

Non-Stationary Analysis of Rotating Systems Through Auto-Regressive Models in the Angle Domain

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Abstract. Several approaches can be well employed for transient analysis under non-stationary conditions, for example, time-frequency methods, Wigner-Ville distributions, recursive least squares (RLS), order tracking techniques, waterfall plots, etc. This paper proposes a new approach for analysis of rotating systems operating in non-stationary conditions by combining classical order tracking techniques and auto-regressive (AR) models. The main idea is very simple and consists to perform an order tracking technique to resample the signals in the non-stationary time domain to stationary angular domain. Once this transformation is realized, the conventional AR models can be identified in a reference condition. This reference angle domain AR (ADAR) model is utilized for non-stationary condition monitoring of the rotating system by analyzing the error predictions in angle domain through the statistical process control plots. In order to illustrate the results, the ADAR model is validated based on experimental signals in a rotor under non-stationary operational condition with the addition controlled of unbalanced masses. The advantages and drawbacks of the proposed approach are presented in details.

Keywords: order tracking, AR model, non-stationary analysis, rotordynamics.

1. INTRODUCTION

The vibration measurements in the real-world rotating systems, normally contain multiples orders and a rich dynamics which represent a large types of phenomena, as for instance, unbalance, misalignment, rubbing, cracks, rattle noise, etc. The classical method for damage diagnosis in the rotating machinery has been based primarily on analysis of vibration signals during the steady state of operation, with the application of appropriate tools for analysis of signal shape, such as discrete Fourier transform (DFT). It is very difficult to perform a correct analyze by using only the data in stationary conditions. Generally, the signals measured from run-up or run-down tests and changes of operational conditions are more useful and with enormous information about the dynamics contained in the system. However, in these conditions, the common analysis methods in time (AR, ARMA models, etc) (Silva *et al.*, 2007) or frequency domain (Fourier Analysis) must be used cautiously. Precession, unbalance, misalignment, natural frequency, fissures and cracks are phenomena in stationary operation regimes that are not so easily distinguishable, being difficult to classify the damage in the machine (Goldman and Muszynska, 1999.), these limitations are also presented by Miranda *et al.* (2002). Based on this concept, the industry need to use more intensive techniques to analyze the process behavior in non-steady condition.

The time-frequency methods, as for instance Wigner-Ville distribution, Wavelet, Waterfall, etc. has found many practical applications. Alarcon *et al.* (2011) employed of the Wigner-Ville distribution (WVD) to identify high and low orders in the transient induction motor. In their work, it was shown that the WVD has a good time-frequency resolution at lower as higher order, beyond the Wigner-Ville distribution is faster than other algorithms and exponential have an excellent frequency resolution. However, it would be possible the appearance of interference caused by auto-correlation process performed by this transformation when applied to the analysis of multicomponent signals in frequency, which can mask the evolution of some relevant harmonics in the plan or time-frequency energy at frequencies appear non-existent. González *et al.* (2010) made a comparison between techniques using DFT and auto-regressive (AR) models for the signal processing techniques combined with the power spectral density (PSD) and spectral coherence (SC). D'Elia *et al.* (2010) propose a new approach for analyzing non-stationary signals consisting of the synchronization of both the spectral correlation density (SCD) as the cyclic modulation (CMS) in order to obtain two-dimensional functions in order maps versus maps frequently. In all these types of methods, the damage of rotating systems are detected in a non-parametric way by analyzing a plot.

The fundamental frequency corresponding to the reference speed is called the basic order. Most rotating machines are usually composed of several rotating elements such as gears, chains, belts for transmission, etc. (Wang and Heyns, 2011). The change of rotation speed leads to non-stationarity of the signal, which becomes difficult to interpret with classical methods. Therefore, Order Tracking (OT) methods have been developed and updated for the analysis of signals in rotating machines. Currently, there are several different classes available of OT techniques for monitoring. For example, order tracking computational technique (OTC) uses a resampling process that leads to a spectrum in the order domain.

