

LAMINAR DIFFUSION FLAME FORMATION IN TWO PHASE JETS

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Abstract. *In this work, diffusion flames formed by coaxial jets of gas-particle two-phase flows composed by a fuel (gas) jet with solid particles, and an external jet of oxidant with or without particles are considered. An analytical solution is developed to estimate the fuel and the oxidant concentrations as well as the flame position. The flame shapes are obtained from analytical models and there are compared with data from experimental results for a number of laminar gas jets configurations. The focus of this study is to analyse the characteristics of the effect soot particles on the flame. The particles presented effects that modify the combustion process. For example, the heat transfer by radiation in the vicinity of the flame is strongly affected by particles. This work is part of a project to study the formation of diffusion flames in swirl injector.*

Keywords: *diffusion flame, two-phase combustion, two-phase jets, Burke-Schumann limit*

1. INTRODUCTION

The formation of diffusion flames by coaxial jets of fuel and oxidant with separated flows is much employed in aerospace and energy generation equipment. In industrial or aircraft engines applications, the diffusion flame have turbulent behavior. But the initial studies of combustion theoretical analysis of diffusion flame are done considering laminar behavior (Kim and Williams, 1997). The Burke and Schumann (1928) work, presented an analytical solution for gaseous fuel and oxidant coaxial jets. In this work, the laminar flame is defined as fuel and oxidant separation surface with infinitely thin thickness. The mass diffusion transport is limited by thermal transport, where the Lewis number is unity (Kuo, 1986).

Two-phase reactive flow is present in most combustion processes (Pai, 1977). In gaseous diffusion combustion processes, another phase (droplets or particles) affect the physical processes. As the cohesive forces between molecules of liquids or solids are more intense, the diffusion process is controlled by gas phase. But the diffusion gaseous process intensity is strongly affected by droplets or particles volume occupation in the mixture. When the reactive process involves the liquid or solid phase, with source of mass and heat the combustion process is analyzed as spray phenomenon, Boltzmann type analysis (Kuo, 1986). In this work the reactive combustion process is restricted to the gas phase, affected by other phase.

Normally hydrocarbon combustion is soot formation process (McEnally *et al.*, 1998). The soot particles are composed by solid carbon with density of the 10^3 order, constituting a second phase into combustion process (Travelho, 1987). In this work, the soot process formation is not studied, the analysis is concentrated in soot particles effect on the mass fraction of fuel and oxidant and on the temperature in the vicinity of the flame. The soot particles affect the diffusion and the heat transfer processes, occupying volume in the mixture and absorbing heat from the reactive process. This leads to a definition of effective diffusion for the two-phase gas reactive flow. This effective diffusion is controlled by volume and mass fraction or, in physics variable terms, by gas and mixture densities.

For easy the solution of conservation equations, the thermophoretic effect (Keh and Chen, 1997) on this soot particles in the first analysis is not considered, and the mixture is homogeneous.

This work is part from study of ignition and extinction limits process in two-phase gas reactive flow analysis of swirl injector for gas turbine/rocket hydrocarbon fuel application. The main core of this work is flight safety, in limit situation of soot formation, as rich flame.

2. MATHEMATICAL DEVELOPMENT

An analysis of diffusion flame in two-phase flow has been made which has led to influence mechanism on shape flame.

A mathematical development of the theory is given based on volume fraction of solid particles. Here the flame is analyzed as discontinuous surface among fuel-particles mixture and oxidant-particles mixture.

The main objectives in this work are the flame shape modified from volume occupation of particles into mixture and the flame temperature.

2.1. Physical Model

The diffusion flame formed from the two-phase gas reactive flow is shown in Fig. 1. The physical model consists of two concentric tubes. Two streams of gas with solid particles flows through the two tubes. The gas that flows through the inner tube of radius r_i is fuel. The oxidant flows through the outer tube of radius r_e . The two streams come into

contact, forming the flame. The radial distance is represented by r and the vertical distance by z from the internal exit tube, where $z=0$.

