# ON NONLINEAR DYNAMICS AND CONTROL OF A BLOCK, MAGNETICALLY LEVITED, EXCITED BY A NON-IDEAL ENGINE

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***Abstract.*** *This paper studies the nonlinearities in dynamics of a magnetically levitated non-ideal body excited by an unbalanced rotor with limited power. These nonlinearities lead the motion of the structure to the Sommerfeld Effect. For this reason the motor's near or in resonance frequency. When the structure achieves the resonance condition, the best part of the energy is consumed to generate large amplitude vibration, with no sensitive change in the motor frequency. In this paper, it is discussed how to drive the system to resonance condition and to avoid the "energy sink" that occurs with the Sommerfeld effect.*

***Keywords****: Non-ideal vibrations, Resonance, Sommerfeld effect*

## 1. INTRODUCTION

Since the first ages of humanity, there is a need of travelling large and small distances among different places. According to this search, new, modern and advanced ways of locomotion emerge every day. The first problem of ground transportation is how to move a large number of people. The creation of the first public transport service occurred only in 1829, in London. This creation is traditionally attributed to Georges Shillibeer. The first idea of the train occurred in 1901, in France.

According to Moon (2004), the suspension of objects and people with no visible means of support is fascinating to most of the people, even in times of high technologies. Dynamical systems without gravity effects are a dream, common to generations of thinkers, since Benjamin Franklin to Robert Goddard. A method found to solve this issue is the MAGnetic LEVitation (MAGLEV). **Magnetic levitation**, **maglev**, or **magnetic suspension** is a method by which an object is suspended with no support, other than magnetic fields. Magnetic pressure is used to counteract the effects of the gravitational and any other accelerations (Braunbeck and Free, 1939).

The modern development of magnetic levitation transportation systems, known as MagLev, started in the late 60s as a natural consequence of the development of low-temperature superconducting wire and the transistor and chip-based electronic control technology. MagLev matured to the point that Japanese and German technologists got ready to market these new high speed levitated machines in the 80s (Moon, 2004). Since the discovery of superconductors, according to (Moon, 1987), magnetic levitation has become a symbol of new technologies. In addition, a transport system for magnetic levitation captures the imagination of the movement of levitated bodies; however, there is a great lack of understanding the magnetic levitation.

Until the late nineteenth century, mathematical models of vibrating dynamic systems did not take into account the influences of system behavior on the sources of vibration. Mathematical models of vibrating dynamic systems that take into account the influences of system behavior on the sources of vibration are called non-ideal dynamical systems; an equation that describes the interaction of the power supply with the driven system must be added. Thus, as a first characteristic, the non-ideal vibrating systems have one more degree of freedom than the corresponding ideal system.

The jump phenomenon in the vibration amplitude and the increase of the power required by the source to operate next to the system resonance are both manifestations of non-ideal problems, and are generally referred in literature as Sommerfeld Effect. This phenomenon suggests that the vibratory response of the non-ideal system emulates an “energy sink” in the regions next to the system resonance, by transferring the power from the source to vibrations of the support structure, instead of speeding the driver machine up. In other words, one of the problems confronted by mechanical engineers is how to drive a system through a system resonance.  A revision of non-ideal problems was published (Balthazar et. al., 2003);

The aim of this paper is to study the interaction between a magnetically levitated block and an unbalanced engine with limited power. The paper is organized as follows: in section 2 a mathematical model that represents the system is showed, by modeling the block behavior and from the motor interactions. In section 3, results and numerical simulations are showed, and in section 4 conclusions and discussions are presented.

## 2. MATHEMATICAL MODEL

It is known that a mathematical model of any dynamic system is essential to analyze the dynamics of the system and when necessary, to design a control method to the system. Through the simplified model in figure 1, we obtain the equations of motion of the system by Lagrange’s method. Next, the dynamic’s behavior of the system and the Sommerfeld effect are modeled.

