A LEVEL SET NUMERICAL APPROACH FOR THE FLUID-STRUCTURE INTERACTION PROBLEM

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Abstract. *The present work aims at development of a versatile formulation via the Galerkin finite element method (GFEM) to approaches fluid-structure interaction problems. This topic is very interesting to the national oil industry (for example large submerged structures as Manifold, PLET and PLEM). The study of theoretical and computational fundamentals to fluid-structure interaction problems is associated to technical and scientific developments (the fluid problem, the solid problem, the problem of motion of the fluid subdomain, coupling problem, for instance). The initial idea to describe free surfaces and interfaces by level set functions. The fluid problem (flow) is approached by Eulerian formalism, while the solid (or solid structure) is described according to Lagrangian formalism. The use of level set functions enables to obtain a compact weak form of the problem. Hence, improving discretization and solution of resulting system. The Augmented Lagrange Multiplier Method is used to fluid-structure coupling.*

Keywords: *Fluid-Structure interaction, FEM, Level Set functions, Manifold , PLET, PLEM.*

1. INTRODUCTION

The numerical simulation of mechanical behavior of solids and industrial fluids is widely used in companies for viability verification, improvement and optimization of projects. The scientific origins of fluid-structure interaction (FSI) are associated to aerospace and nuclear industries, among others. The FSI problems have proposed some challenges to scientific community, specifically related to applied mathematics and computational mechanics. Some references can be found in specialized literature (in scope of nuclear , civil, aerospace, ocean, chemical, and mechanical engineering) as (Bisplinghoff *et al.*, 1983), (Roma *et al.*, 1999), (Fung, 2000), (Dowell *et al.*, 1995), (Dowell and Ilgamov, 1988), (Wilson and Khalvati, 1983), (Chen and Taylor, 1990), (Page, 1982), (Blevins, 1994), (Paidoussis, 1998), (Paidoussis, 2004) and (Paidoussis and Issid, 1974).

The main idea of this methodology is the description of fluid-solid interface for level set functions (Osher and Fedkiw (2003) and Sethian (1999)). This approach ensures concise and compact weak formulation of the problem. It is important to point out that the level set functions provides an elegant way to couple the fluid and solid media in problem formulation (see Park *et al.* (2001); Chessa and Belytschko (2003); Chessa *et al.* (2002, 2003)). This methodology shares some of the advantages of the methods called fictitious volumes (see Glowinski *et al.* (2001) and Patankar *et al.* (1984)) that were initially developed to approach fluid flow interaction with rigid particles. Subsequently, the fictitious volumes technique were extended to deformable solids. Nevertheless, this technique was not suitable for fluid-structure interaction in thin solid structures. Other methods for FSI as the immersed boundary method will be mainly used in the case of incompressibility of fluid medium (Peskin (2002)). This approach has some limitations as the velocity of solid domain is computed by sum of the fluid domain velocity with a superimposed relative velocity.

In this work the Augmented Lagrange Multiplier Method combined with level set functions provide a consistent approach to fluid-structure interaction in coupled systems. In this sense, the problem formulation is two-dimensional (plane stress, infinitesimal strain and axial symmetry) as first analysis and some possible applications are presented. It is important to comment that this work is the first part of an undergraduate scientific initiation project.

2. MATHEMATICAL STRATEGY FOR FSI PROBLEM

Among several possible strategies for approach of FSI problems, will be adopted here, due to its robustness, the method referred to as Eulerian-Lagrangian (Eulerian description for the fluid and Lagrangian for solid/structure). In contrast to the aforementioned technique can highlight the method called arbitrary Eulerian-Lagrangian (ALE) in which it is not necessary movement of the mesh on the domain of the fluid to match the motion of fluid-structure interface, for details one can observe the following works [(Belytschkoand and Kennedy, 1978), (Farhat *et al.*, 2003), (Farhat *et al.*, 2001), (Hu *et al.*, 2001)]. You can also comment that the method used here achieves better results in problems that are considered large topological changes in the field solid / structure. In these applications the LEA methods fail.

2.1 Problem Identification

Consider the domain $\Omega_{CD} = \Omega_F \cup \Omega_S \subset \mathbb{R}^2$ comprising the areas open to the fluid medium Ω_F and the solid structure. Ω_S. Denoting by $\mathbf{x}$ Eulerian spatial coordinates, $\mathbf{v}^F(\mathbf{x}, t)$ the velocity field of fluid domain denoted and $\mathbf{x} = \mathbf{x}^S(\mathbf{X}, t)$

the motion of solid domain. As $X \in \Omega_{S_o}$ the reference configuration of solid domain. The velocity field is given by:

$$\mathbf{v}^S(\mathbf{X},t) = \frac{\partial \mathbf{x}^S(\mathbf{X},t)}{\partial t}. \tag{1}$$

or

$$\mathbf{v}^S(\mathbf{x},t) = \mathbf{v}^S(\mathbf{X},t) \circ \left(\mathbf{x}^S(\mathbf{X},t)\right)^{-1}. \tag{2}$$

However, in discretization, the description of the material is Lagrangian. The external stresses in boundary of fluid domain Γ_τ^F are designated by τ^F. Aiming just written a more simplified of the equations at this moment, it is assumed that no external tractions prescribed in boundary of the solid domain.