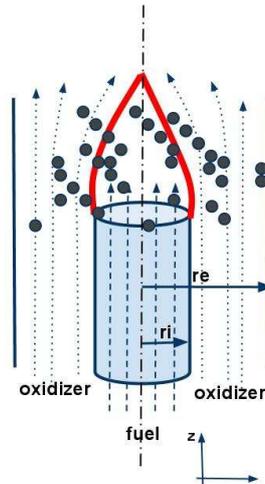


Figure 1. Physical schema of the burner.

The solid particles formation is not considered, and the mixture is homogeneous. The laminar flame is formed by two mixture flow with same mean velocity in coaxial cylindrical tubes. In general, the velocity of the gas may be different from that of the solid particles. We may assume that the velocity of gas and that of solid particles are the same. If we consider the mixture of gas and solid particles, one of the most important parameters is the volume fraction. The volume fraction is defined as follows:

$$f = \frac{V_g}{V} \quad (1)$$

where V_g is the volume of gas in an element of mixture and V is total volume of an element of mixture. As we consider gas and solid particles, the ratio f represents the fraction of volume in which solid particles is void. The fraction of volume occupied by the solid particles is the complement to V .

The density of the mixture is defined as

$$\rho = \frac{m_g + m_p}{V_g + V_p} = \frac{m}{V} \quad (2)$$

where subscript g refers to value of the gas, the subscript p to the value of the solid particles and without subscript refers to the mixture. The species density is defined as follows:

$$\rho_g = \frac{m_g}{V_g} \quad (3)$$

and

$$\rho_p = \frac{m_p}{V_p} \quad (4)$$

The partial density of a species in the mixture is defined as follows:

$$\bar{\rho}_g = f \frac{m_g}{V_g} \quad (5)$$

and

$$\bar{\rho}_p = f \frac{m_p}{V_p} \quad (6)$$

The mass flux of fuel in the mixture through a normal stationary plane is done as follows:

$$\dot{\mathbf{m}}_f = f \rho_f \mathbf{u}_g - \rho_g D \nabla (f Y_f) \quad (7)$$

where \mathbf{u}_g is the velocity vector of gas, D is the mass diffusivity in binary system fuel-oxidant and ρ_f is the species density of fuel. The mass fraction of fuel is defined as follows:

$$Y_f = \frac{\rho_f}{\rho_f + \rho_o} = \frac{\rho_f}{\rho_g} \quad (8)$$

where ρ_o is the species density of oxidant.

The equation of continuity for fuel in a two-phase flow, in vector notation, may be written as:

$$\frac{\partial(f \rho_f)}{\partial t} + \nabla(\dot{\mathbf{m}}_f) = f \omega_f \quad (9)$$

Substituting Eq.(7) into Eq.(9), we obtain

$$\frac{\partial(f \rho_f)}{\partial t} + \nabla(f \rho_f \mathbf{u}_g) = \nabla \cdot \rho_g D \nabla (f Y_f) - f \omega_f \quad (10)$$

for a fluid of constant mass density ρ_g . Now ω_f has a negative sign because it represents the rate of consumption of mass of fuel.

Under the assumption that the velocity of gas and that of solid particles are the same, i.e.,

$$\mathbf{u}_g = \mathbf{u}_p = \mathbf{u} \quad (11)$$

(where \mathbf{u} we may define as velocity vector of the mixture) the Eq. (8) becomes:

$$\frac{\partial(f \rho_f)}{\partial t} + \nabla(f \rho_f \mathbf{u}) = \nabla \cdot \rho_g D \nabla (f Y_f) - f \omega_f \quad (12)$$

For cylindrical geometry under axisymmetric condition, the above species continuity equation in two-phase flow becomes:

$$\frac{\partial(f \rho_f)}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[D \rho_g r \frac{\partial}{\partial r} (f Y_f) \right] - \left[\frac{1}{r} \frac{\partial}{\partial r} (r f \rho_f u_r) + \frac{\partial}{\partial z} (u_z f \rho_f) \right] - f \omega_f \quad (13)$$

where $\mathbf{u} = u_r \hat{\mathbf{r}} + u_\theta \hat{\boldsymbol{\theta}} + u_z \hat{\mathbf{z}}$.