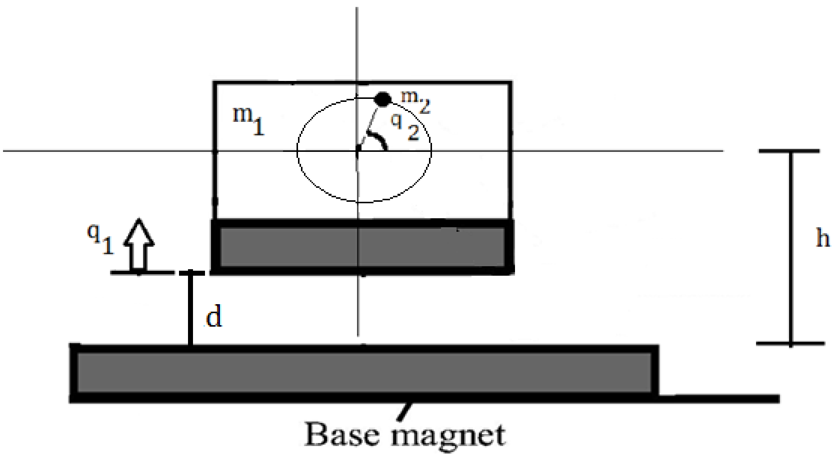


Figure 1. Simplified model of a Non-ideal Maglev

Observing the model, $m_1$ represents the block mass, $m_2$ is an unbalanced mass, $q_1$ is the vertical displacement, $q_2$ is the angular displacement of the rotor that expresses the action of the source of energy on the oscillating system (the angular velocity of the motor that is not constant). The parameter $r$ is the eccentricity, $m_2$ is the mass of unbalanced shaft of the electric motor. $J_2$ is the moment of inertia of the rotor, the function $R(q_2, \dot{q}_2, \ddot{q}_1, r)$ represents the action of the oscillating system on the source of energy, the function $H(\dot{q}_2)$ is the resistive torque applied to the motor and the function $L(\dot{q}_2)$ is the driving torque of the source of energy (motor).

Note that the inductance is usually much smaller than the mechanical constant time of the system and, when in stationary regime, we can take $L(\dot{q}_2)$ as (linear) $M(\dot{q}_2) = L(\dot{q}_2) - H(\dot{q}_2) = \mu_1 - \mu_2\dot{q}_2$, where $\mu_1$ is related to voltage applied across to the armature of the DC motor, that is, a possible control parameter of the problem and $\mu_2$ is a constant for each model of DC motor considered. $f(q_1, \dot{q}_1)$ is the nonlinear and non-conservative part of the restoring force, while $\frac{\partial U(q_1)}{\partial q_1}$ is its conservative part (U is the potential, or strain energy).

From figure 1, follow the equations:

For the block

$$X_1 = 0 \qquad \dot{X}_1 = 0$$
$$Y_1 = h + q_1 \qquad \dot{Y}_1 = \dot{q}_1 \tag{1}$$

And for the DC motor

$$X_2 = rcos(q_2) \qquad \dot{X}_2 = -\dot{q}_2 rsen(q_2)$$
$$Y_2 = h + q_1 + rsen(q_2) \qquad \dot{Y}_2 = \dot{q}_1 + \dot{q}_2 r\cos(q_2) \tag{2}$$

Then, we obtain the kinetic (T) and potential (V) energies,

$$T = \frac{\dot{q}_1^2}{2}(m_1 + m_2) + \dot{q}_1\dot{q}_2 m_2 \operatorname{rcos}(q_2) + \frac{m_2 {\dot{q}_2}^2 r^2}{2} + \frac{J_2\dot{q}_2^2}{2} \tag{3}$$

$$V = g[m_1 q_1 + m_2(q_1 + rsen(q_2))] \tag{4}$$

The governing equations of motion

$$\begin{cases} (m_1 + m_2) + m_2 r\ddot{q}_2\, cos(q_2) - m_2 r\dot{q}_2^2 sen(q_2) = -c_1\dot{q}_1 - K_{mag}(d - q_1)^2 - g(m_1 + m_2) \\ \ddot{q}_1 m_2 rcos(q_2) - \dot{q}_1\dot{q}_2 m_2 rsen(q_2) + \ddot{q}_2 m_2 r^2 + J_2\ddot{q}_2 = M(\dot{q}_2) - gm_2 rcos(q_2) \end{cases} \tag{5}$$

Rewriting (5) on matrix form we will have

$$\begin{bmatrix} 1 & \frac{m_2 r\, cos(q_2)}{m_1 + m_2} \\ rcos(q_2) & \frac{J_2 + m_2 r^2}{m_2} \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} = \begin{Bmatrix} -\frac{c_1}{m_1+m_2}\dot{q}_1 - \frac{K_{mag}(d-q_1)^2}{m_1+m_2} + \frac{m_2 r\dot{q}_2^2 sen(q_2)}{m_1+m_2} + g \\ \frac{\mu_1}{m_2} - \frac{\mu_2}{m_2}\dot{q}_2 - grcos(q_2) + \dot{q}_1\dot{q}_2 rsen(q_2) \end{Bmatrix} \tag{6}$$

Adding,

$$\begin{Bmatrix} -\dfrac{c_1}{m_1+m_2}\dot{q}_1 - \dfrac{K_{mag}(d-q_1)^2}{m_1+m_2} + \dfrac{m_2 r\dot{q}_2^2 sen(q_2)}{m_1+m_2} + g \\ \dfrac{\mu_1}{m_2} - \dfrac{\mu_2}{m_2}\dot{q}_2 - grcos(q_2) + \dot{q}_1\dot{q}_2 rsen(q_2) \end{Bmatrix} = \begin{Bmatrix} G_1 \\ G_2 \end{Bmatrix}$$

$$A = \begin{bmatrix} 1 & \dfrac{m_2 r\cos(q_2)}{m_1+m_2} \\ rcos(q_2) & \dfrac{J_2+m_2r^2}{m_2} \end{bmatrix}$$

$$B = \begin{bmatrix} G_1 & \dfrac{m_2 r\cos(q_2)}{m_1+m_2} \\ G_2 & \dfrac{J_2+m_2r^2}{m_2} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & G_1 \\ rcos(q_2) & G_2 \end{bmatrix}$$