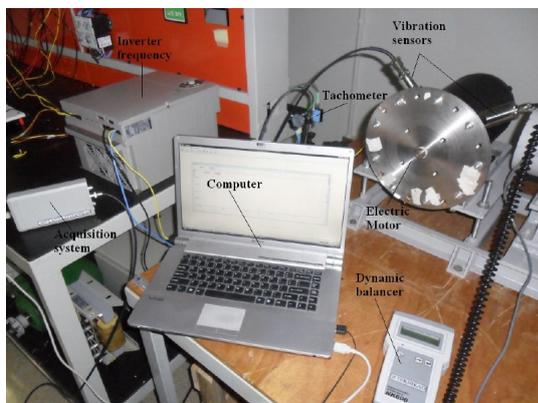
Gabor order tracking (GOT) and order tracking Vold-Kalman (VKF-OT) filters allow the separation of the orders. Pan *et al.* (2007) compare the method GOT with the VKF-OT for different situations of transient machines, and conclude that the GOT method applied to the construction of maps of rotating machinery orders, do not require that the information of the angular velocity of the axis is known during the analysis. Moreover, the technique is not restricted to applications in the order, and can be used to highlight and reconstructs the waveform spectral components of transient interest. Guo

and Tan (2009) show that the method GOT is superior to VKF-OT in the reconstruction of the waveform with the use of independent component analysis (ICA). Wang and Heyns (2011) have proposed a new proposal for use of VKF-OT association with the OTC by combining the advantage of the OTC non-stationary data transform data into stationary and method VKF-OT in order to separate the desired order of harmonics and other noises. So, it would require a supervised human to observe visually the existence of a damage, or by training neural networks or other artificial intelligent method. This may not be feasible to be implemented quickly in an industrial environment.

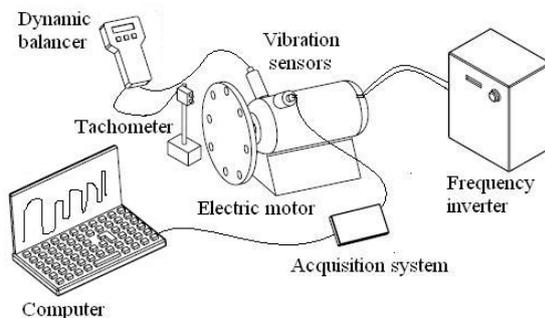
In this sense, the present paper proposes a new approach through the resampled signal processing in the angle domain instead of conventional OT methods performing a post-processing with spectral analysis. The paper is organized as follows. First of all, the experimental setup used in the tests is described. Next, a basic overview in order tracking by using digital resampling is briefly presented. A reference autoregressive model (AR) in healthy condition in the angle domain is then identified, called ADAR model. The reference ADAR model is used for monitoring the rotation machine operating in several non-stationary conditions, by analyzing the prediction error in the angle domain using statistical process control. The experimental results are presented and discussed in details. Finally, the final remarks show the feasibility and applicability of the approach proposed and the further researches directions seeking practical use in the industry.

2. EXPERIMENTAL TEST BENCH DESCRIPTION

The stationary and non-stationary data used in this paper are obtained from a experimental test setup presented in Fig. 1(a). This experimental setup consists of the followings equipments and sensors: a frequency inverter with a nominal capacity of 15 kW, a three-phase electric motor with power of 2 kW and nominal voltage of 220 V, a disk with diameter of 250 mm, mass of 3.950 kg and several holes with distance each one of 20° (see Fig.2(a)), a RPM gauge, an accelerometer with sensitivity of 105 mV/g and frequency range of 10 kHz attached to the motor housing, a dynamic balance equipment, a data acquisition board model SDAV from Teknikao® for measurement the signals and a software for control and processing the experimental signals. After the signals are recorded by using a sampling rate of 250 Hz and 2048 samples, all the analysis are performed through the Matlab®. Figure 1(b) illustrates a schematic diagram of the experimental setup.



(a) Experimental test bench.



(b) Schematic diagram of the test bench.

Figure 1. Experimental test setup.

First of all, the rotating disk is balanced by using a NK 600 commercial equipment in accordance with the requirements of NBR 10082 (ABNT, 1987). The disk was balanced with a rotational speed fixed at 1790 RPM and after the procedure a RMS vibration level of 1.05 mm/s is reached, within the insurance limit.

After the balancing procedure, two sets of tests are performed. The first one is by measurement the vibration in steady condition with rotational speed of 1795 RPM. The second one, is a run-up test when a linear ramp from 300 to 1295 RPM is applied to the electrical motor. Examples of the type of the signals obtained are shown in Fig. 3.

