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MULTI-OBJECTIVE OPTIMIZATION APPLIED ON SINGLE-STAGE AXIAL TURBINE PRELIMINARY DESIGN USING LOSSES MODELS

Victor Fujii Ando, victor.ando@gmail.com
Jesuíno Takachi Tomita, takachi@ita.br
João Roberto Barbosa, barbosa@ita.br
Technological Institute of Aeronautics. DCTA-ITA-IEM Grupo de Turbinas, São José dos Campos-SP, CEP 12228-900

Nelson Manzanares Filho, nelson@unifei.edu.br
Universidade Federal de Itajubá

Abstract. *A single-stage axial flow turbine was designed based on literature data. Additionally, losses models proposed by Ainley & Mathieson (1951) and Dunham & Came (1970), used for meanline prediction, were implemented in a FORTRAN program. Normally, in the preliminary design, several parameters should be wisely adopted by designer, requiring wide experience. To support the designer in the choice of some parameters aiming at higher efficiency an optimization tool can be used. The procedure involves the search of parameters, such as aspect ratio, blade thickness to chord ratio and gas outlet angle, using a Multi-Objective Genetic Algorithm, concerning the consistency of the values of losses coefficient and isentropic efficiency between the initial design and the designs derived from the models.*

Keywords: *Axial turbine, preliminary design, optimization, models of losses*

1. INTRODUCTION

The Center for Reference on Gas Turbine and Energy (CRTGE – Centro de Referência em Turbinas a Gás e Energia) staff comes from Technological Institute of Aeronautics (ITA) and from Institute of Aeronautics and Space (IAE). Both institutes are subordinated to the Department of Aerospace Science and Technology (DCTA) from the Brazilian Air Force. The development of technologies related to gas turbines is lengthy and costly, thus the companies treat results and correlations derived from experiments as confidential, because they represent key elements on their competitiveness. The research conducted within CRTGE is based upon information of public domain, although still relatively restricted, and upon many years of experience of its members. The centerpiece of the research developed in CRTGE is on numerical simulation of performance.

The study on preliminary design optimization intends to provide a better understanding of the impact of certain geometrical and aerodynamic parameters on the achievement of higher efficiencies, more appropriate dimensions. Even though the theme of research on optimization in engineering is not internationally new, it is relatively recent among the groups that work with gas turbines and aim at the development of project technology and project of those machines.

2. OBJECTIVE

The objective of this article is to show a basic procedure of optimization of the preliminary design of a single stage axial flow turbine, based on losses models. In this work, the historically relevant models of Ainley and Mathieson (1951) and Dunham and Came (1970) were used. The derived graphical correlations from those works were converted into a set of polynomials and the procedure was written in FORTRAN code. The optimization procedure searched for higher efficiencies using a Multi-Objective Genetic Algorithm, as it will be henceforward described.

3. NOMENCLATURE

There exist many distinct nomenclatures concerning geometry and aerodynamics aspects of gas turbines. The nomenclature used in this work is the same as from Saravanamutto (1996) and is indicated in Fig. 1. Gas flow angles related to absolute velocity are denoted by α and angles related to relative velocity are denoted by β. Indexes 1, 2, 3 refer to NGV inlet, rotor inlet and rotor outlet, respectively. Absolute velocity is given by *C* and relative velocity by *V*.

4. ELEMENTARY THEORY

The losses model of Ainley and Mathieson (1951) is based on extensive gathered data from turbine tests. It provides an estimation of profile losses, secondary losses and tip clearance losses. Through further tests, Dunham and Came (1970) revised the method from Ainley and Mathieson and accounted for Mach number in the profile loss. Furthermore, it simplified the calculation of secondary and tip clearance losses by eliminating the empirical function λ. Moreover, the Dunham and Came loss model provides better results than the original model developed by Ainley and Mathieson when applied in axial turbine design with low aspect ratio of blades.

The profile loss is due to the frictional loss along the blade surface, given by the boundary layer viscous shear.

The secondary flow loss occurs due to the vorticity formed by the encounter of flows with different velocities. The outcome is a structured flow field, but the kinetic energy is lost, since it cannot be converted to useful work. Examples of secondary flows commonly found in turbines are the horseshoe vortex and the tip vortex. Both flows are not part of the main flow.