And

$$\Delta = \frac{(J_2+m_2r^2)}{(m_2)} - \frac{{m_2}^2 r^2\cos^2(q_2)}{(m_1+m_2)}$$

It follows that

$$\begin{cases} \ddot{q}_1 = \dfrac{\dfrac{J_2+m_2r^2}{m_2}G_1 - \dfrac{m_2r\cos(q_2)}{m_1+m_2}G_2}{\Delta} \\ \\ \ddot{q}_2 = \dfrac{G_2 - rcos(q_2)G_1}{\Delta} \end{cases} \tag{7}$$

In state space:

$$\begin{cases} \dot{y}_1 = y_2 \\ \\ \dot{y}_2 = \dfrac{\dfrac{J_2+m_2r^2}{m_2}G_1 - \dfrac{m_2r\cos(q_2)}{m_1+m_2}G_2}{\Delta} \\ \\ \dot{y}_3 = y_4 \\ \\ \dot{y}_4 = \dfrac{G_2 - rcos(q_2)G_1}{\Delta} \end{cases} \tag{8}$$

Next, some numerical simulations are presented:

## 3. NUMERICAL SIMULATIONS AND RESULTS

In this section, the results of numerical simulations are obtained using software Matlab®, with integrator ode113, Adams-Bashforth-Moulton PECE solver algorithm with variable step-length.

The table 1 shows the numerical values for used parameters,

Table 1. Parameters:

| *Descriptions* | *symbol* | *Value* |
|---|---|---|
| Mass of Block | $m_1$ | 1 *kg* |
| Unbalanced mass | $m_2$ | 0.1 *kg* |
| Eccentricity | $r$ | 0.5 *m* |
| Gap | $d$ | 1.5 *m* |
| Parameter control | $\mu_1$ | variable *N.m* |
| Parameter control | $\mu_2$ | 1 N.m/Rad/s |
| Damped | $c_1$ | 0.05 *N.s/m* |
| Gravity | $g$ | 9.8 $m/s^2$ |
| Stiffness | k | 0 or 0.5 *N/m* |
| Moment of inertia | $J_2$ | 0.37 $kg.m^2$ |

### 3.1. Sommerfeld effect

When the rotor tension ($\mu_1$) are near by the resonance region through Fig. 2, the "jump" is obtained:

