1. Non-linear least squares

In this optimization routine, data from only one test type is used: simple tension/compression, pure shear or equibiaxial tension. The experimental values of the strains are used in conjunction with the chosen hyperelastic model's strain energy function to obtain theoretical values for the stress (using Eqs. (2), (3) or (4), according to the type of experimental results used).

HyperFit retains a certain number of points among the given stress-strain data and applies a non-linear least squares method using the Matlab function *lsqcurvefit* in order to determine the hyperelastic model's constants (Mathworks, 2010a). The constants that make the difference between the theoretical curve and the test data (in the least squares sense) less than an user-defined tolerance are taken as the optimized constants.

3.2.2. Compromise optimization

This optimization technique, proposed by Hoss (2009), relies on a linear combination using constants from different optimizations. In fact, three vectors of constants are obtained from calibrations using data from each deformation mode individually (using the aforementioned non-linear least squares method): simple tension constants ($\mathbf{C_T}$), pure shear constants ($\mathbf{C_P}$) and equibiaxial tension constants ($\mathbf{C_B}$). Afterwards, a new vector of constitutive constants $\overline{\mathbf{C}}$ is obtained from the previous results:

$$\overline{\mathbf{C}} = w_T\mathbf{C_T} + w_P\mathbf{C_P} + w_B\mathbf{C_B} \quad (5)$$

where w_T, w_P and w_B represent the user-defined weights for each constant vector. The user can fine-tune the relative importance of each deformation mode on the results, possibly improving the global quality of the curve fit. This procedure can be advantageous if the analyst does not have full confidence on certain data, e.g. the results of a tensile test, but does not want to completely discard it. In this case, a relatively small weight should be assigned to these results.

3.2.3. Multi-criteria optimization

Even though the non-linear least squares method described in Section 3.2.1 generally gives reasonable curve fits for the main deformation mode (the one from which the experimental data comes), Hoss (2009) and Stumpf (2009) have shown that the predictions for other deformation modes can sometimes be very poor.

With that in mind, Stumpf (2009) proposed an optimization method where the constants are determined using data from all (available) tests *simultaneously* using a multi-criteria cost function. This is done by defining an error function for each type of test whose data is to be used: e_T, e_P and e_B. The errors are taken as the squared difference between experimental and theoretical values of stress. The global error function E is then simply the sum of the error functions of each individual type of test considered:

Table 1. Incompressible hyperelastic strain energy functions currently implemented in HyperFit.