The tip clearance loss consists of the spillage and leakage that occurs in either an open blade, which must have a clearance to the casing, or in seals in stators without clearance. The leakage flow does not contribute to the process of energy transfer between fluid and blade. Hence, this energy is dissipated causing internal losses.

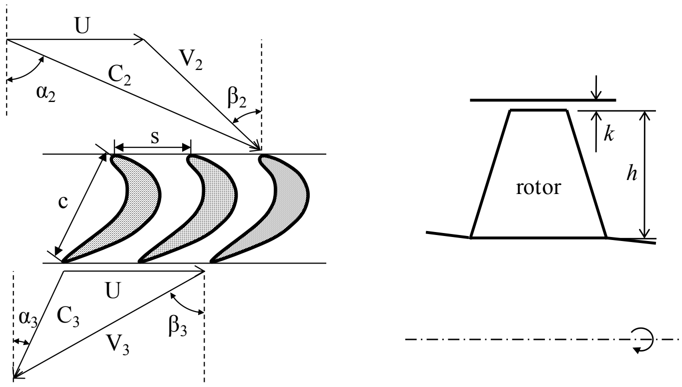


Figure 1. Nomenclature used in this work.

4.1. Ainley and Mathieson Loss Model

The profile loss is considered only for zero incidence in this work, although Ainley and Mathieson show how to account for different incidence angles (this is because only design-point is evaluated). The profile loss is given as a function of gas and flow angles, as well as space-to-chord ratio and thickness-to-chord ratio. It is calculated as if it were an intermediate case between a blade having $\alpha_1 = 0$ and $\alpha_1 = \alpha_2$, for Nozzle Guide Vane (NGV) or $\beta_1 = 0$ and $\beta_1 = \beta_2$ for rotor. For NGV and rotor, respectively:

$$Y_{pN(i=0)} = \left\{ Y_{pN(\beta_1=0)} + \left(\frac{\alpha_1}{\alpha_2}\right)^2 \left[Y_{pN(a_1=\alpha_2)} - Y_{pN(\alpha_1=0)} \right] \right\} \tag{1}$$

$$Y_{pR(i=0)} = \left\{ Y_{pR(\beta_2=0)} + \left(\frac{\beta_2}{\beta_3}\right)^2 \left[Y_{pR(\beta_2=\beta_3)} - Y_{pR(\beta_2=0)} \right] \right\} \tag{2}$$

Both $Y_{pN(a_1=\alpha_2)}$ and $Y_{pN(\alpha_1=0)}$ are functions derived from experimental data. The graphs are given on their original works. Second order or third order polynomials as function of space to chord ratio are fitted to them, so the method can be automated without the need of an operator to evaluate points in the graphs.

The secondary and tip clearance losses are given, for the NGV, by:

$$Y_{sN} + Y_{kN} = \left[\lambda + B\left(\frac{k}{h}\right) \right] \left[\frac{C_L}{(s/c)} \right]^2 \left[\frac{\cos^2 \alpha_2}{\cos^3 \alpha_m} \right], \tag{3}$$

where k is the tip clearance, h is the blade height, (s/c) is the pitch-to-chord ratio, B is 0.5 for row with radial tip clearance and 0.25 for shrouded seal, and:

$$\lambda = f\left[\frac{(A_2/A_1)^2}{1+(s/c)} \right], \tag{4}$$

where A_1 is the flow area at inlet to rotor blade row and A_2 is the flow area at exit from blade row. Function (4) is, similarly, given by a graphical correlation and converted into a polynomial. And:

$$\frac{C_L}{s/c} = 2\left(\tan\alpha_1 - \tan\alpha_2\right)\cos\alpha_m, \tag{5}$$

$$\alpha_m = \arctan\left[\frac{\tan\alpha_1 + \tan\alpha_2}{2}\right]. \tag{6}$$

4.2. Dunham and Came Loss Model

The update provided by Dunham and Came can be summarized as some corrections to the Ainley and Mathieson model. Firstly, the profile loss receives the influence of Mach number, as follows:

$$Y_{p,DC} = Y_{p,AM} \times \left[1 + 60\left(M_{out,rel} - 1\right)^2\right] \tag{7}$$