3. ORDER TRACKING BACKGROUND

The signal of the angular velocity is of great importance in the analysis in angle domain and order domain. Thus, if this signal is not well estimated, all the analysis can be erroneous (Andre *et al.*, 2010). One can obtain the angular velocity by using a RPM gauge. However, this paper used a combination of Wigner-Ville distribution (WVD) with search technique for peaks for selecting some points in the WVD. Finally, the instantaneous frequency estimated is computed by using these points by applying an interpolation technique. Figure 4(a) presents the WVD obtained from non-stationary signal reference shown in Fig. 3(b). Figure 4(b) illustrates the instantaneous frequency estimated for this signal. The

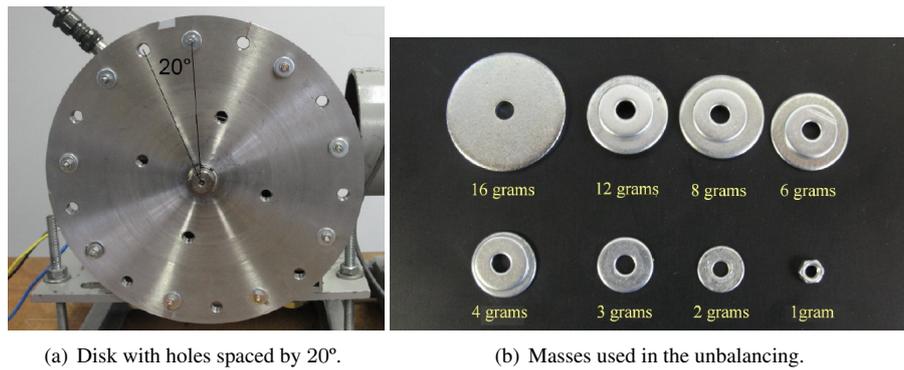


Figure 2. Details of the disk and the masses.

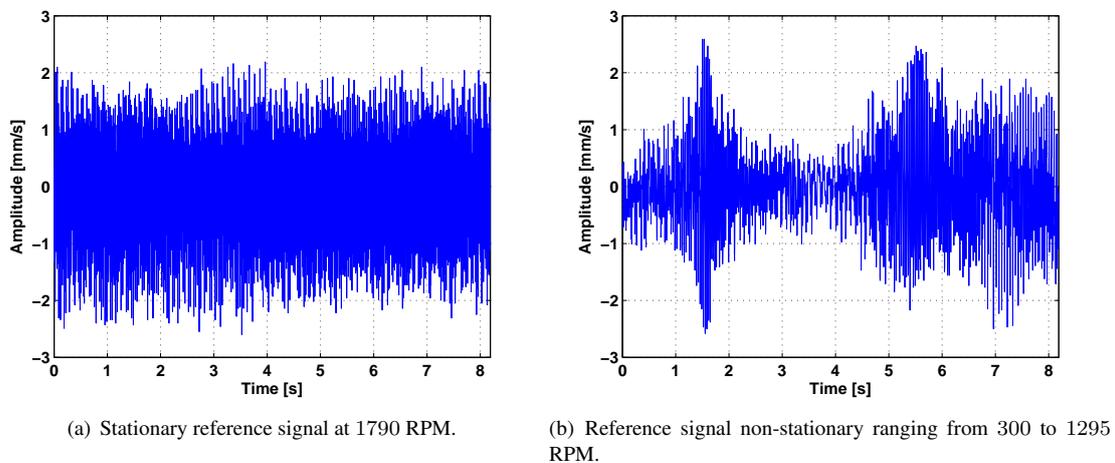


Figure 3. Examples of the reference signals stationary and non-stationary.

angular velocity is given by $\dot{\theta} = \omega \cdot 60$, where $\dot{\theta}$ is the angular velocity in RPM and ω is the instantaneous frequency in Hz.

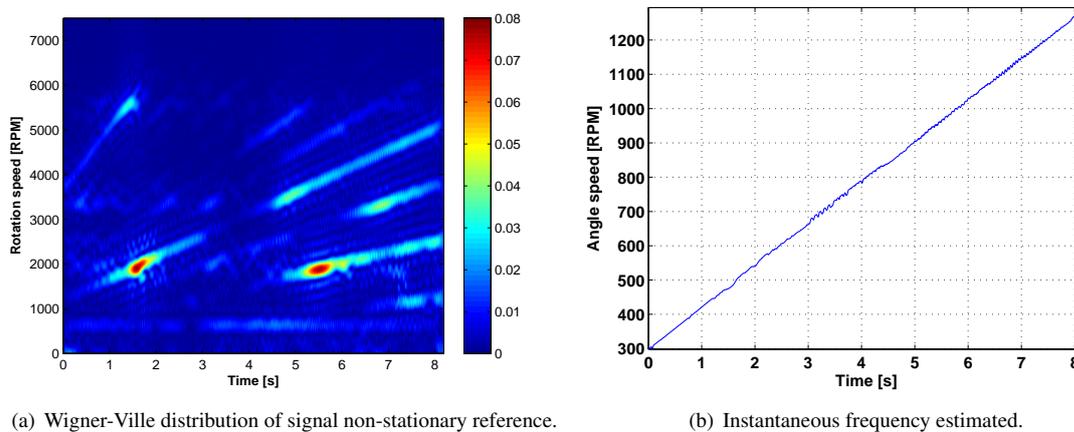


Figure 4. Estimation of the instantaneous frequency by time-frequency analysis.