The interface boundary $\mathbf{\Gamma}_{FSI}$ between fluid and solid domains is defined by a Level Set Level Set-ϕ (or implicit function):

$$\mathbf{\Gamma}_{FSI}(t) = \left\{\mathbf{x} \in \Omega_{CD} | \phi(\mathbf{x},t) = 0\right\}, \tag{3}$$

and $\phi(\mathbf{x},t) = 0$ corresponds to the interface of the domains. Hence, for a instant $t \in \mathbb{R}_+$ we have:

$$\begin{aligned} &\phi(\mathbf{x},t) > 0 \Rightarrow \mathbf{x} \in \Omega_F; \\ &\phi(\mathbf{x},t) < 0 \Rightarrow \mathbf{x} \in \Omega_S. \end{aligned} \tag{4}$$

In this sense the normal vector of the boundary of the solid domain can be defined as follows:

$$\hat{\mathbf{n}}^S = \frac{\nabla\phi}{\|\nabla\phi\|},\ (\hat{n}_i^S = \phi_{,i}), \tag{5}$$

where the notation $(.)_{,i}$ designates derivative with respect to x_i,thus in this region the normal to the fluid domain is given by $\mathbf{n}^F = -\mathbf{n}^S$, could have the following definition

$$\phi(\mathbf{x},t) = \min_{\tilde{\mathbf{x}} \in \Gamma_{FSI}} \left(\|\mathbf{x} - \tilde{\mathbf{x}}\|\right) \mathbf{sign}\left[\hat{\mathbf{n}}^S \cdot (\mathbf{x} - \tilde{\mathbf{x}})\right]. \tag{6}$$

The implicit functions are given in terms of conventional norms, such as the Euclidean norm. It should also comment that when another interface is required in the computational domain studied, the implicit function must attain the value, $\phi(\mathbf{x},t) = 0$, which describes the boundary of fluid domain in contact with other domains. We also observe that in the case of interfaces "not smooth" can determine the normal vector according to the technique described in Sethian (1999).

The problem called *strong form* can be stated as:

Problem 1. *Determine* $\mathbf{u}(\mathbf{x},t)$*, for each* $t \in \mathbb{R}_+$*, such that*

$$\nabla \cdot \sigma^{\mathbf{S}}(\mathbf{x},t) + \rho^S(\mathbf{x})\,\mathbf{b}^S(\mathbf{x},t) = \rho^S(\mathbf{x})\,\ddot{\mathbf{u}}(\mathbf{x},t), \forall \mathbf{x} \in \Omega_S; \tag{7}$$

$$\nabla \cdot \sigma^{\mathbf{F}}(\mathbf{x},t) + \rho^F(\mathbf{x})\,\mathbf{b}^F(\mathbf{x},t) = \rho^F(\mathbf{x})\,\ddot{\mathbf{u}}(\mathbf{x},t), \forall \mathbf{x} \in \Omega_F; \tag{8}$$

$$\sigma^S(\mathbf{x},t) \cdot \hat{\mathbf{n}}^S(\mathbf{x},t) = -\lambda, \forall \mathbf{x} \in\ \Gamma_{FSI}; \tag{9}$$

$$\sigma^F(\mathbf{x},t) \cdot \hat{\mathbf{n}}^F(\mathbf{x},t) = \lambda, \forall \mathbf{x} \in\ \Gamma_{FSI}; \tag{10}$$

$$\sigma^F(\mathbf{x},t) \cdot \hat{\mathbf{n}}^F(\mathbf{x},t) = \tau^F(\mathbf{x},t), \forall \mathbf{x} \in\ \Gamma_\tau^F; \tag{11}$$

$$\dot{\mathbf{u}}^S(\mathbf{x},t) = \dot{\mathbf{u}}^F(\mathbf{x},t), \forall \mathbf{x} \in \Gamma_{FSI}; \tag{12}$$

$$\dot{\rho}^F + \nabla \cdot \left(\rho^F \dot{\mathbf{u}}^F(\mathbf{x},t)\right) = 0, \forall \mathbf{x} \in \Omega_F. \tag{13}$$

with $\tau^F \in [H^{\frac{1}{2}}(\Gamma_\tau^F)]^2$ *e* $\mathbf{b}^{(\cdot)} \in [L_2(\Omega_{(\cdot)})]^2$, $\lambda \in [L_2(\Gamma_{FSI})]^2$ *(Lagrange multipliers), where the Cauchy stress tensor is designated by the letter* $\sigma^{(\cdot)}$ *e* $\rho^{(\cdot)}$ *designates the density. As a matter of ease of writing consider the omission of the boundary conditions associated with zero speed.*