	Hyperelastic model	Incompressible strain energy function
1	2 term Mooney-Rivlin	$W = C_{10}(I_1-3)+C_{01}(I_2-3)$
2	3 term Mooney-Rivlin	$W = C_{10}(I_1-3)+C_{01}(I_2-3)+C_{11}(I_1-3)(I_2-3)$
3	5 term Mooney-Rivlin	$W = C_{10}(I_1-3)+C_{01}(I_2-3)+C_{20}(I_1-3)^2+C_{11}(I_1-3)(I_2-3)$
4	9 term Mooney-Rivlin	$W = C_{10}(I_1-3)+C_{01}(I_2-3)+C_{20}(I_1-3)^2+$ $+C_{30}(I_1-3)^3+C_{11}(I_1-3)(I_2-3)$
5	Third degree polynomial	$W = C_{10}(I_1-3)+C_{01}(I_2-3)+C_{11}(I_1-3)(I_2-3)+C_{20}(I_1-3)^2$ $+C_{02}(I_2-3)^2+C_{21}(I_1-3)^2(I_2-3)+C_{12}(I_1-3)(I_2-3)^2$
6	Neo-Hookean	$W = \frac{\mu}{2}(I_1-3)$
7	2 term Yeoh	$W = C_{10}(I_1-3)+C_{20}(I_1-3)^2$
8	3 term Yeoh	$W = C_{10}(I_1-3)+C_{20}(I_1-3)^2+C_{30}(I_1-3)^3$
9	5 term Yeoh	$W = C_{10}(I_1-3)+C_{20}(I_1-3)^2+C_{30}(I_1-3)^3+C_{40}(I_1-3)^4+C_{50}(I_1-3)^5$
10	5 term Arruda-Boyce	$W = \mu\sum_{i=1}^{5}\frac{c_i}{\lambda_L^{2i-2}}(I_1^i-3^i)$, $c_1=\frac{1}{2}$, $c_2=\frac{1}{20}$, $c_3=\frac{11}{1050}$, $c_4=\frac{19}{7050}$, $c_5=\frac{519}{673750}$
11	Gent	$W = -\frac{\mu}{2}(I_L-3)\ln\left(1-\frac{I_1-3}{I_L-3}\right)$
12	2 term Ogden	$W = \sum_{i=1}^{2}\frac{\mu_i}{\alpha_i}\left(\lambda_1^{\alpha_i}+\lambda_2^{\alpha_i}+\lambda_3^{\alpha_i}-3\right)$
13	3 term Ogden	$W = \sum_{i=1}^{3}\frac{\mu_i}{\alpha_i}\left(\lambda_1^{\alpha_i}+\lambda_2^{\alpha_i}+\lambda_3^{\alpha_i}-3\right)$
14	Veronda-Westmann	$W = C_1\left(e^{\alpha(I_1-3)}-1\right)-C_2(I_2-3)$
15	Humphrey-Yin	$W = C_1\left(e^{C_2(I_1-3)}-1\right)$
16	Hart-Smith	$W = \frac{C_1 e^{C_3(I_1-3)^2}}{2}+3C_2\ln I_2$
17	Peng-Landel	$W = C_1\sum_{i=1}^{3}\left[\lambda_i-1-\ln\lambda_i-\frac{1}{6}(\ln\lambda_i)^2+\frac{1}{18}(\ln\lambda_i)^3-\frac{1}{216}(\ln\lambda_i)^4\right]$
18	Knowles	$W = \frac{\mu}{2b}\left[\left(1+\frac{b(I_1-3)}{n}\right)^n-1\right]$
19	Martins	$W = C_1\left(e^{C_2(I_1-3)}-1\right)+C_3\left(e^{C_4(\lambda_f-1)^2}-1\right)$
20	Pucci-Saccomandi	$W = -\frac{1}{2}\mu J_L\ln\left(1-\frac{(I_1-3)}{J_L}\right)+C_2\ln\left(\frac{1}{3}I_2\right)$
21	Gregory	$W = \frac{A}{2-n}\left(I_1-3+C^2\right)^{1-n/2}+\frac{B}{2+m}\left(I_1-3+C^2\right)^{1+m/2}$
22	Edwards-Vilgis	$W = \frac{\mu}{2}\left[\frac{(J_L+2)(J_L-3)(I_1-3)}{J_L(J_L-I_1+3)}+\ln\left(1-\frac{I_1-3}{J_L}\right)\right]$
23	David De-Thomas	$W = \frac{A}{2-n}\left(I_1-3+C^2\right)^{1-n/2}+k(I_1-3)^2$

Table 1 (continued). Incompressible hyperelastic strain energy functions currently implemented in HyperFit.