The NGV secondary loss is estimated as (for rotor, the angles should be taken analogously as from Ainley and Mathieson):

$$Y_{s,DC} = 0.0334\left(\frac{c}{h}\right)\left(\frac{\cos\alpha_2}{\cos\alpha_1}\right)\left\{\left[\frac{C_L}{(s/c)}\right]^2\left[\frac{\cos^2\alpha_2}{\cos^3\alpha_m}\right]\right\} \tag{8}$$

And the NGV tip clearance loss (analogously for rotor):

$$Y_{k,DC} = B_{DC}\frac{c}{h}\left(\frac{k}{c}\right)^{0.78}\left\{\left[\frac{C_L}{(s/c)}\right]^2\left[\frac{\cos^2\alpha_2}{\cos^3\alpha_m}\right]\right\}, \tag{9}$$

where constant B_{DC} is 0.47 for plain tip clearance and 0.37 for shrouded.

5. COMPUTATIONAL PROGRAM

The computational program was written in FORTRAN. The basic structure is shown in Fig.2.

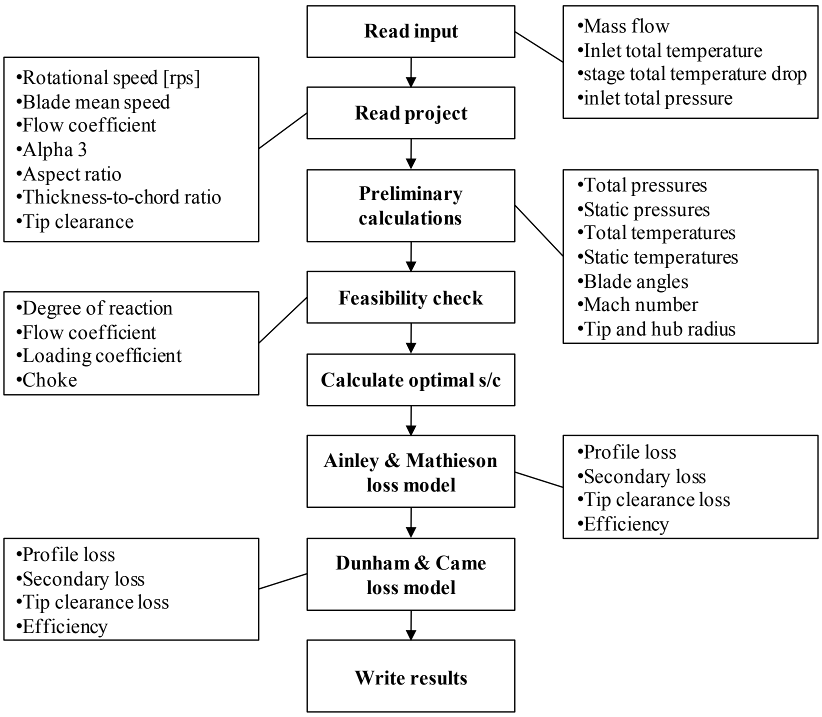


Figure 2. Computational program overview.

6. MULTI-OBJECTIVE GENETIC ALGORITHM (MOGA) OPTIMIZATION

Genetic algorithms (GAs) are search and optimization procedures based on the mechanics of natural selection and natural genetics. Fundamentally, a GA works by creating a population, which reproduces, mutates, suffers crossing over and are then is selected according to a survival criterion. In summary, a simple GA is composed by three operators: Reproduction, Crossover and Mutation. The essential structure of a GA algorithm is given by Deb (2002) in Fig. 3.

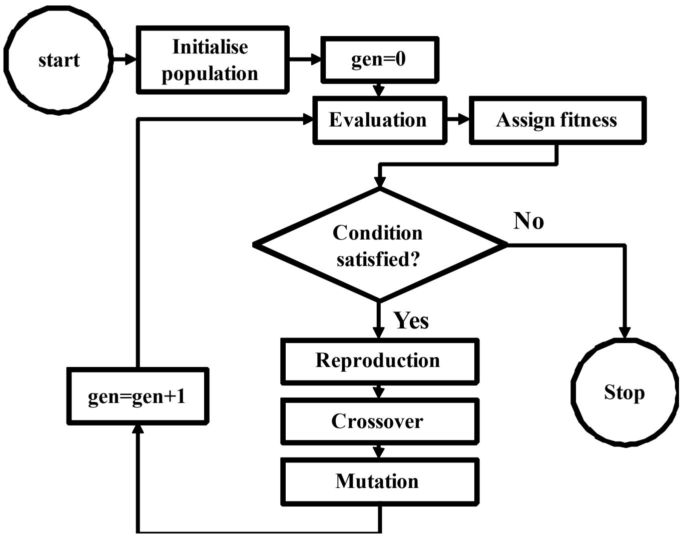


Figure 3. Genetic Algorithm, according to Deb (2002).