Here we assumed that axial diffusion is neglected, since it is considered to be small in comparison with radial term, i.e.,

$$\frac{\partial^2}{\partial r^2} (f Y_f) \gg \frac{\partial^2}{\partial z^2} (f Y_f) \quad (14)$$

Working the convective terms of Eq. (13) as

$$\frac{1}{r} \frac{\partial}{\partial r} (r f \rho_f u_r) + \frac{\partial}{\partial z} (u_z f \rho_f) = \rho_g u_r \frac{\partial}{\partial r} (f Y_f) + \rho_g u_z \frac{\partial}{\partial z} (f Y_f) \quad (15)$$

considering steady-state and $u_z \gg u_r$, the Eq. (13) becomes

$$\frac{1}{r} \frac{\partial}{\partial r} \left[D \rho_g r \frac{\partial}{\partial r} (f Y_f) \right] - \rho_g u_z \frac{\partial}{\partial z} (f Y_f) = f \omega_f \quad (16)$$

The previous density of mixture definition Eq. (2), with Eq. (3), (4), (5) and (6) becomes:

$$\rho = f \rho_g + (1-f) \rho_p \quad (17)$$

and the mixture mass fraction may be shows as:

$$\kappa = \frac{f \rho_g}{\rho} = \frac{m_g}{m_g + m_p} \quad (18)$$

As the previous discuss, the diffusion of mass is restricted to the gas phase and the mixture is homogeneous, the Eq. (16) becomes:

$$\frac{1}{r} \frac{\partial}{\partial r} \left[r f \rho_g D \frac{\partial}{\partial r} (\kappa Y_f) \right] - \rho u_z \frac{\partial}{\partial z} (\kappa Y_f) = f \omega_f \quad (19)$$

Assuming steady-state, constant pressure, small radial velocity component and negligible axial heat conduction, the form of the homogeneous mixture energy equation is

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\lambda}{C_p} \frac{\partial}{\partial r} (C_p T) \right) - \rho u_z \frac{\partial}{\partial z} (C_p T) = -Q \quad (20)$$

where C_p is the specific heat of the mixture at constant pressure, T is the temperature of the mixture, λ is the coefficient of heat conductivity of the mixture and Q is the heat source of the mixture.

We may assume the thermal equilibrium, i.e., the temperature of the gas and that the solid particles are the same. Following Pai (1977), the mixture coefficients above are defined as:

$$C_p = \frac{f \rho_g C_{p_g} + (1-f) \rho_p C_{p_p}}{f \rho_g + (1-f) \rho_p} \quad (21)$$

$$\lambda = f \lambda_g + (1-f) \lambda_p \quad (22)$$

where subscript g refers to value of the gas, the subscript p to the value of the solid particles.

As previous discuss, the reactive process is restricted to the gas phase, since heat source of the mixture is:

$$Q = f \omega_f \frac{\rho_g}{\rho} H_r \quad (23)$$

where H_r is the fuel specific heat of reaction.

2.2. The Burke-Schumann Flame

The Burke-Schumann flame is defined based on the ratio thermal energy and mass diffusion, into condition the infinitely thin flame. Initially, we should to define the Lewis number from mixture as

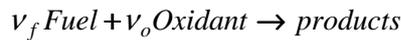
$$Le = \frac{\lambda/C_p}{f \rho_g D} \quad (24)$$

Considering that in the Shvab-Zel'dovich formulation we used the hypothesis $Le=1$, the definition of the Eq. (24) may be analyzed from volume fraction. Considering the Eq. (21), (22) and (24), the ratio of mixture and gas Lewis numbers becomes:

$$Le = Le_g \frac{\left(1 + \frac{1-f}{f} \frac{\lambda_p}{\lambda_g}\right) \left(1 + \frac{1-f}{f} \frac{\rho_p}{\rho_g}\right)}{\left(1 + \frac{1-f}{f} \frac{\rho_p C_{p_p}}{\rho_g C_{p_g}}\right)} \quad (25)$$

Normally, the typical values of the reasons involved in the Eq. (25) are around 10^3 for ratio of coefficients of heat conductivity, 10^4 for ratio of densities and around 10^{-1} for specific heat coefficients ratio. The value of volume fraction of solid particles in the mixture is around 10^{-6} , in according (Travelho, 1987) and (McEnally *et al.*, 1998). With this, the values of the ratio Le/Le_g are approximately unit.