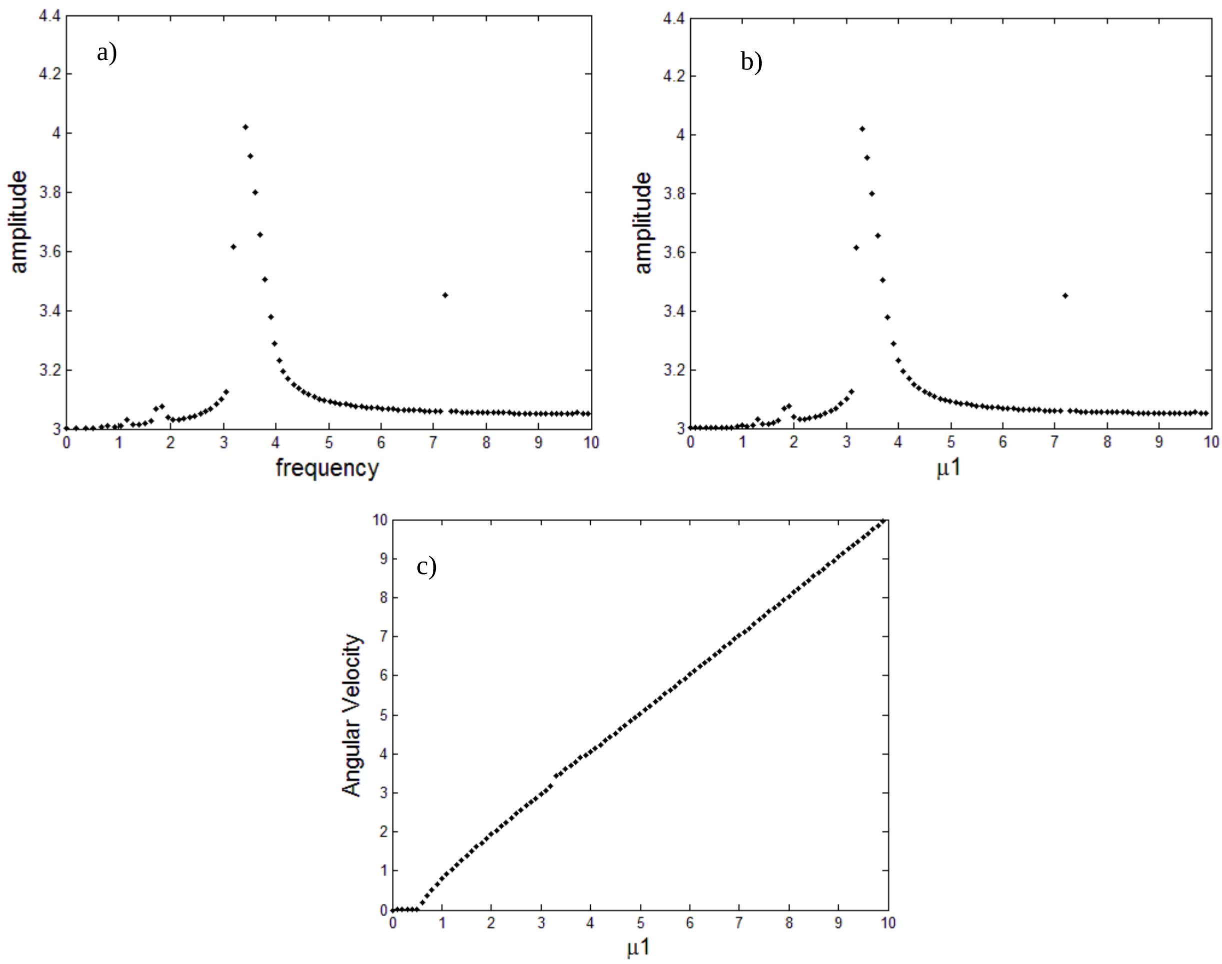


Figure 2 – Sommerfeld Effect a) amplitude vs. frequency means b) amplitude vs. tension ( $\mu_1$) c) angular velocity vs tension ( $\mu_1$)

The next step is to show the time history of the displacement and the phase portrait of the block before, through and after the jump in order to observe the behavior of the dynamical amplitude in these tree regions. Moreover we need to observe the same phenomena when the initial position of the block is below or above equilibrium position.

In next figures, with simulations on the initial position $[0, 0, 0, 0]$ follows that the block tends to stabilize in $q_1 = 3$ , the same occurs on the initial position $[4.5, 0, 0, 0]$.

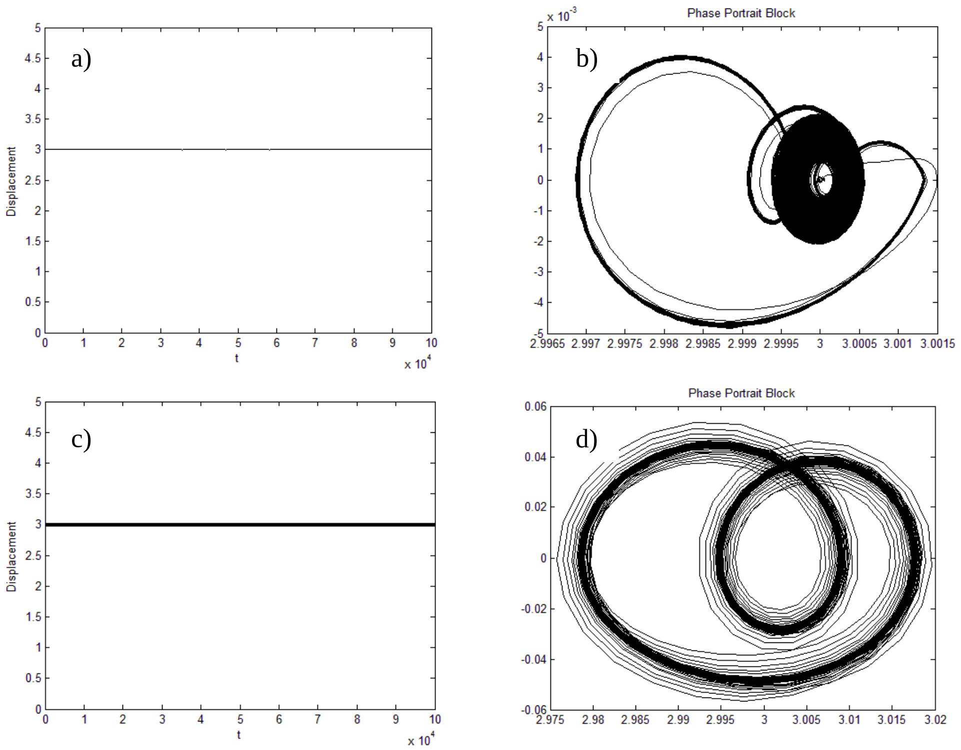


Figure 3: Initial position [0, 0, 0, 0] before jump a) displacement $\boldsymbol{\mu_1 = 0.5}$ b) phase portrait $\boldsymbol{\mu_1 = 0.5}$ c) displacement $\boldsymbol{\mu_1 = 1.6}$ d) phase portrait $\boldsymbol{\mu_1 = 1.6}$ .