In the frequency spectrum (waterfall) computed by discrete Fourier transform, the periodic signals are observed as peaks distributed over the frequency axis. However, in an order spectrum (order map), obtained by the discrete Fourier transform applied to a signal in the angular domain, the periodic signals also appear as peaks, but are aligned with respect to different orders. The order tracking (OT) methods are characterized by resampling the signals with a constant angular variation $\Delta\theta$, i.e., with the same number of points per cycle, independently the angular velocity is or not constant. The

most popular order tracking method is the computational digital resampling with a constant angular displacement signal sampled with sampling rate fixed provide by a signal of angular velocity (Fyfe and Munck, 1997). In that case, a signal with fixed time period Δt and variant frequency become a signal with a constant angular interval $\Delta\theta$.

The digital resampling uses a series of interpolations to generate a synchronous signal with angular velocity and constant angular increments, as shown in Figure (5(b)). Andre *et al.* (2010) show comparative tests of digital resampling method with analogical order tracking, by analyzing experimental run-up with linear and quadratic angular variation.

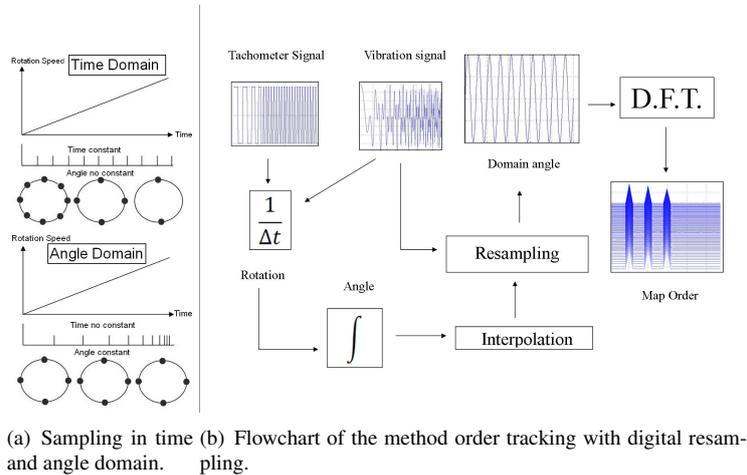


Figure 5. Representation of flowchart Order tracking

From the signal of angular velocity can be obtained the angular displacement for each time step integration:

$$\theta(t) = \int_{t_1}^{t_2} \omega(t) dt \quad (1)$$

where $\theta(t)$ is the instantaneous angular displacement and $\omega(t)$ is the rotational speed. The numerical integration is computationally solved by Simpson's rule simple (Barroso *et al.*, 1987):

$$\int_{t_1}^{t_2} \omega(t) dt = \frac{t_2 - t_1}{6} \left[\omega(t_1) + 4\omega\left(\frac{t_1 + t_2}{2}\right) + \omega(t_2) \right] \quad (2)$$

The resampling time is the time when the signal has a constant number of points per cycle (constant angular displacement $\Delta\theta$) and is obtained by interpolation of the angle-time curves at points equally spaced angles. Once obtained the values for the angular variation of time constant, an interpolation technique is performed for the signal analysis time with constant angular variation to obtain $t = f(\theta)$, this operation is obtained with the use the function *interp1* in the Matlab®.

4. AUTO-REGRESSIVE MODEL IN THE ANGLE DOMAIN (ADAR)

The autoregressive model in the domain angle (ADAR) is a result of applying a classical AR model in a reference signal in the domain angle obtained through the order tracking by digital resampling from time domain signals measured in the rotor in non-stationary and healthy condition.