In single-objective problems, the fitness of an individual is the objective function itself. To treat multi-objective problems, a new approach is required. Instead of analyzing each objective, and perhaps attributing multiple fitness values, the fitness is related to the rank of each solution, i.e., is based on domination criterion.

Additionally, MOGA does also secure diversity among non-dominated solutions, by means of a niche count factor. In order to detail this procedure, the sharing function is defined as follows:

$$Sh(d) = \begin{cases} 1 - \left(\dfrac{d}{\sigma_{share}} \right)^{\alpha}, & \text{if d} \le \sigma_{share}; \\ 0, & \text{if d} > \sigma_{share}. \end{cases}, \tag{10}$$

where d is the distance between any two solutions in the population and σ_{share} is a distance, which determines a region wherein the sharing function assumes a value greater than zero. The sharing functions assumes values in the interval [0,1]. Next, the niche count is calculated. This function is the sum of all the sharing functions of solution i, including i itself:

$$nc_i = \sum_{j=1}^{N} Sh\left(d_{ij}\right). \tag{11}$$

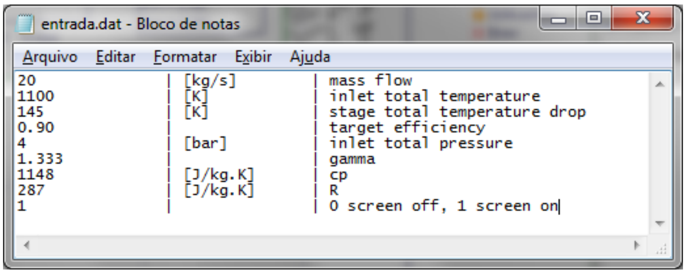
For a multi-objective optimization problem, the distance between any two solutions i and j in a rank is calculated as:

$$d_{ij} = \sqrt{\sum_{k=1}^{M} \left(\frac{f_k^{(i)} - f_k^{(j)}}{f_k^{\max} - f_k^{\min}} \right)}, \tag{12}$$

where $f_k^{\max}$ and $f_k^{\min}$ are the maximum and minimum objective functions values of the k-th objective. For the solution i, the distance is calculated for every solution j, which includes i, with the same rank. Additionally, the summation upper limit N in Eq. (11) is the number of solutions in rank i.

7. METHODOLOGY

The single-stage turbine preliminary design starts with the definition of some project parameters. The input file is shown in Fig. 4. The values are from the initial non-optimized turbine, from Saravanamuttoo (1996). The mass flow, inlet total temperature, stage total temperature drop, target efficiency and inlet total pressure are fixed in the design.




```
20          | [kg/s]      | mass flow
1100        | [K]         | inlet total temperature
145         | [K]         | stage total temperature drop
0.90        |             | target efficiency
4           | [bar]       | inlet total pressure
1.333       |             | gamma
1148        | [J/kg.K]    | cp
287         | [J/kg.K]    | R
1           |             | 0 screen off, 1 screen on
```

Figure 4. FORTRAN code input file.

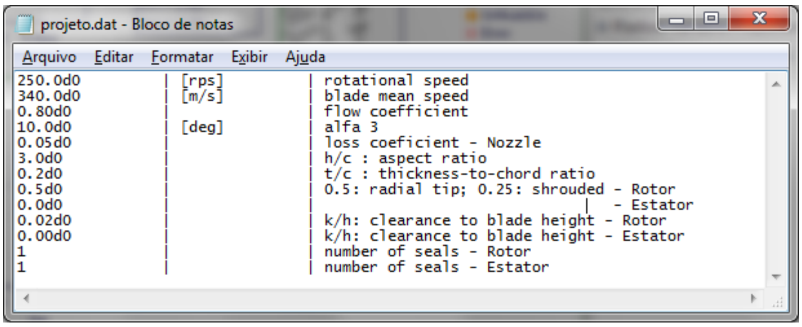
Further calculations are carried with the support of the project input file, which was created separately in order to allow a more editable file then the design input file, which has the design constraints. The project input file is show in Fig. 5. Again, it is the project file of the non-optimized initial turbine. With the initial design information and project variables, most of the aerothermodynamics can be calculated with few preliminary guesses.