The equations 7 and 8 refer to the equations of momentum for the solid and fluid domains respectively. The eq. 11 matches with the boundary condition of traction for the fluid domain. The eq. 13 is the continuity equation, and equations 9 and 10 matches with the condition

$$\sigma^F(\mathbf{x},t)\cdot\hat{\mathbf{n}}^F(\mathbf{x},t) + \sigma^S(\mathbf{x},t)\cdot\hat{\mathbf{n}}^S(\mathbf{x},t) = \mathbf{0}, \forall \mathbf{x} \in \Gamma_{FSI}. \tag{14}$$

For the Augmented Lagrangian Method the equations 3 and 4, could be rewritten as

$$\sigma^S(\mathbf{x},t)\cdot\hat{\mathbf{n}}^S(\mathbf{x},t) + \lambda(\mathbf{x},t) + \beta\left(\dot{\mathbf{u}}^S(\mathbf{x},t) - \dot{\mathbf{u}}^F(\mathbf{x},t)\right) = \mathbf{0}, \forall \mathbf{x} \in \Gamma_{FSI}; \tag{15}$$

$$\sigma^F(\mathbf{x},t)\cdot\hat{\mathbf{n}}^F(\mathbf{x},t) - \lambda(\mathbf{x},t) - \beta\left(\dot{\mathbf{u}}^S(\mathbf{x},t) - \dot{\mathbf{u}}^F(\mathbf{x},t)\right) = \mathbf{0}, \forall \mathbf{x} \in \Gamma_{FSI}. \tag{16}$$

Setting up, at this point, the following sets for each $t \in \mathbb{R}_+$

$$\begin{aligned}
Kin_{\dot{u}}(\Omega_{CD}) &= \left\{\dot{\mathbf{u}} : \Omega_{CD} \to \mathbb{R}^2 \mid \dot{\mathbf{u}} \in \left[H^1(\Omega_{CD})\right]^2 \text{ , } \dot{\mathbf{u}}(\mathbf{x},t) = \mathbf{0} \text{ in } \mathbf{x} \in \Omega_{CD}\right\};\\
Kin_{\rho}(\Omega_F) &= \left\{\rho : \Omega_F \to \mathbb{R}_+ \mid \rho(\mathbf{x},t) \in H^1(\Omega_F), \text{ in } \mathbf{x} \in \Omega_F\right\};\\
Kin_{\lambda}(\Gamma_{FSI}) &= \left\{\lambda : \Gamma_{FSI} \to \mathbb{R} \mid \lambda(\mathbf{x},t) \in L_2(\Gamma_{FSI}), \text{ in } \mathbf{x} \in \Gamma_{FSI}\right\}.
\end{aligned}$$

denoting, for each $t \in \mathbb{R}_+$,

$$\begin{aligned}
\digamma_m\left(\dot{\mathbf{u}}^F, \dot{\mathbf{u}}^S, \lambda; \delta\mathbf{v}^F, \delta\mathbf{v}^S, \delta\lambda\right) = &\int_{\Omega_{CD}} \left\{\left[\rho^F\left(\mathbf{b}^F - \ddot{\mathbf{u}}^F\right)\cdot\delta\mathbf{v}^F - \sigma^F : \nabla\delta\mathbf{v}^F\right] H(\phi)\right.\\
&+ \left.\left[\rho^S\left(\mathbf{b}^S - \ddot{\mathbf{u}}^S\right)\cdot\delta\mathbf{v}^S - \sigma^S : \nabla\delta\mathbf{v}^S\right] H(-\phi)\right\} d\Omega\\
&+ \int_{\Gamma_\tau^F} \tau^F \cdot \delta\mathbf{v}^F d\Gamma + \delta\left[\int_{\Gamma_{FSI}} \lambda \cdot \left(\dot{\mathbf{u}}^F - \dot{\mathbf{u}}^S\right) d\Gamma\right]\\
&+ \frac{\beta}{2}\delta\left[\int_{\Gamma_{FSI}} \left\|\dot{\mathbf{u}}^F - \dot{\mathbf{u}}^S\right\|^2 d\Gamma\right],\\
&\forall\left(\delta\mathbf{v}^F, \delta\mathbf{v}^S, \delta\lambda\right) \in Kin_{\dot{u}}(\Omega_{CD}) \times Kin_{\dot{u}}(\Omega_{CD}) \times Kin_{\lambda}(\Gamma_{FSI}).
\end{aligned} \tag{17}$$