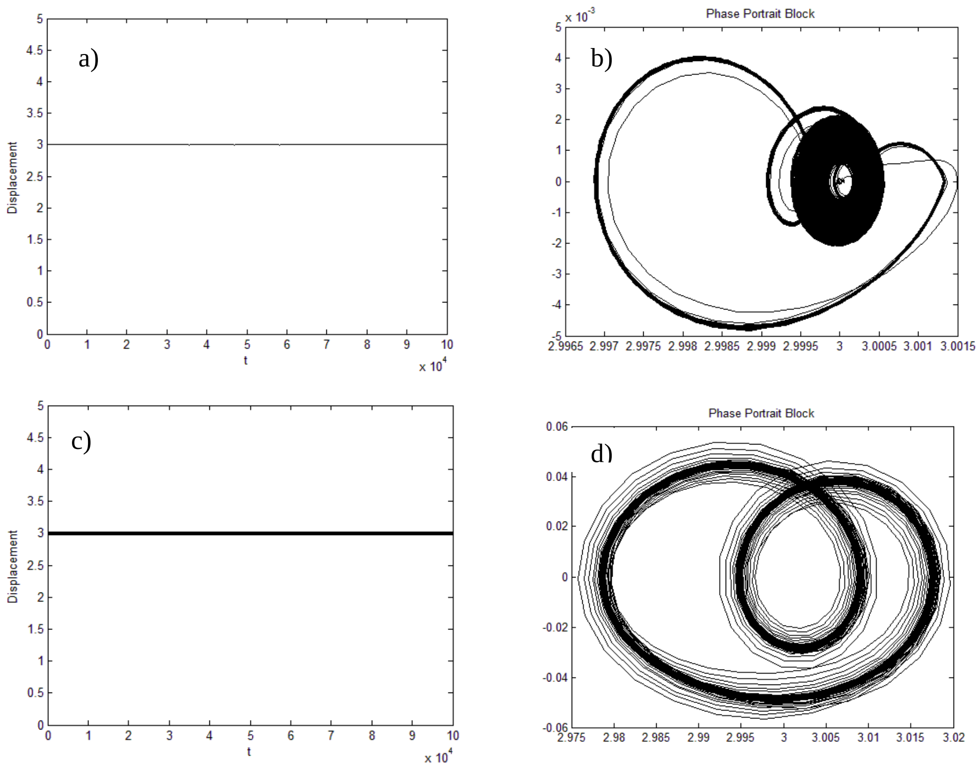


Figure 4: initial position [4.5, 0, 0, 0] before jump a) displacement $\boldsymbol{\mu_1 = 0.5}$ b) phase portrait $\boldsymbol{\mu_1 = 0.5}$ c) displacement $\boldsymbol{\mu_1 = 1.6}$ d) phase portrait $\boldsymbol{\mu_1 = 1.6}$

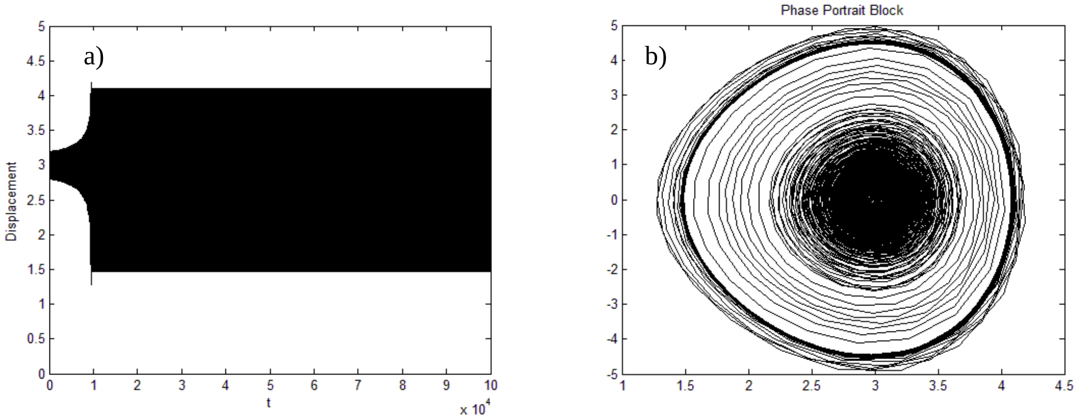


Figure 5: initial position [0, 0, 0, 0] in jump a) displacement $\boldsymbol{\mu_1 = 3.2}$ b) phase portrait $\boldsymbol{\mu_1 = 3.2}$