The chemical reaction may be described as follows, once the solid phase reaction does not participate, i.e.:



where ν_f and ν_o are the stoichiometric coefficients.

Introducing the Lewis number definition for mixture in Eq. (19) with Eq. (18) and dividing by fuel gas mass in the reaction ($w_f \nu_f$), this gives:

$$\frac{u_z}{\kappa D} \frac{\partial}{\partial z} (\alpha_f) - \frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial}{\partial r} (\alpha_f) \right] = \frac{f \omega_f}{w_f \nu_f} \quad (26)$$

where $\alpha_f = -\kappa Y_f / (w_f \nu_f)$.

Similarly, the Eq. (26) can be written for the oxidant as:

$$\frac{u_z}{\kappa D} \frac{\partial}{\partial z} (\alpha_o) - \frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial}{\partial r} (\alpha_o) \right] = \frac{f \omega_o}{w_o \nu_o} \quad (27)$$

where $\alpha_o = -\kappa Y_o / (w_o \nu_o)$.

For energy equation, Eq. (20), with Eq. (23) we have the similar form:

$$\frac{u_z}{\kappa D} \frac{\partial}{\partial z} (\alpha_T) - \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} (\alpha_T) \right) = \frac{f \omega_f}{w_f \nu_f} \quad (28)$$

where $\alpha_T = -f C_p T / (\kappa H_r w_f \nu_f)$.

Following Kuo (1986), the profile of the mass fraction of fuel and of oxidant, and the profile of temperature in diffusion flame formed from coaxial two-phase flow jets is obtained combining the Eq. (26) and Eq. (27). The non-homogeneous term is eliminated by linear combination of α_f , α_o and α_T as:

$$\beta = \alpha_f - \alpha_o, \beta = \alpha_T - \alpha_f, \dots \quad (29)$$

The equation for β variable, in non-dimensional form is given, following Kuo (1986), as:

$$\frac{\partial \gamma}{\partial \eta} = \frac{1}{\xi} \frac{\partial}{\partial \xi} \left(\xi \frac{\partial \gamma}{\partial \xi} \right) \quad (30)$$

where $\xi = r/re$, $\eta = \{(z\kappa D)/(u_z re^2)\}$ and $\gamma = \{(\beta w_f v_f)/(Y_f)_{z=0}\}$ for mass fraction of fuel.

The boundary conditions for γ , following Kuo (1986), in coaxial two-phase flow jets for mass fraction of fuel and of oxidant are:

$$\gamma = \begin{cases} 1 & \text{at } \eta = 0 \quad 0 \leq \xi < \frac{ri}{re} \\ -\left(\frac{w_f v_f}{w_o v_o}\right) \left(\frac{Y_o}{Y_f}\right)_{z=0} & \text{at } \eta = 0 \quad \frac{ri}{re} \leq \xi < 1 \end{cases} \quad (31)$$

and

$$\frac{\partial \gamma}{\partial \xi} = 0 \quad \text{at } \xi = 1 \quad \text{and} \quad \xi = 0 \quad \eta > 0 \quad (32)$$