and

$$\digamma_c\left(\dot{\mathbf{u}}^F, \rho^F; \delta\rho^F\right) = \int_{\Omega_{CD}} \left[\dot{\rho}^F + \nabla\cdot\left(\rho^F\dot{\mathbf{u}}^F(\mathbf{x},t)\right)\right]\delta\rho^F H(\phi)\, d\Omega, \forall\delta\rho^F \in Kin_\rho(\Omega_F), \tag{18}$$

which has the state equations for the pressure in a fluid medium

where the function $H(z)$ is the Heaviside step function, which is equal to 1 if z is positive and vanishes otherwise. In order to enforce the continuity of the velocity on the interface, the above includes a Lagrange multiplier field λ and a penalty with penalty parameter scalar β.

The equation (17) is the weak form of the continuity equation. This equation is not needed in the solid because a Lagrangian description is used there and an algebraic equation $\rho^S J^S = \rho_0^S$ suffices to verify conservation of mass.

To show that the above weak form implies the strong right way, we need the following identities:

$$H_{,i}(\phi) = \delta(\phi)\phi_{,i} = -\delta(\phi)\hat{n}_i^F \tag{19}$$

and

$$H_{,i}(-\phi) = -\delta(-\phi)\phi_{,i} = -\delta(\phi)\hat{n}_i^S \tag{20}$$

from which it follows:

$$\int_{\Omega_{CD}} f H_{,i}(\phi) d\Omega = -\int_{\Omega_{CD}} f\delta(\phi)\hat{n}_i^F d\Omega = -\int_{\Gamma} f\hat{n}_i^F d\Gamma \tag{21}$$

and

$$\int_{\Omega_{CD}} f H_{,i}(-\phi) d\Omega = -\int_{\Omega_{CD}} f\delta(\phi)\hat{n}_i^S d\Omega = -\int_{\Gamma} f\hat{n}_i^S d\Gamma \tag{22}$$

Eq. (19) is simply the chain rule combined with (5). The eq. (21) is based on (19) and the fact that the Dirac delta function of the level set gives a boundary integral, i.e. a surface integral in three dimensions or a line in two dimensions.

The second term of the first integral of (17) can be transformed as follows by integration by parts and using the condition that $\delta\dot{\mathbf{u}} = 0$ on Γ_{CD}:

$$\begin{aligned} \int_{\Omega_{CD}} \delta\ddot{\mathbf{u}}^F \sigma^F H(\phi) d\Omega &= \int_{\Omega_{CD}} \{(\delta\dot{\mathbf{u}}^F\sigma^F)_{,j} - \delta\dot{\mathbf{u}}^F\sigma^F_{,j} H(\phi) - \delta\dot{\mathbf{u}}^F\sigma^F H(\phi)_{,j}\} d\Omega \\ &= \int_{\Gamma_{out}} \delta\dot{\mathbf{u}}^F\sigma^F\hat{\mathbf{n}}^F d\Gamma - \int_{\Omega_F} \delta\dot{\mathbf{u}}^F\sigma^F_{,j} d\Omega + \int_{\Gamma_{FSI}} \delta\dot{\mathbf{u}}^F\sigma^F\hat{\mathbf{n}}^F d\Gamma, \end{aligned} \tag{23}$$

where Γ_{out} denotes the entire exterior boundary of the fluid. In the above, the second and third lines follow by using (19) and (21).

Similarly in the solid:

$$\int_{\Omega_{CD}} \delta\ddot{\mathbf{u}}^S \sigma^S H(-\phi) d\Omega = -\int_{\Omega_S} \delta\dot{\mathbf{u}}^S\sigma^S_{,j} d\Omega + \int_{\Gamma_{FSI}} \delta\dot{\mathbf{u}}^S\sigma^S\hat{\mathbf{n}}^S d\Gamma, \tag{24}$$

Substituting (23) and (24) into (17), yields:

$$\begin{aligned} &\int_{\Omega_F} (\mathbf{b}^F + \sigma^F_{,j}) d\Omega\delta\dot{\mathbf{u}}^F + \int_{\Omega_S} (\mathbf{b}^S + \sigma^S_{,j})\delta\dot{\mathbf{u}}^S d\Omega - \int_{\Gamma^F_{out}} \sigma^F\hat{\mathbf{n}}^F\delta\dot{\mathbf{u}}^F d\Gamma + \int_{\Gamma^F_\tau} \tau^F\delta\dot{\mathbf{u}}^F d\Gamma \\ &+ \int_{\Gamma_{FSI}} \delta\lambda(\dot{\mathbf{u}}^F - \dot{\mathbf{u}}^S) d\Gamma + \int_{\Gamma_{FSI}} \{\delta\dot{\mathbf{u}}^F(-\sigma^F\hat{\mathbf{n}}^F + \lambda + \beta(\dot{\mathbf{u}}^F + \dot{\mathbf{u}}^S))\} d\Gamma \\ &+ \int_{\Gamma_{FSI}} \{\delta\dot{\mathbf{u}}^S(-\sigma^S\hat{\mathbf{n}}^S + \lambda + \beta(\dot{\mathbf{u}}^F + \dot{\mathbf{u}}^S))\} d\Gamma = 0 \end{aligned} \tag{25}$$