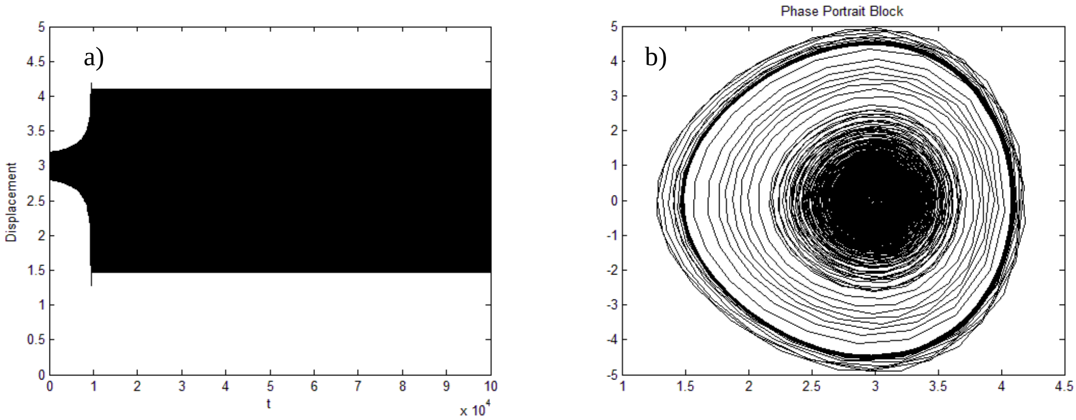


Figure 6: initial position [4.5, 0, 0, 0] in jump a) displacement $\boldsymbol{\mu_1 = 3.2}$ b) phase portrait $\boldsymbol{\mu_1 = 3.2}$

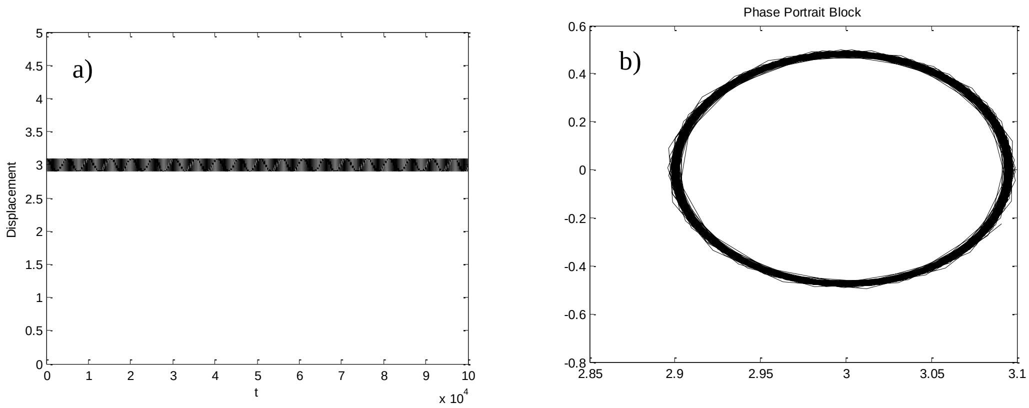


Figure 7: initial position [0, 0, 0, 0] after jump a) displacement $\boldsymbol{\mu_1 = 4.9}$ b) phase portrait $\boldsymbol{\mu_1 = 4.9}$

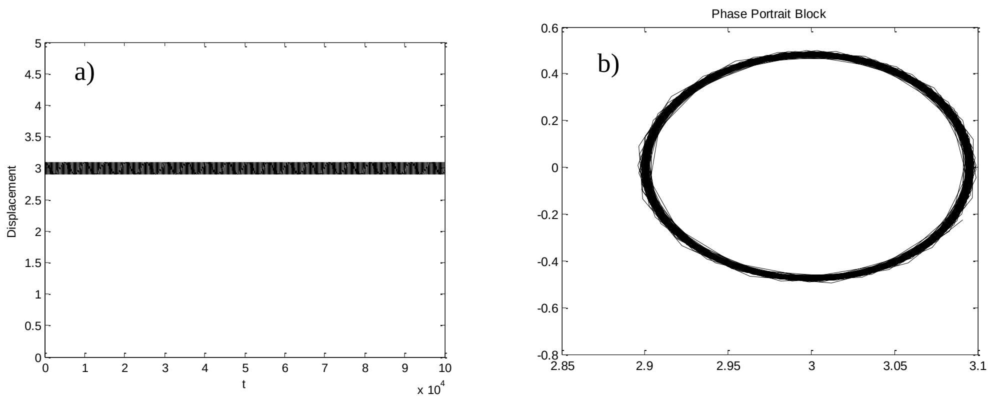


Figure 8: initial position [4.5, 0, 0, 0] after jump a) displacement $\boldsymbol{\mu_1 = 4.9}$ b) phase portrait $\boldsymbol{\mu_1 = 4.9}$

**3.2. The system with the stiffness parameter:**

In order to decrease the amplitude on resonance region, a spring is inserted in the system. Then the equation (5) is:

$$\begin{cases} (m_1+m_2)+m_2r\ddot{q}_2\cos(q_2)-m_2r\dot{q}_2^2sen(q_2)=-c_1\dot{q}_1-\mathrm{K_{mag}}(\mathrm{d}-\mathrm{q}_1)^2-kq_1-\ g(m_1+m_2) \\ \ddot{q}_1m_2rcos(q_2)-\dot{q}_1\dot{q}_2m_2rsen(q_2)+\ddot{q}_2m_2r^2+J_2\ddot{q}_2=M(\dot{q}_2)-gm_2rcos(q_2) \end{cases} \tag{9}$$

Obtaining the following results (figure 9).