A standardization of data to remove various trends and effects is performed. A signal in the angle domain $g(k)$, $k = 1, 2, \dots, n$ can be obtained in each measurement point. The procedure can be implemented with MIMO case, but in this paper it was presented based only in the SISO case. In the first stage, each angle series, $g(k)$ is normalized to remove trends (Wirsching *et al.*, 1995):

$$x(k) = g(k) - m(g) \quad (3)$$

Since $x(k)$ is the signal in the k -th standardized moment and $m(\cdot)$ is the operators mean, respectively, of the sequence $g(k)$. A reference AR model with order lag of p can be written as (Ljung, 1998):

$$A(q)x(k) = e_{ref}(k) \quad (4)$$

where $e_{ref}(k)$ is the reference error between the measured signal and the output signal of the prediction model. The order p of this model in general is not known a priori. Several methods can be used to estimate this value with the Akaike information criterion (AIC) and the final prediction error Akaike (FPE).

The polynomial $A(q)$ is the polynomial in the lag operator q^{-1} written as:

$$A(q) = 1 + a_1q^{-1} + a_2q^{-2} + \dots + a_pq^{-p} \quad (5)$$

where a_i are the coefficients of the polynomial $A(q)$. The order p means the number of past samples needed to make the operations of regression in each angle k . $q^{-1}x(k)$ corresponds to the angle delay in a position of the output signal, for example $q^{-1}x(k) = x(k - 1)$. The set of coefficients in Eq. (5) can be estimated by minimizing the power of each prediction error that leads to the Yule-Walker equations or by using least square method.

The monitoring system is accomplished by obtaining new data vectors in unknown conditions in healthy or damaged states. This new sequence $y(k)$ has the same size as the reference $x(k)$. Thus, the unknown data $y(k)$ are filtered by the reference ADAR model:

$$A(q)y(k) = e_{unk}(k) \quad (6)$$

The signal $e_{unk}(k)$ is the prediction error in unknown condition. The statistical process control (SPC) can be used based on $e_{ref}(k)$ to compute the control limits (Montgomery, 1996). By comparing the number of outside points (outliers) with these limits with the outliers when the unknown error $e_{unk}(k)$ is computed, a damage decision can be performed with a statistical confidence. A similar approach can be implemented with the signals in the time domain, but, in this condition is necessary to use signal with constant rotational speed (steady condition).

The basic procedure for damage detection with ADAR model is composed of some steps illustrated in Fig. 6. The Matlab® has several effective commands in the system identification toolbox that can be employed to identify the reference ADAR model proposed in this paper.