Hyperelastic model		Incompressible strain energy function
24	Gent-Thomas	$W = C_1\left(I_1-3\right)+3C_2\ln I_2$
25	Yeoh-Fleming	$W = \frac{A}{B}\left(1-e^{-B(I_1-3)}\right)-C_{10}\left(I_L-3\right)\ln\left(1-\frac{I_1-3}{I_L-3}\right)$
26	4 term H. Bechir *et al.*	$W = \sum_{n=1}^{2}\sum_{r=1}^{2}C_n^r\left(\lambda_1^{2n}+\lambda_2^{2n}+\lambda_3^{2n}-3\right)^r$
27	6 term H. Bechir *et al.*	$W = \sum_{n=1}^{3}\sum_{r=1}^{2}C_n^r\left(\lambda_1^{2n}+\lambda_2^{2n}+\lambda_3^{2n}-3\right)^r$
28	3 term Hartmann-Neff	$W = \alpha\left(I_1^3-3^3\right)+C_{10}\left(I_1-3\right)+C_{01}\left(I_2^{3/2}-3\sqrt{3}\right)$
29	5 term Hartmann-Neff	$W = \alpha\left(I_1^3-3^3\right)+C_{10}\left(I_1-3\right)+C_{01}\left(I_2^{3/2}-3\sqrt{3}\right)+C_{20}\left(I_1-3\right)^2+C_{02}\left(I_2^{3/2}-3\sqrt{3}\right)^2$
30	7 term Hartmann-Neff	$W = \alpha\left(I_1^3-3^3\right)+C_{10}\left(I_1-3\right)+C_{01}\left(I_2^{3/2}-3\sqrt{3}\right)+C_{20}\left(I_1-3\right)^2+$ $+C_{02}\left(I_2^{3/2}-3\sqrt{3}\right)^2+C_{30}\left(I_1-3\right)^3+C_{03}\left(I_2^{3/2}-3\sqrt{3}\right)^3$
31	Modified Yeoh	$W = C_{10}\left(I_1-3\right)+C_{20}\left(I_1-3\right)^2+C_{30}\left(I_1-3\right)^3+\frac{\alpha}{\beta}\left(1-e^{-\beta(I_1-3)}\right)$
32	Van Der Waals	$W = \mu\left[-\left(\lambda_m^2-3\right)\left[\ln\left(1-\eta\right)+\eta\right]-\frac{2a}{3}\left(\frac{I_1-3}{2}\right)^{3/2}\right],\ \ \eta=\sqrt{\frac{\left(1-\beta\right)I_1+\beta I_2-3}{\lambda_m^2-3}}$
33	Fung	$W = \frac{\mu}{2b}\left(e^{b(I_1-3)}-1\right)$
34	Horgan-Saccomandi	$W = -\frac{\mu}{2}J_L\ln\left(\frac{J_L^3-J_L^2I_1+J_LI_2-1}{\left(J_L-1\right)^3}\right)$
35	Kilian	$W = -\mu J_L\left[\ln\left(1-\sqrt{\frac{I_1-3}{J_L}}\right)+\sqrt{\frac{I_1-3}{J_L}}\right]$
36	3 parameter Gent	$W = \frac{\mu}{2}\left[-\alpha\left(I_L-3\right)\ln\left(1-\frac{I_1-3}{I_L-3}\right)+\left(1-\alpha\right)\left(I_2-3\right)\right]$
37	Low strain Hoss-Marczak	$W = \frac{\alpha}{\beta}\left(1-e^{-\beta(I_1-3)}\right)+\frac{\mu}{2b}\left[\left(1+\frac{b\left(I_1-3\right)}{n}\right)^n-1\right]$
38	High strain Hoss-Marczak	$W = \frac{\alpha}{\beta}\left(1-e^{-\beta(I_1-3)}\right)+\frac{\mu}{2b}\left[\left(1+\frac{b\left(I_1-3\right)}{n}\right)^n-1\right]+C_2\ln\left(\frac{1}{3}I_2\right)$
39	Improved Hart-Smith	$W = \frac{C_1e^{C_3(I_1-3)^n}}{n}+3C_2\ln\left(I_2\right)$
40	Takamizawa-Hayashi	$W = -c\ln\left[1-\left(\frac{I_1-2}{J_L}\right)^2\right]$
41	Yamashita-Kawabata	$W = C_{10}\left(I_1-3\right)+\frac{C_3}{N+1}\left(I_1-3\right)^{N+1}$
42	Amin	$W = C_{10}\left(I_1-3\right)+\frac{C_3}{N+1}\left(I_1-3\right)^{N+1}+\frac{C_4}{M+1}\left(I_1-3\right)^{M+1}$

$$E(\mathbf{C}) = e_T + e_P + e_B = \sum_{i=1}^{n_T}\left(t_{T_i} - t_{E_i}\right)^2 + \sum_{j=1}^{n_P}\left(t_{T_j} - t_{E_j}\right)^2 + \sum_{i=1}^{n_B}\left(t_{T_k} - t_{E_k}\right)^2 \tag{6}$$

where n_T, n_P and n_B are the number of experimental points considered in single tension, pure shear and equibiaxial tension, respectively, t_T are the theoretical values of the stresses (originating from the model's strain energy function) and t_E are the experimental values of the stresses. Thus, the optimization problem can be otherwise stated as finding the material constants vector $\mathbf{C}$ that minimizes the error function E. The solution is considered an optimum when its global error is smaller than an user-defined tolerance.