To the equation of state for the fluid

$$\begin{aligned} &p(\rho^F) = \rho^F RT \text{ (for perfect fluids);} \\ &p(\rho^F) = \rho^F_o c^2 \left(1 - \frac{\rho^F_o}{\rho^F}\right), \text{ com } \rho^F_o - \text{initial density (for water).} \end{aligned} \tag{26}$$

So the problem can be written as

Problem 2. *Determine* $(\mathbf{v}^F, \mathbf{v}^S, \lambda, \rho^F) \in Kin_{\dot{u}}(\Omega_{CD}) \times Kin_{\dot{u}}(\Omega_{CD}) \times Kin_\lambda(\Gamma_{FSI}) \times Kin_\rho(\Omega_F)$ *for each* $t \in \mathbb{R}_+$, *such that*

$$\mathcal{F}\left(\dot{\mathbf{u}}^F, \dot{\mathbf{u}}^S, \lambda, \rho^F; \delta\mathbf{v}^F, \delta\mathbf{v}^S, \delta\lambda, \delta\rho^F\right) = (\mathcal{F}_m, \mathcal{F}_c)^T = \mathbf{0}, \tag{27}$$

$\forall \left(\delta\mathbf{v}^F, \delta\mathbf{v}^S, \delta\lambda, \delta\rho^F\right) \in Kin_{\dot{u}}(\Omega_{CD}) \times Kin_{\dot{u}}(\Omega_{CD}) \times Kin_\lambda(\Gamma_{FSI}) \times Kin_\rho(\Omega_F)$.

Denoting $\mathbf{u} = \left(\mathbf{v}^F, \mathbf{v}^S, \lambda, \rho^F\right) \in Kin_U = Kin_{\dot{u}}(\Omega_{CD}) \times Kin_{\dot{u}}(\Omega_{CD}) \times Kin_{\lambda}(\Gamma_{FSI}) \times Kin_{\rho}(\Omega_F)$ has

Problem 3. *Determine* $\mathbf{u} \in Kin_U$ *for each* $t \in \mathbb{R}_+$*, such that*

$$F\left(\mathbf{u}; \delta\mathbf{u}\right) = \mathbf{0}, \tag{28}$$

$\forall \delta\mathbf{u} \in Kin_U$.

To solve the problems defined in equations 27 and 28 can be used the Newton's method.

2.2 Discretization

The fluid and solid velocities are discretized by using the following approximations for the velocity field:

$$\dot{\mathbf{u}}^F(\mathbf{x}, t) = N_I^F(\mathbf{x}) V_{iI}^F(t) H(\phi(\mathbf{x}, t)), \tag{29}$$

The fluid and solid velocities are discretized by using the following approximations for the velocity field:

$$\dot{\mathbf{u}}^S(\mathbf{X}, t) = N_I^S(\mathbf{X}) V_{iI}^S(t), \tag{30}$$

where V_{iI}^F and V_{iI}^S are respectively, the nodal velocities for the fluid and solid and N_I^F and N_I^S are respectively, the shape functions for the fluid and for the solid. All repeated upper case indices are repeated over their range, which, for example, are the fluid nodes and the solid (or structural) nodes in (29) and (31) respectively. Note that the velocities are approximated by shape functions that are expressed in terms of the spatial coordinates for the fluid, but are a function of the material coordinates for the solid. The shape functions for the fluid and solid may be different. The density is approximated by the same shape functions as the velocity for the fluid:

$$\rho^F(\mathbf{x}, t) = N_I^F \rho_I^F(t) H(\phi(\mathbf{x}, t)). \tag{31}$$

The Lagrange multipliers are discretized by the following:

$$\lambda(\zeta, t) = N_I^{\lambda}(\zeta) \Lambda_{iI}(t), \tag{32}$$

where Λ_{iI} are the nodal values of the Lagrange multipliers, and ζ are the local coordinates that describe the fluid–solid interface. The mesh for the Lagrange multipliers must conform to the interface $\phi = 0$ and must satisfy the Babuska–Brezzi stability condition. We choose linear shape functions in the following; we have not investigated its stability as well as the interface consistency as described in Park *et al.* (2001). The mesh for the Lagrange multipliers is the set of points which are the intersection points between the structure and the fluid mesh .