The solution of this problem (Kuo, (1986); Burke and Schurmann (1928)) in Bessel functions series is:

$$\gamma = \left\{ 1 - \left(\frac{w_f v_f}{w_o v_o}\right) \left(\frac{Y_o}{Y_f}\right)_{z=0} \right\} \left(\frac{ri}{re}\right)^2 + \left(\frac{w_f v_f}{w_o v_o}\right) \left(\frac{Y_o}{Y_f}\right)_{z=0} + 2 \left\{ 1 - \left(\frac{w_f v_f}{w_o v_o}\right) \left(\frac{Y_o}{Y_f}\right)_{z=0} \right\} \left(\frac{ri}{re}\right) \times \sum_{n=1}^{\infty} \frac{1}{\phi_n} \frac{J_1\left(\frac{ri}{re} \phi_n\right)}{[J_0(\phi_n)]^2} J_0\left(\phi_n \frac{r}{re}\right) e^{-\phi_n^2 \frac{z\kappa D}{u_z re^2}} \quad (33)$$

The expression showed in the Eq. (33) is similar to that of coaxial gas flow jets, i.e., the Burke-Schurmann problem. But the solid phase effect is introduced in the exponential factor as mass fraction of gas into mixture κ .

The mass fraction of gas into mixture ($\kappa = f \rho_g / \rho$) on the flame shape is obtained with condition $\gamma = 0$. The flame shape curves for coaxial two-phase flow jets, with reactive gas phase of methane and oxygen, and methane and air are showed in the Fig. 2.

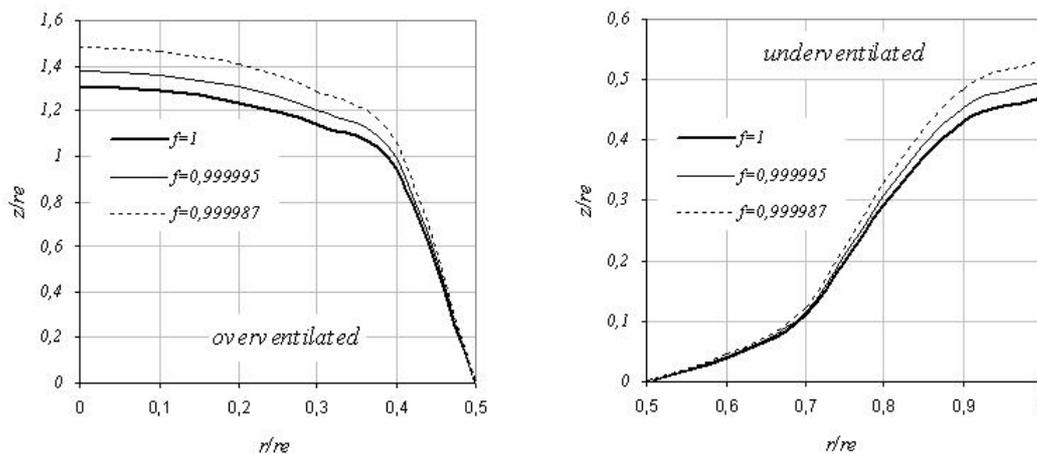


Figure 2. Shape of flame in two-phase jets flow gas reactive.

The curves of Fig. 2 were established for $\rho_g/\rho_l = 10^{-4}$, with $f = 1; 0,999995$ and $0,999987$. This values correspond to the mass fraction of gas in the mixture $\kappa = 1; 0,9$ and $0,885$. In consequence, the Le/Le_g is closed to unit. The left curve correspond to the over ventilated flame and the right curve under ventilated flame. Following Burke and Schurmann (1928), the over ventilated flame is obtained from methane-oxygen reaction and the under ventilated flame correspond to the methane-air reaction. In this analysis the non dimensional diffusion velocity $\{(D/re)/u_z\} = 0,125$.

For temperature profile, following Kuo (1986), $\gamma = \{\beta/[-(C_p T)_{z=0}/(H_r w_f v_f) + (Y_f)_{z=0}/(w_f v_f)]\}$ and the boundary conditions becomes:

$$\gamma = \begin{cases} 1 & \text{at } \eta = 0 \quad 0 \leq \xi < \frac{ri}{re} \\ \frac{1}{\frac{(Y_f)_{z=0}}{C_{p_g} T_0 / H_r} - 1}} & \text{at } \eta = 0 \quad \frac{ri}{re} \leq \xi < 1 \end{cases} \quad (34)$$

and

$$\frac{\partial \gamma}{\partial \xi} = 0 \quad \text{at } \xi = 1 \quad \text{and} \quad \xi = 0 \quad \eta > 0 \quad (35)$$