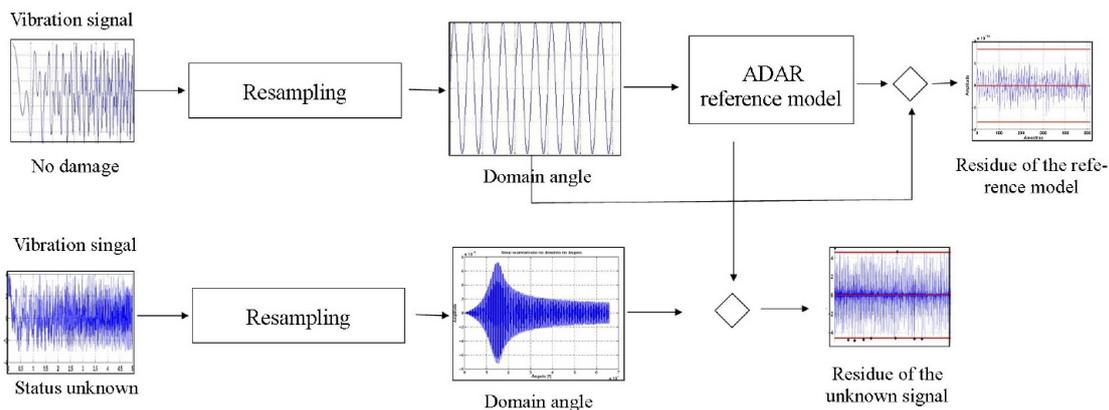


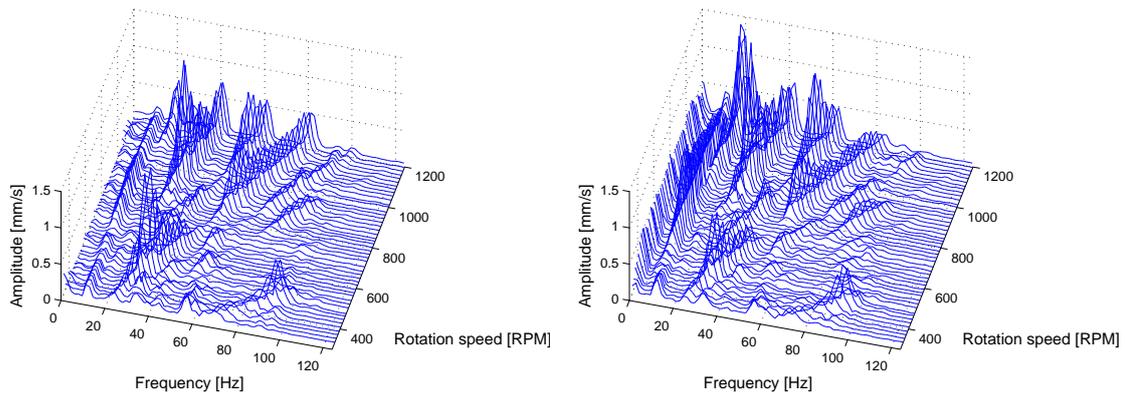
Figure 6. Flowchart for implementation of the model ADAR.

5. EXPERIMENTAL RESULTS

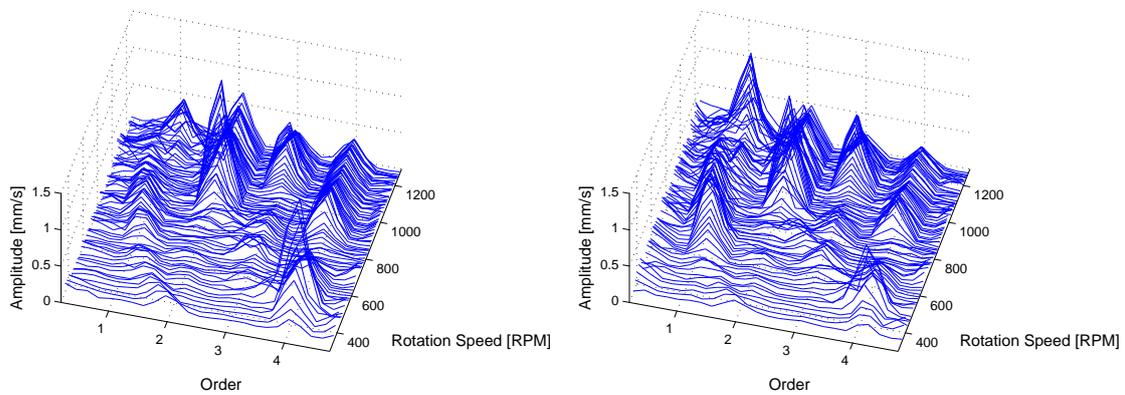
In order to illustrate the ADAR model, it were measured different signals when the level of unbalance is varying in a controlled way by adding mass in specific positions in the rotating disk. The healthy condition in the bench test corresponds to balanced rotating system through a commercial dynamic balancer with a RMS level of 1.05 mm/s. The damage conditions tests are performed by adding unbalanced masses known of 1, 2, 4, 8 and 16 grams in the same angular positions (Fig. 2(b)). These data are recorded in steady test with constant speed of 1790 RPM and run-up tests by varying linearly its angular velocity from 300 to 1295 RPM. False positive tests for each domain (time and angle) are also recorded to verify the approach. All tests are used to illustrate the procedure proposed by comparing the diagnosis through the NBR 10082, AR model in the time domain and ADAR model in the angle domain. Thus, a data set of progressive damages are recorded for analysis with each unbalance mass with two tests (steady and run-up conditions).

Figure 7 presents the order maps and waterfalls to the condition with unbalance mass of 0 and 16 grams. Figure 7(c) shows that the 1st order has low amplitude, however in Fig. 7(d) with unbalance mass of 16 grams the amplitudes increased that corresponds to severe unbalanced condition. It is worthing to observe that in the waterfall, the spectral components are varying the rotation which appear as curved lines and natural frequencies as vertical lines indicating constant frequency. On the other hand, in order maps, the natural frequencies appear as curved lines of negative slope, and the components with time varying frequency are seen as vertical lines. So, the order maps indicate the individual order relationship between vibration and rotational speed of the machine.

After estimating the instantaneous frequency, the nonstationary signal in time domain is resampled to the angle domain using the method of Order Tracking. Figure 8(a) presents the resampled signal for first order in the domain angle. Figure 8(b) shows the vibration signal in a range of 360° (one revolution of the shaft).