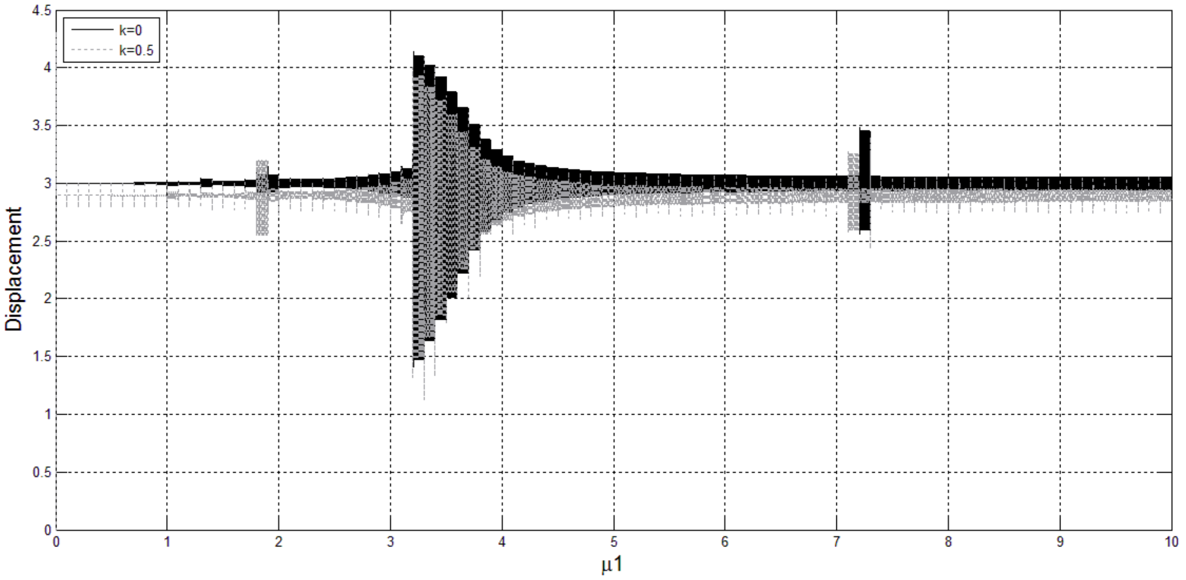


Figure 9: the comparative system with the spring (gray) and without spring (black)

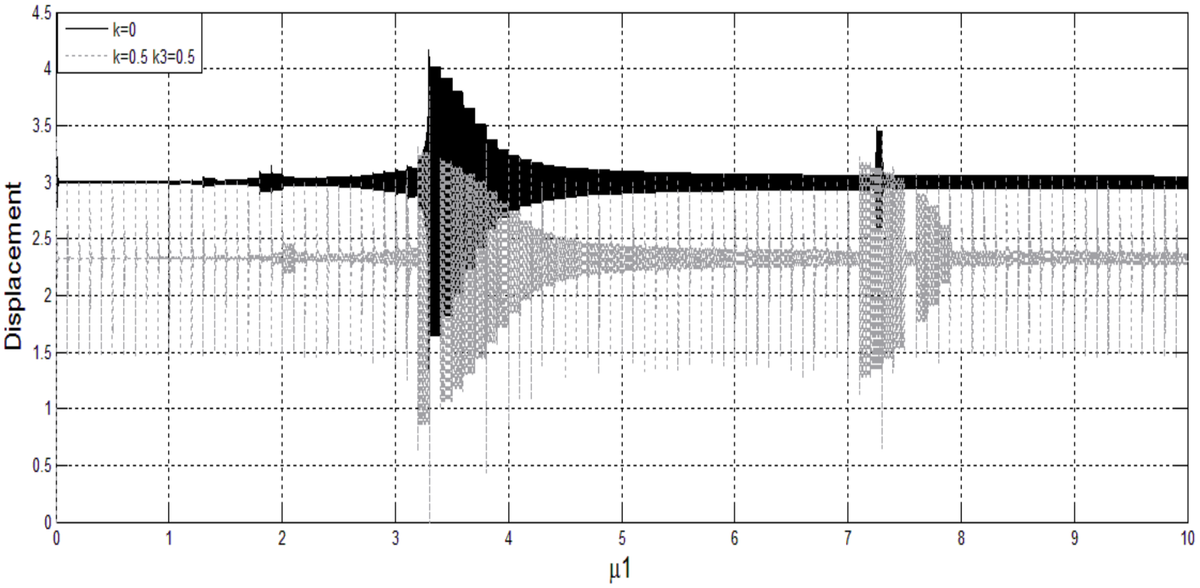


Figure 10: the comparative system with the cubic spring (gray) and without spring (black)

We observe through figure 10 an increasing of the amplitude of block.