Substituting (29), (30) and (32) into (17) yields:

$$\begin{aligned}
&-\int_{\Omega_{CD}} \rho^F N_J^F N_I^F \dot{V}_{iI}^F H(\phi) d\Omega - \int_{\Omega_{CD}} \rho^F N_J^F \dot{\mathbf{u}}^F \ddot{\mathbf{u}}^F H(\phi) d\Omega - \int_{\Omega_{CD}} N_{J,j}^F \sigma^F H(\phi) d\Omega \\
&-\int_{\Omega_{S_0}} \rho^S N_J^S N_I^S \dot{V}_{iI}^S H(\phi) d\Omega_0 - \int_{\Omega_{S_0}} \frac{\partial N_J^S}{\partial X_j} \sigma^S d\Omega_0 + \int_{\Gamma_\tau^F} N_J^F \tau^F d\Gamma + \beta \int_{\Gamma_{FSI}} N_J^F (N_I^F V_{iI}^F - N_I^S V_{iI}^S) d\Gamma \\
&-\beta \int_{\Gamma_{FSI}} N_J^S (N_I^F V_{iI}^F - N_I^S V_{iI}^S) d\Gamma + \int_{\Gamma_{FSI}} N_J^F N_I^{\lambda} \Lambda_{iI} d\Gamma - \int_{\Gamma_{FSI}} N_J^S N_I^{\lambda} \Lambda_{iI} d\Gamma \\
&+\int_{\Gamma_{FSI}} N_J^{\lambda} (N_I^F V_{iI}^F - N_I^S V_{iI}^S) d\Gamma = 0
\end{aligned} \tag{33}$$

Dropping the indicator function on the solid domain (using a Lagrangian description), the integral can always be expressed in terms of the initial domain. The gravity is omitted in the discretized form of the equations. Substituting (31) into (18), yields the discretized form of the continuity equation:

$$\int_{\Omega_{CD}} N_J^F N_I^F \dot{\rho}_I^F H(\phi) d\Omega + \int_{\Omega_{CD}} N_J^F (\rho^F \dot{\mathbf{u}}^F)_{,i} H(\phi) d\Omega = 0 \tag{34}$$

3. APLICATIONS

In this first approach aims to make the study of fluid-structure interaction formulation, that is a known problems in the oil industry. In most cases such problems have a strong committed relationship involving costly and even dangerous. Some of the proposed feasible study would be a numeric evaluation of the structural behavior of submerged equipment as described below.

3.1 Manifold, PLET e PLEM

For example, the design of large subsea structures used in drilling for oil, as Manifold (see Fig.1), PLET and PLEM, require the correct determination of the loads involved in transport, installation and operation. For transportation and operation of these loads can be defined with little difficulty. But in the process of installation, beyond dynamic components of motion of the boat, waves and currents, there are also hydrodynamic components. The installation can be separated into two distinct steps: entry into the water (Splash Zone) and with the equipment fully submerged. Documents created by companies such as DNV (Det Norske Veritas) assist in determining these charges, however, depends on factors to set important values as the added mass. This is also important to assess the dynamic status of equipment, along with the damping provided by sea water.

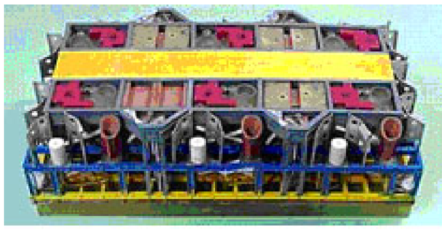

Figure 1. Manifold being installed (left) and Manifold Model for testing (right) (font: (Petroleo, TN))

The added mass is often drawn from literature that provides factors for geometries behaved. Thus, the equipment is simplified, turned into cylinder, blocks and/or plates. The values obtained are usually higher than the reality and structure will be oversized and therefore uneconomic the equipment and increase the installation cost, which might even derail the project.

Another way to obtain these values is by testing with reduced models that also provide dynamic conditions. That is, provides condition of loading for the equipment. The problem with this solution is cost and, especially, the time required to complete the test. Moreover, if the equipment has changed considerably in its mass or geometry, the test should be completely redone, which rarely occurs.

3.2 Dynamic of Risers

It is notorius the importance of offshore oil exploration, especially in Brazil, and Risers are essential structures for this activity. A good understanding of the dynamics of these bodies and the stresses they are submitted, has generated a fertile field of research, aiming at the understanding of very complex phenomena/effects that determine the design and the life time. Also comments that, at présent, the theoretical models,Can an even comment on that at present the theoretical modeling, both analytical and numerical, and experimental techniques are employed in a complementary way.