The solution of this problem (Kuo, (1986); Burke and Schurmann (1928)) in Bessel functions series is:

$$\gamma = \left\{ 1 + \frac{1}{1 - \frac{(Y_f)_{z=0}}{C_{p_g} T_0 / H_r}} \right\} \left(\frac{ri}{re} \right)^2 + \frac{1}{\frac{(Y_f)_{z=0}}{C_{p_g} T_0 / H_r} - 1} + 2 \left\{ 1 + \frac{1}{1 - \frac{(Y_f)_{z=0}}{C_{p_g} T_0 / H_r}} \right\} \left(\frac{ri}{re} \right) \times \sum_{n=1}^{\infty} \frac{1}{\phi_n} \frac{J_1\left(\frac{ri}{re} \phi_n\right)}{[J_0(\phi_n)]^2} J_0\left(\phi_n \frac{r}{re}\right) e^{-\phi_n^2 \frac{z \kappa D}{u_z re^2}} \quad (36)$$

Substituting in Eq. (36) the γ definition in temperature problem, with the condition $Y_f = 0$ in the front flame we have general expression for flame temperature. With $f=1$ for pure gas and the general expression of flame temperature, we may obtain flame temperature ratio, among temperature of flame of two-phase flow jets and the temperature of flame of pure gas flow jets, i.e.:

$$\frac{T_f}{T_{f_g}} = \frac{\kappa C_{p_g}}{f C_p} = \frac{1}{f \left(1 + \frac{1-f}{f} \frac{C_{p_s} \rho_s}{C_{p_g} \rho_g} \right)} \quad (37)$$

where T_f is flame temperature of mixture and T_{f_g} flame temperature of pure gas. The Fig. 3 shows temperature ratio for typical values of 10^4 for ratio of densities and around 10^{-1} for specific heat coefficients ratio.

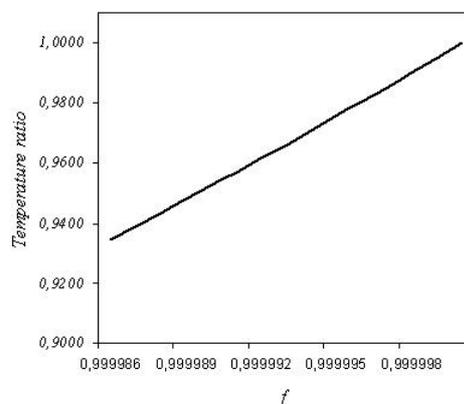


Figure 3. Flame temperature ratio.

3. CONCLUSIONS

This work has showed the main effect of solid particles in the shape and temperature of diffusion flame formed from coaxial jets of two-phase flow.

An important result is represented by Eq. (25), that shows relationship with respect the Lewis number of mixture and Lewis number of pure gas. This model work with volume fraction of gas closed to unit, providing Lewis number unit. In despite this, facilities in mathematical solution of equations are obtained.

The Fig. 2 shows the increased on height of the flame. This effect is due to the diffusion velocity decrease caused by particles in the flow. The top hat flame shape for over ventilated case is due to the axes scale difference in the graphic. The result for $f = 1$ is similarity to that of Burke and Schurmann (1928) which was strongly compared with experimental results. The physical explanation is associated with diffusion time necessary for diffusion of fuel and oxidant inlet front of flame. For volume fraction of gas values $f = 1; 0,999995$ and $0,999987$, the variation of flame height may arrive around 15%.

The Fig.3 shows the decreased on temperature of the flame. This effect is due to heat absorption by solid particles in the flow. For volume fraction of gas values $f = 1; 0,999995$ and $0,999987$, the variation of flame height may arrive around 7%.

The new step in this direction is the numerical solution of equations with general Lewis number values and experimental construction of two-phase flow burner. An important point is referenced to hydrocarbon burning with soot particles formation. In this focus, for aeronautical engine applications this work may to contribute.

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