This methodology is implemented in Matlab using the function *fminsearch* (Mathworks, 2010b), based on the Nelder-Mead minimization method (Stumpf, 2009).

3.3. Goodness of fit estimator

Measuring fit quality means evaluating differences between theoretical and experimental stress-strain curves. This may be achieved by simple visual inspection, making it a subjective and somewhat unreliable task. When choosing a hyperelastic model among the dozens available, the analyst usually lacks a direct, objective mean of comparing their performance in a particular curve fit.

It may seem natural to calculate the relative errors E_{r_i} between the theoretical and the experimental curves along the whole deformation range of the curve fit:

$$E_{r_i} = \frac{\left|t_{e_i} - t_{t_i}\right|}{t_{e_i}} \text{ for } i = 1, 2, \ldots, n \tag{7}$$

where t_{e_i} and t_{t_i} are the experimental and numerical stress values, respectively, and n is the number of experimental data points. However, this measure does not take into account the relative magnitudes of the stresses along the deformation range, tending to exaggerate the errors on the low strain region of the curves. Moreover, working with a set of error values (one for each point in the fitted curve) is cumbersome when dealing with multiple models.

A simple correlation coefficient proves to be a more useful parameter to estimate fitting quality. Nevertheless, the classical correlation coefficient (Pearson's correlation coefficient) is unsuitable for nonlinear regressions (Press *et al.*, 1992). Since there is no general equivalent to Pearson's coefficient in nonlinear correlations, Hoss (2009) devised a specific non-linear goodness of fit estimator for hyperelastic curve fitting:

$$R_{NL} = 1 - \frac{S_{reg}}{S_{Stot}} \qquad S_{reg} = \sum_{i=1}^{n}\left(t_{e_i} - t_{t_i}\right)^2 \qquad S_{Stot} = \sum_{i=1}^{n}\left(t_{e_i} - \overline{t_e}\right)^2 \tag{8}$$

where $\overline{t_e}$ is the mean stress value of the experimental data. This means that R_{NL} is calculated using the sum of the squared differences between fitted and experimental stresses (S_{reg}) normalized by the sum of the squared differences between experimental stresses and their mean value (S_{Stot}). The better the curve fits, the smaller this ratio becomes and R_{NL} approaches unity. Unlike the Pearson's coefficient, it is possible for R_{NL} to have negative values for particularly bad curve fittings. The fact that this coefficient is a function of the mean stress value makes it strongly dependent on the deformation range intended. This is a welcome characteristic, since a visually good overall fitting for a large deformation range may hide locally inaccurate fittings for specific values of deformation. The proposed R_{NL} estimator will capture these deviations, unlike Pearson's coefficient.

In order to assess the quality of its fittings, HyperFit calculates multiple values of R_{NL}. The deformation range where the curve fit is applied is divided into parts of increasing size (from 0 to 50%, from 0 to 100% and so on). For each interval, HyperFit runs the optimization process finding a temporary set of material constants (valid for the current range) and calculating the corresponding value of R_{NL} specifically for this range. This is repeated until the whole deformation range of the original curve fit is covered. The value of R_{NL} retained by HyperFit is the smallest one from all the values previously calculated.

To justify this procedure, suppose that one tries to analyze a certain rubber component known to stretch up to 500% under operation. Naturally, the analyst does a curve fitting using experimental strain values ranging from 0 to 500%, obtaining a good curve fit for a certain hyperelastic model (meaning that R_{NL} is near unity). In spite of that, this model may lead to bad fitting for strain ranges, say, between 100% and 250%. The near unity value of R_{NL} for the whole

500% strain range would mislead the analyst into assuming that this model successfully captures the global material behavior. However, when HyperFit calculates the multiple values of R_{NL} and retains only the smallest of them, it is evident that this model is not a good choice.