The dynamics of Risers involves issues such as structural mechanics of risers, pipelines, mooring lines, dynamic positioning of platforms and floating units, as well as nonlinear dynamics and hydrodynamics applied. The deep water offshore engineering has been the main stage of operation of these lines of research, the evolution of the national oil industry has required the analysis of stresses of different natures in submerged structures.

The transport of fluid and power between platforms or vessels on the surface and the fields of exploration on the ocean floor is made by structures called Risers, which are submerged in a inside water and assist in drilling and in the production of wells. The classification of these structures is rigid Risers and flexible Risers (see Fig.2).

The rigid vertical Risers do not support large curvatures, and in general, are used primarily in drilling wells (and also in production of wells), has a tensioner at the top end (to avoid buckling) and a articulated joint at the bottom that allows maximum inclination around the 10°. If there are major problems with the movement of the platform, the Riser can be hydraulically disconnected from the well. Risers, catenary-shaped, has also been used in deep water exploration.

The flexible Risers, mainly used in producing wells, are designed to have great flexibility and thus support the stress of the platform and the hydrodynamic loads. So these structures must have high axial rigidity and low flexural strength.

The drilling activity assisted by rigid vertical Risers is characterized by the guided installation of drill , by specific devices, at the wellhead. In this Riser is injected the called “drilling mud” (made of clay, water and chemicals) that aims to lubrication and cooling of the drill and provides support to the Riser and prevent the rise of undersirable oil. Furthermore, in the activity of the production, the Riser’s function is to transport the fluids products of well to the platform.

Whatever the activity (drilling or production) the Risers are subject to internal stresses (hydrostatic pressure of the fluid carried inside) and external (surface waves and ocean currents that can cause mats vortices around the structure) com-

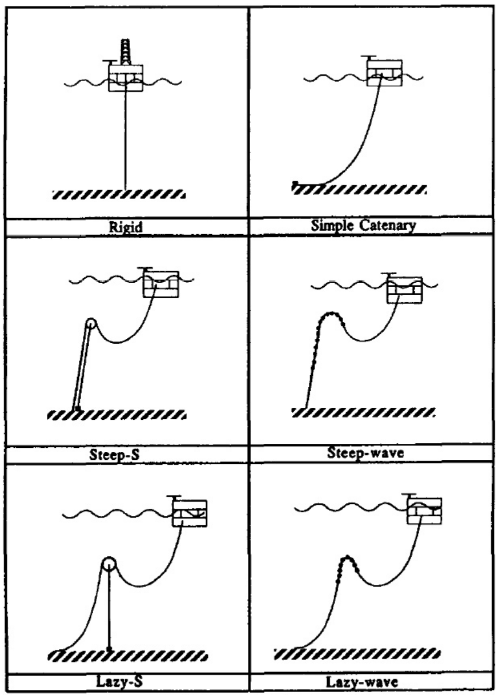

Figure 2. Examples of Riser. (fonte: Patel and Sayed (1995))

plex in nature (for details [(Meneghini, 1993), (Ferrari, 1999), (Jeong and Hussain, 1995), (Blevins, 1990),(Petroleo, TN)]).

In this sense, a very appropriate approach to the problems arranged in section 2.1 is the numerical simulation, since has not an exact analytical solution and explicit to the mentioned coupled problems (due to the complexity of his models). Allied to this fact, the numerical-computational approach can take advantage of the geometry of 3D CAD models already available and thus accrete gain of time and decrease the final cost of a series of procedures. Thus, the numerical analysis of structural dynamics and fluid flow become of paramount importance in these applications fluid-structure interaction, providing an important tool of estimation and analysis the complex phenomena involved.

4. CONCLUSION

This work proposes a formulation to FSI problems with intention of oil industry future applications as two problems that are shown on offshore platforms. The proposed formulation shows a compact form of the weak formulation of the problem that turn discretization process less complex. Considering the next steps of this research, in computational sense, three distinct stages are named: the fluid problem, the solid structure and the fluid-structure coupling. Initially we will intend to create a basic structure for FSI problems (elastic solid and incompressible fluid, monophase and Newtonian) which actually is in development in MATLAB. After this, we proceed to the enrichment of this structure (a priori and posteriori error estimators, adaptivity mesh, improved conditioning of the discretized operators, sensitivity analysis and applications to classical FSI problems).

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6. REFERENCES

Belytschkoand, T. and Kennedy, J.M., 1978. “Computer models for subassembly simulation”. *Nucl. Engrg. Des.*, Vol. 49, pp. 17–38.

Bisplinghoff, R.L., Ashley, H. and Halfman, R.L., 1983. *Aeroelasticity*.

Blevins, R., 1990. *Flow-induced Vibration*. 2nd ed.

Blevins, R., 1994. *Flow-Induced Vibration*.