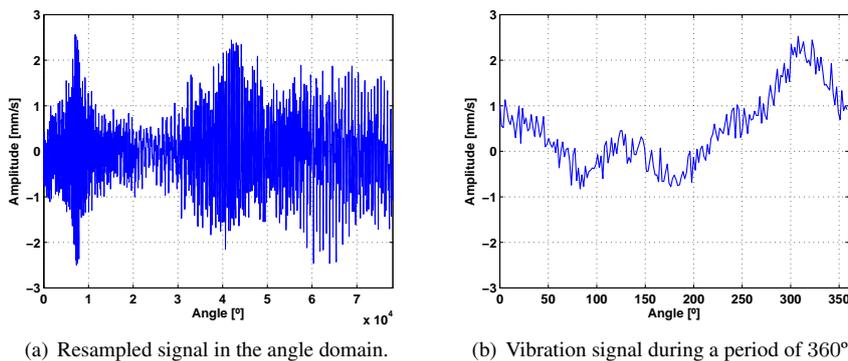


(a) Waterfall of the vibration signal in healthy condition, with 150 samples by block, overlap of 25% of the blocks, window Hanning. (b) Waterfall of the vibration signal with unbalance mass of 16 grams, with 150 samples by block, overlap of 25% of the blocks, window Hanning.



(c) Order map of the vibration signal in healthy condition, with 1000 samples by block, overlap of 25% of the blocks, window Hanning. (d) Order map of the vibration signal with unbalance mass of 16 grams, with 1000 samples by block, overlap of 25% of the blocks, window Hanning.

Figure 7. Waterfall and order map of the signal in healthy and damage condition (unbalance mass).



(a) Resampled signal in the angle domain. (b) Vibration signal during a period of 360°.

Figure 8. Resampled signals in the angle domain.

The reference signals are standardized to avoid influence of environmental and operational condition. After this process, it is identified a model with order based on Akaike information criterion (AIC) to represent the models in the angle domain and time domain. Figures 9(a) and 9(b) show the AIC for the ADAR and AR model, respectively. After analyzing the AIC in Fig. 9, the ADAR can be represented with two parameters. On the other hand, in the AR model is necessary 6 parameters. Clearly, the ADAR model has a order lower than the AR model, consequently, it is easier to be implemented in a further microchip or PIC, for example. Next eq. illustrates the ADAR and AR model identified:

$$A_{ADAR}(q) = 1 - 1.891q^{-1} + 0.9109q^{-2} \tag{7}$$

$$A_{AR}(q) = 1 - 0.3426q^{-1} + 1.108q^{-2} - 0.4614q^{-3} + 1.081q^{-4} - 0.3722q^{-5} + 0.9469q^{-6} \tag{8}$$

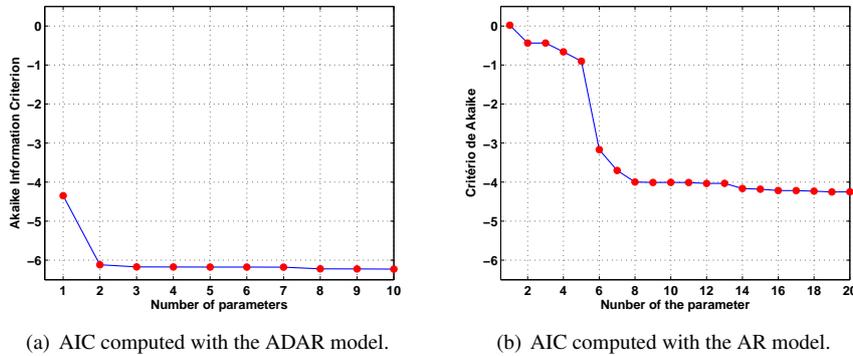


Figure 9. Akaike information criterion (AIC) plot.

In order to validate the AR and ADAR model, the residual analysis is used with other healthy data set to improve the procedure of validation. Figures 10(a) and 10(b) show the correlation function between the output signal and the prediction error provided by AR and ADAR models, respectively. Clearly, the results show that the errors correspond to white noise process. So, the AR and ADAR models can be used as reference model in healthy condition.

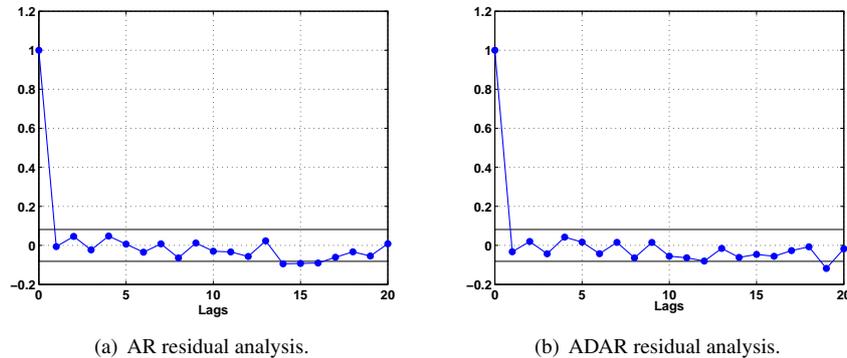


Figure 10. Residual analysis for validation.