4. CASE STUDY

In order to illustrate the different optimization procedures and fit quality measures exposed above, an elementary case study is developed in this section. Data from Jones and Treloar (1975) from simple tension, pure shear and equibiaxial tension tests is analyzed. Suppose that the analyst is interested in finding the hyperelastic model among the following ones that better fits this data (the numbers refer to Tab. 1): 3 term Mooney-Rivlin (2), 3 term Yeoh (8), 3 term Ogden (13) and Pucci-Saccomandi (20).

The results of a curve fit on the 0 to 350% strain range using the multi-criteria optimization method presented in Section 3.2.3 are shown in Figure 3. The goodness of fit estimators R_{NL} are shown for each test type and for each model selected in the analysis.

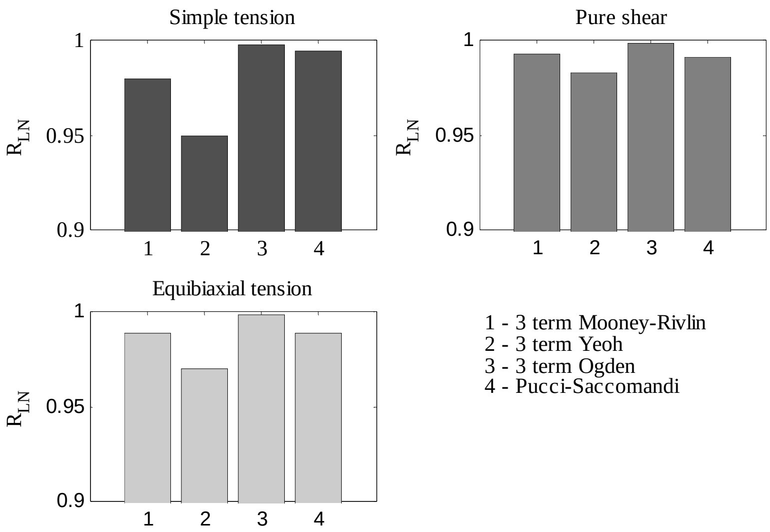


Figure 3. Goodness of fit estimators for the four case study models

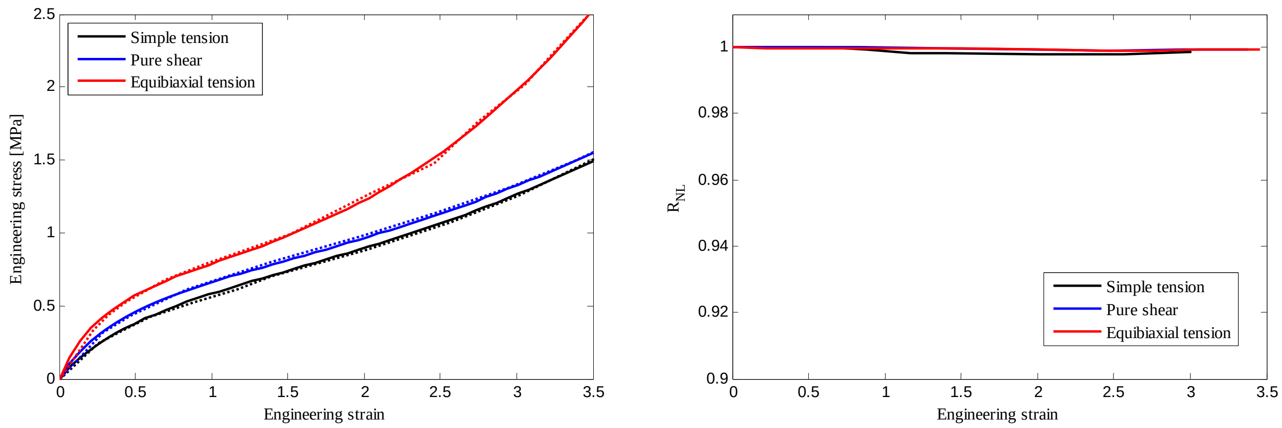
The values of R_{NL} are relatively close to unity in all cases but that does not necessarily means that all curve fittings are satisfactory. To illustrate this, the results yielded by the 3 term Ogden model and by the 3 term Yeoh model are shown in Figs. 4 and 5, respectively, along with the corresponding values of R_{NL} as a function of the strain range. The dotted lines represent experimental data while the continuous lines are the theoretical stress-strain curves obtained from the material constants found by HyperFit. While the difference of 0,035 between the best and the worst values of R_{NL} found in this analysis may seem irrelevant, a careful inspection of Figs. 4(a) and 5(a) shows that it is in fact very significant. To avoid misjudging, the vertical scale of the R_{NL} plots was adjusted accordingly.