After the design input, as well as the project parameters are determined, the program checks the thermodynamic feasibility and if the turbine is choked. After that, it proceeds with diverse calculations and with the losses calculation from each model. With the total losses from the rotor and the stator, the efficiency of the turbine can be estimated and should be compared to the initial guess. The efficiency is estimated as:

$$\eta_s \approx \left(1 + \frac{Y_R \left(\frac{V_3^2}{2c_p} \right) + Y_N \left(\frac{T_3}{T_2} \right) \left(\frac{C_3^2}{2c_p} \right)}{T_{01} - T_{03}} \right)^{-1} \tag{13}$$

If the efficiencies do not match, it implies that the preliminary turbine configuration is different from the one which results from the losses calculation. Therefore, the calculated efficiency and the target efficiency should be tried until they are approximately the same.

In order to attend the efficiency match requirement and concurrently increase its value, the Multi-Objective Genetic Algorithm was used. Through the variation of project parameters, as flow coefficient, α_3, aspect ratio and thickness to chord ratio with a 16 individuals and 50 generations MOGA, efficient designs arose.




```
250.0d0     | [rps]   | rotational speed
340.0d0     | [m/s]   | blade mean speed
0.80d0      |         | flow coefficient
10.0d0      | [deg]   | alfa 3
0.05d0      |         | loss coeficient - Nozzle
3.0d0       |         | h/c : aspect ratio
0.2d0       |         | t/c : thickness-to-chord ratio
0.5d0       |         | 0.5: radial tip; 0.25: shrouded - Rotor
0.0d0       |         |                                | - Estator
0.02d0      |         | k/h: clearance to blade height - Rotor
0.00d0      |         | k/h: clearance to blade height - Estator
1           |         | number of seals - Rotor
1           |         | number of seals - Estator
```

Figure 5. Project input file.

7.1. Non-optimized initial turbine results

The results file from the initial non-optimized turbine comprises 116 lines, which summarizes the results shown in Fig. 2. The lines regarding the loss models from the initial non-optimized turbine are shown in Fig. 6.

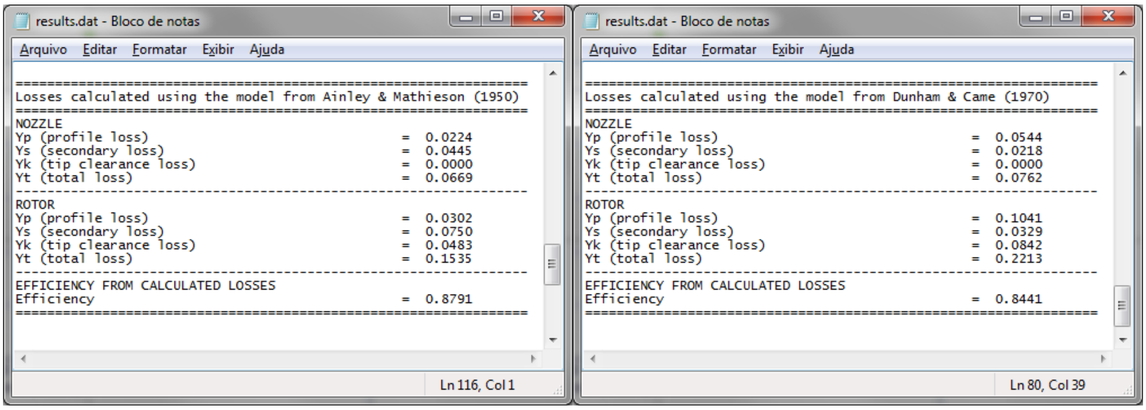


Figure 6. Results from the initial non-optimized turbine.