### 3.3. Evaluation of the Lyapunov exponents

There is no chaos in this problem, as show in figure 11(Nayfeh and Balachandran, 1995).

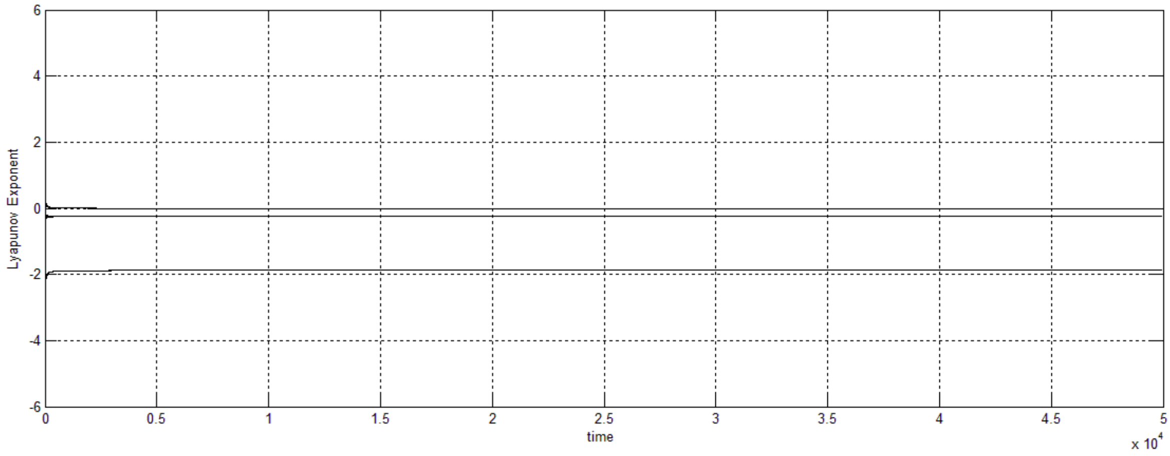


Figure 11: Lyanunov exponents

### 3.4 The behavior of the system taking into account the energy

We remark that the DC motor accelerates to reach near resonant conditions; a considerable part of its output energy is consumed to generate large amplitude motions of the structure and not to increase its own angular speed. This phenomenon is showed in figures 2, 5, 6, 9 and 12. In figure 5a the DC motor gets stuck at resonance. Only after increasing its voltage, it occurs the decrease amplitude. The figure was obtained according to (Kerschen 2005).

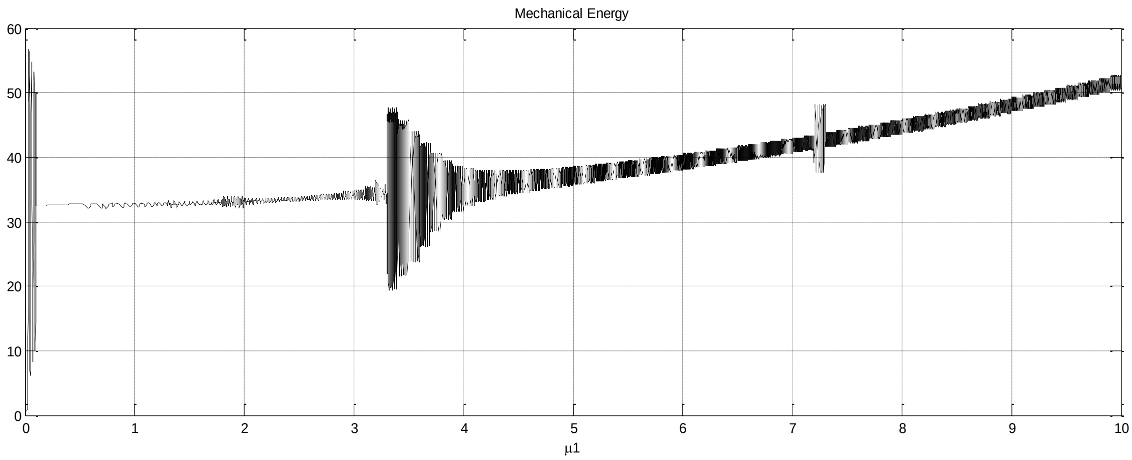


Figure 12: Total mechanical energy

### 4. CONCLUSIONS

The Non-Ideal vibrating System proposed in this paper showed that the energy source has influence over the structure.

Due to the existence of the Sommerfeld effect, the maximum amplitude occurred in regions where the jump phenomenon happened (Palacios 2009).

For some parameters of the system, the motor can get stuck at resonance, not having enough power to reach higher rotation regimes (figures 5a, 6a, 9 and 12).

In future papers, a strategy for controlling and reducing the Sommerfeld effect can be implemented in order to keep the motions of the system in a low vibration. Moreover insert a parametric pendulum as an energy sink.

## 5. ACKNOWLEDGEMENTS

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