The 3 term Ogden model clearly allows very good fits for all three test types curves while the 3 term Yeoh model has the worst performance in all cases. For both models R_{NL} is almost unity in low strain ranges, but for the Yeoh model its value lowers considerably as the strain range increases. It is clear that R_{NL} greatly simplifies the choice of the most convenient model for a particular material.

Globally, the multi-criteria optimization produced very good results for most models in every test type. To illustrate the difference between both optimization techniques, the results of a non-linear least squares optimization to the same data using the 3 term Ogden model are shown in Fig. 6. In Fig. 6(a), the optimization is made using only the simple tension data, while in Fig. 6(b), it is made using the equibiaxial tension data.

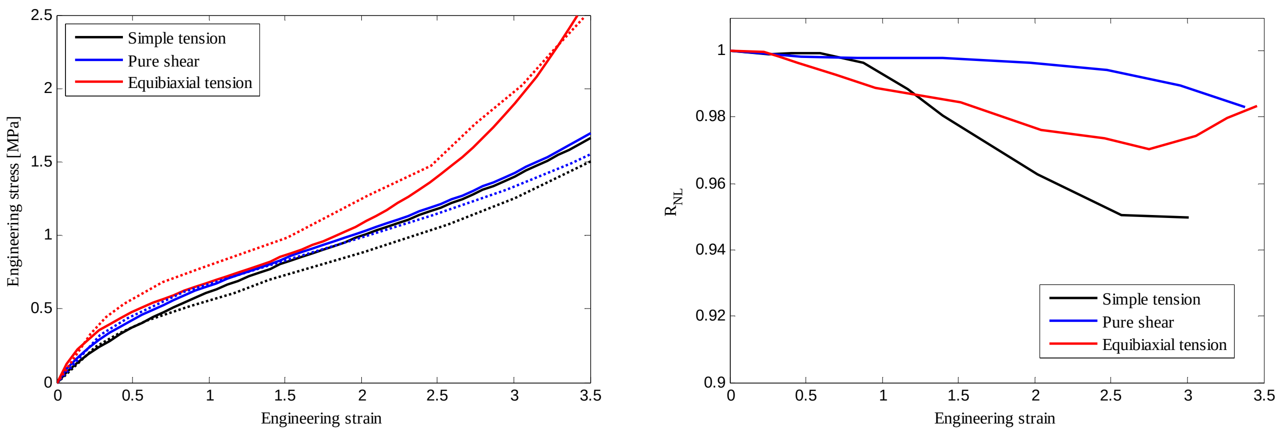
The fit quality is extremely high for the curve corresponding to the experimental data used. However, the capability of predicting other deformation modes is considerably lower while the results obtained using multi-criteria optimization were far superior.

Clearly, when experimental data from multiple test types are available, the multi-criteria optimization tends to produce better overall results than the non-linear least squares method applied to a single test type. The latter is more appropriate, however, when the analyst knows that most efforts in a component correspond to the type used in the curve fitting (e.g. considering the tensile test data when the component is mainly loaded in tension).