7.2. Optimization setup

The optimization workflow was designed in the commercial software modeFRONTIER and is shown in Fig. 6. The initial population set in the Design of Experiments (DOE) is given by the Latin Square. Four levels were chosen, resulting in 16 designs.

The chosen optimizer was the MOGA-II with the settings given by Tab.1. The input parameters and their intervals, as well as the step are show in Tab.2.

Table 1. MOGA-II settings.

Optimizer	MOGA-II
Number of generations	50
Probability of directional cross-over	0.5
Probability of selection	0.05
Probability of mutation	0.1
DNA string mutation ratio	0.05
Elitism	Enabled
Number of concurrent design evaluations	10
Evaluate repeated designs	No

Table 2. Input parameters range and step.

Parameter	Lower bound	Upper bound	Step
Target efficiency	0.7000	0.9500	0.0005
Flow coefficient	0.500	1.000	0.005
Alpha 3	5.00	13.00	0.02
Aspect ratio (h/c)	2.50	4.50	0.01
Thickness to chord ratio (t/c)	0.1000	0.3000	0.0005

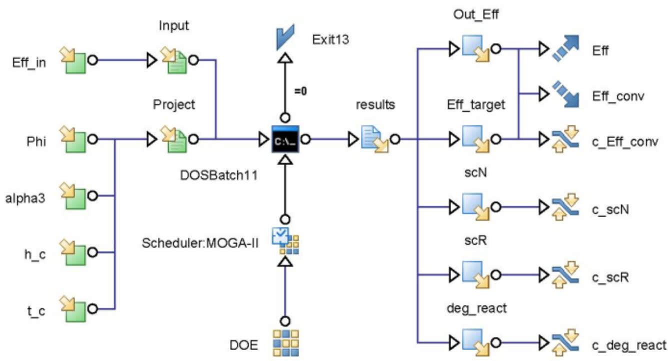


Figure 7. Optimization workflow in modeFRONTIER.

8. RESULTS AND ANALYSIS

Figure 8 displays the objective space in regions of high efficiencies and low target-to-obtained efficiency quadratic differences. The Bubble 4D graph is useful to display multi-dimensional functions. In Fig. 7, the axes are the aforementioned objectives, the bubble color is the thickness-to-chord ratio and its diameter is the aspect ratio.

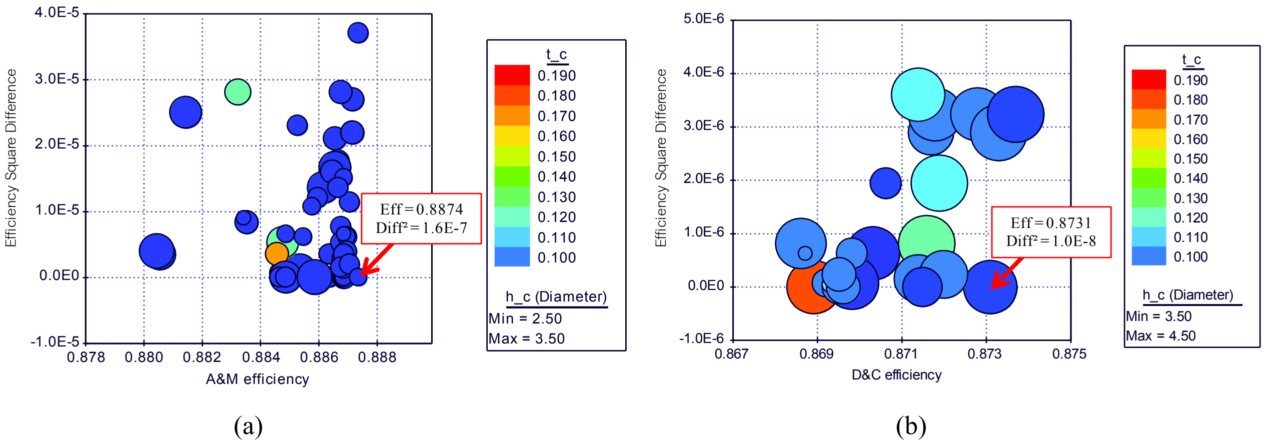


Figure 8. Optimization result highlighting optimum designs: (a) considering the efficiency of Ainley and Mathieson model and (b) Dunham and Came model.