Chen, H.C. and Taylor, R., 1990. “Vibration analysis of fluid-solid systems using a finite element displacement formulation”. *International Journal for Numerical Methods in Engineering*, Vol. 29, pp. 683–698.

Chessa, J. and Belytschko, T., 2003. “An extended finite element method for two-phase fluids”. *ASME J. Appl. Mech.*, Vol. 70, pp. 10–17.

Chessa, J., Smolinski, P. and Belytschko, T., 2002. "The extended finite element method (xfem) for solidification problems". *Int. J. Numer. Methods Engrg.*, Vol. 53, pp. 1957–1977.

Chessa, J., Wang, H. and Belytschko, T., 2003. "On the construction of blending elements for local partition of unity enriched finite elements". *Int. J. Numer. Methods Engrg.*, Vol. 57, pp. 1015–1038.

Dowell, E.H., Crawley, E., Jr., H.C., Peters, D.A., Scanlan, R. and Sisto, F., 1995. *A Modern Course in Aeroelasticity*.

Dowell, E.H. and Ilgamov, M., 1988. *Studies in Nonlinear Aeroelasticity*.

Farhat, C., Geuzaine, P. and Brown, G., 2003. "Application of a three-field nonlinear fluid-structure formulation to the prediction of the aeroelastic parameters of an f-16 fighter". *Comput. Fluids*, Vol. 32, pp. 3–29.

Farhat, C., Geuzaine, P. and Grandmont, C., 2001. "The discrete geometric conservation law and the non-linear stability of ale schemes for the solution of flow problems on moving grids". *J. Comput. Phys.*, Vol. 174, pp. 669–694.

Ferrari, J.A., 1999. *Hydridinamics Loading and Response of Offshore Risers*. Ph.D. thesis, Imperial College of Science.

Fung, Y.C., 2000. *Biomechanics: Motion, Flow, Stress, and Growth*.

Glowinski, R., Pan, T.W., Hesla, T.I., Joseph, D.D. and Periaux, J., 2001. "A fictitious domain approach to the direct numerical simulation of incompressible viscous flow past moving rigid bodies:application to particulate flow". *J. Comput. Phys.*, Vol. 169, pp. 363–426.

Hu, H.H., Patankar, N.A. and Zhu, M.Y., 2001. "Direct numerical simulations of fluid-solid systems using arbitrary-lagrangian-eulerian technique". *J. Comput. Phys.*, Vol. 169.

Jeong, J. and Hussain, F., 1995. "On identification of a vortex". *J. Fluid Mechanics*, Vol. 285, pp. 69–94.

Meneghini, J., 1993. *Numerical Simulation of Bluff Body Flow Control Using Vortex Method*. Ph.D. thesis, Faculty of Eng. of the University of London.

Osher, S. and Fedkiw, R., 2003. *Level set methods and dynamic implicit surfaces*.

Page, R.E., 1982. "Method of suppressing paper web flutter".

Paidoussis, M.P., 1998. *Fluid-Structure Interactions-Slender Structures and Axial Flow*. vol. I.

Paidoussis, M.P., 2004. *Fluid-Structure Interactions-Slender Structures and Axial Flow*. vol. II.

Paidoussis, M.P. and Issid, N.T., 1974. "Dynamic stability of pipes conveying fluid". *Journal of Sound and Vibration*, Vol. 33, pp. 267–294.

Park, K.C., Felippa, C.A. and Ohayon, R., 2001. "Partitioned formulation of internal fluid-structure interaction problems by localized lagrange multipliers". *Comput. Methods Appl. Mech. Engrg.*, Vol. 190, pp. 2989–3007.

Patankar, N.A., Singh, P., Joseph, D.D., Glowinski, R. and Pan, T.W., 1984. "A new formulation of the distributed lagrange multiplier fictitious domain method for particulate flows". *Int. J. Multiphase Flow*, Vol. 26, pp. 1509–1524.

Patel, M.H. and Sayed, F.B., 1995. *Review of flexible riser modelling and analysis techniques*. Vol. 17.

Peskin, C.S., 2002. "The immersed boundary method". *Acta Numer.*, Vol. 11, pp. 479–517.

Petroleo, TN. "Magazine TN Petroleo". 20 Feb. 2010 <http://www.tnpetroleo.com.br/images/>.

Roma, A.M., Peskin, C.S. and Berger, M.J., 1999. "An adaptive version of the immersed boundary method". *Journal of Computational Physics*, Vol. 153, pp. 509–534.

Sethian, J.A., 1999. *Level Set Methods and Fast Marching Methods Evolving Interfaces in Computational Geometry, Fluid mechanics, Computer Vision, and Materials Science*.

Wilson, E.L. and Khalvati, M., 1983. "Finite elements for the dynamic analysis of fluid-solid systems". *International Journal for Numerical Methods in Engineering*, Vol. 19, pp. 1657–1668.