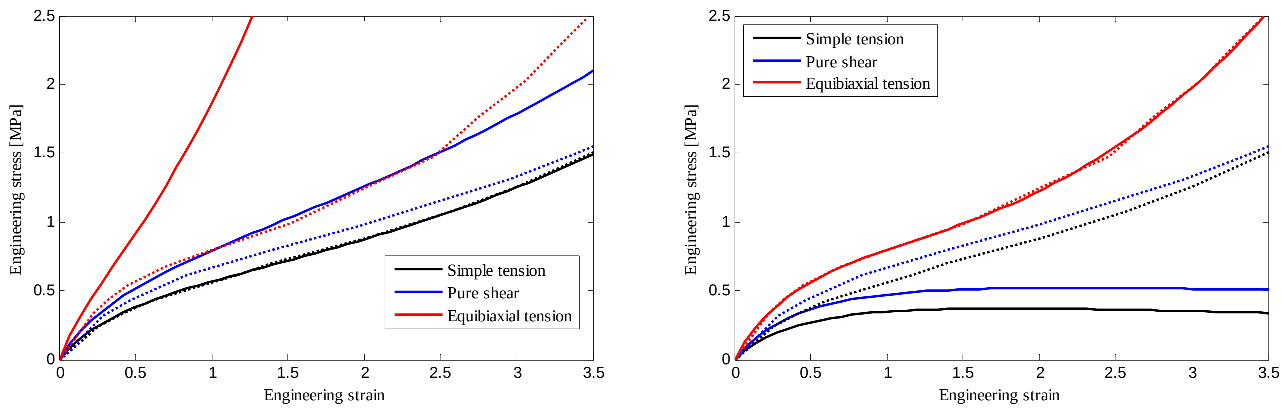
(a) (b)

Figure 4. Multi-criteria optimization with the 3 term Ogden model: (a) experimental/theoretical curve comparison (b) goodness of fit estimator R_{NL}



(a) (b)

Figure 5. Multi-criteria optimization with the 3 term Yeoh model: (a) experimental/theoretical curve comparison (b) goodness of fit estimator R_{NL}



(a) (b)

Figure 6. Non-linear least squares curve fit using the 3 term Ogden model: (a) using only simple tension data (b) using only equibiaxial tension data

5. CONCLUDING REMARKS

This work made clear the benefits of a dedicated research code in the analysis of rubberlike materials. The case study pointed how the code can help the analyst in choosing a hyperelastic constitutive model that correctly represents material behavior.

A research code such as HyperFit has the advantage of automating otherwise cumbersome tasks, allowing quick calibration of models. With the aid of the post-processing tools and its graphical interface, instant and objective comparison of models is possible. The user can analyze dozens of models simultaneously and decide with confidence which one is the most suited to his application. Moreover, the code allows complete control over the routines and parameters of its different optimization techniques, also letting the user implement new strain energy functions. This makes it a flexible research platform that can be used to develop and assess the performance of novel hyperelastic models.

Currently, the software is only capable of dealing with incompressible materials. The inclusion of compressive material behavior and dissipative effects is fundamental to turn it into a complete analysis tool in hyperelastic constitutive models.

6. ACKNOWLEDGEMENTS

The authors would like to thank CETEPO/SENAI-RS Center of Polymers for supporting this work.

7. REFERENCES

GMAp, 2011, “HyperFit Manual” (internal report).

Hoss, L., 2009, "Hyperelastic Constitutive Models for Incompressible Elastomers: Fitting, Performance Comparison and Proposal of a New Model" (in Portuguese), UFRGS, Porto Alegre, 290 p.

Jones, D.F., Treloar, L.R.G., 1975, “The properties of rubber in pure homogeneous strain”, J. Phys. D, Appl. Phys. V. 8: 1285–1304.

Marczak, R., Hoss, L., Gheller Jr., J., 2006, “Elastomer Characterization for Numerical Analysis” (in Portuguese), SENAI-RS, Porto Alegre, 126 p.

Mathworks, 2010a, Matlab Help Files (version R2010a), "lsqcurvefit".

Mathworks, 2010b, Matlab Help Files (version R2010a), "fminsearch".

Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P., 1992, “Numerical Recipes in C”, The Art of Scientific Computing, 2nd Edition, ISBN 0-521-43108-5.

Stumpf, F.T., 2009, "Assessment of a Hyperelastic Model for Incompressible Materials: Analysis of Restrictions, Numerical Implementation and Optimization of the Constitutive Parameters" (in Portuguese), UFRGS, Porto Alegre, 91 p.

8. RESPONSIBILITY NOTICE

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