Note that the aspect ratio range is greater in the optimization of the efficiency calculated with Dunham and Came method. This shows the different behavior of the methods in regard to this parameter. Tables 3 and 4 summarize the results of the losses for both the designs highlighted in Fig. 8. The column "opt. A&M" in Tab. 3 refers to the design highlighted in Fig. 7(a). For this same design, the results using Dunham and Came model is shown beside.

Table 3. Results of the efficiency optimization using the model of Ainley and Mathieson.

Ainley and Mathieson		non-opt.	opt. A&M	Difference	opt. D&C	Difference
NGV	Yp (profile)	0.0224	0.0240	7.1%	0.0214	-4.5%
	Ys (secondary)	0.0445	0.0422	-5.2%	0.0457	2.7%
	Yk (tip clearance)	0.0000	0.0000	-	0.0000	-
	Yt (total)	**0.0669**	**0.0662**	**-1.0%**	**0.0671**	**0.3%**
ROTOR	Yp (profile)	0.0302	0.0255	-15.6%	0.0231	-23.5%
	Ys (secondary)	0.0750	0.0692	-7.7%	0.0773	3.1%
	Yk (tip clearance)	0.0483	0.0519	7.5%	0.0454	-6.0%
	Yt (total)	**0.1535**	**0.1467**	**-4.4%**	**0.1459**	**-5.0%**
EFFICIENCY		**87.9%**	**88.7%**	**0.8%**	**87.9%**	**0.0%**

Similarly for the model of Dunham and Came is organized in Tab.4.

Table 4. Results of the efficiency optimization using the model of Ainley and Mathieson.

Dunham and Came		non-opt.	opt. A&M	Difference	opt. D&C	Difference
NGV	Yp (profile)	0.0544	0.0773	42.1%	0.0521	-4.2%
	Ys (secondary)	0.0218	0.0256	17.4%	0.0144	-33.9%
	Yk (tip clearance)	0.0000	0.0000	-	0.0000	-
	Yt (total)	**0.0762**	**0.1028**	**34.9%**	**0.0664**	**-12.9%**
ROTOR	Yp (profile)	0.1041	0.0980	-5.9%	0.0656	-37.0%
	Ys (secondary)	0.0329	0.0375	14.0%	0.0208	-36.8%
	Yk (tip clearance)	0.0842	0.0936	11.2%	0.0726	-13.8%
	Yt (total)	**0.2213**	**0.2291**	**3.5%**	**0.1589**	**-28.2%**
EFFICIENCY		**84.4%**	**83.5%**	**-0.9%**	**87.3%**	**2.9%**

Table 5 compares the project input parameters for each optimum found. Note the relatively large difference in the aspect ratio.

Table 5. Comparison of the project for each optimized design.

	A&M	D&C	%Difference
flow coefficient	0.7200	0.8400	16.7%
alpha 3	13.00	11.28	-13.2%
aspect ratio	2.600	4.480	72.3%
thickness to chord ratio	0.1000	0.1165	16.5%

9. CONCLUSION

During the preliminary design of a turbomachine, the designer expertise is decisive and has direct impact on the design success. This is especially true because some important parameters, generally geometrical constraints, should be adopted among several numbers of variables that need to be controlled and analyzed during this process.

With the advent of optimization tools, starting with simple models until multi-objective heuristics, many processes became automatic and of valuable helpfulness for engineering design proposals. Naturally, the expertise from design is indispensable mainly during decisions concerning the range to vary specific parameters and, most importantly, on the analysis of the results. It can occur that the heuristic leads to an unrealistic design, which shall be noticed by an experienced designer.

In this work, the results presented have shown an application of the preliminary axial turbine design coupled with an optimization tool that supplied important guidance for design decisions based on highly-efficient solutions. If the designer had to run all the calculated designs without a robust tool as MOGA, some days would be necessary, rather than some minutes. Thus, the cost related with the design process can be reduced; meanwhile, the capabilities can be improved. Nevertheless, the final decision about the best solution taking into account the whole gas turbine requirement is still from the designer.

For multistage axial turbines the number of possibilities and variables to control is higher than the single-stage machine. For this case, certainly the use of MOGA will be very helpful.

10. ACKNOWLEDGEMENTS

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