Thus, the validated model can be used as a filter to extract the prediction error in reference condition and unknown condition (Tab. 1). The AR and ADAR prediction error are arranged in 4 groups with 512 and 5421 samples, respectively. The lower and upper control limits, respectively LCL and UCL, with 95 % of statistical confidence is computed. Fugate *et al.* (2001) show several steps to SPC procedure. The number of samples expected to be outside of the UCL and LCL are = 5% of total samples to the signal in healthy condition, being 25 samples for the AR and 271 for the ADAR model. Figure 11 shows the the statistical process control in healthy condition.

It is performed a false-positive test with AR and ADAR models, in order to test if the models detect damages in situations that do not exist. Figure 12 presents the tests of the AR and ADAR models respectively.

Once the control limits are estimated, the error predictions, computed by AR and ADAR models for different levels of unbalance mass applied in the rotating disk, are identified. If the number of outliers is upper than 25 in the AR model or 271 with the ADAR model, the rotor is in damage condition with the severity associated directly with the number of outliers. Figure 11 present the results of the SPC by using the error predictions computed with AR and ADAR models for some unbalance mass. The level of RMS is shown with the signal in steady condition. In all cases, the steady (AR model) and run-up (ADAR model) have the same rotational speed discussed before.

Table 1 presents the method to detect damages in the rotating machines by RMS with the NBR 10082. The table 2 presents the comparison among the methods AR and ADAR models, respectively. By analyzing the tables, it is observed that the ADAR model identified the changes in the signal with the increase of 4 grams in the disk, the AR model detected changes in the condition of the machine when a unbalance mass of 4 grams is added. On the other hand, the method

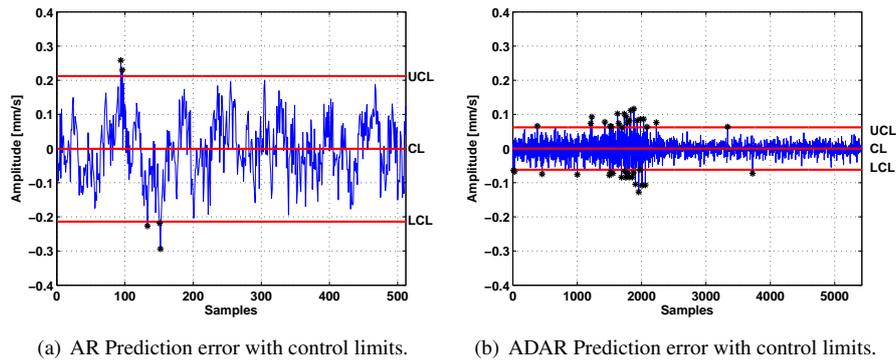


Figure 11. Prediction error considering the signal in the healthy condition.

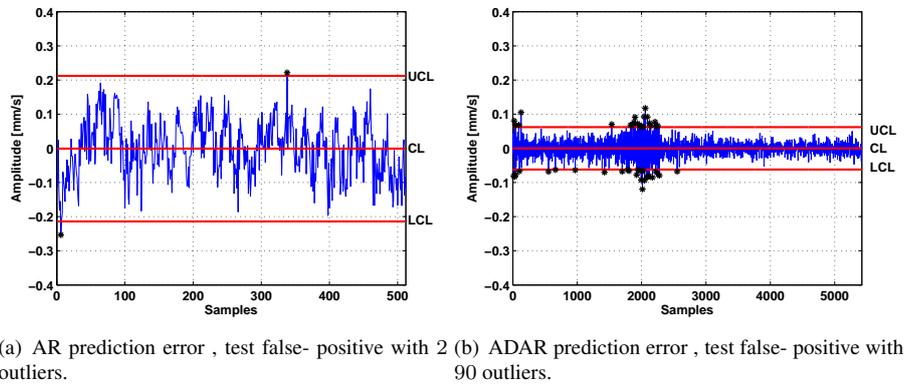


Figure 12. AR and ADAR prediction error for the false-positive test.

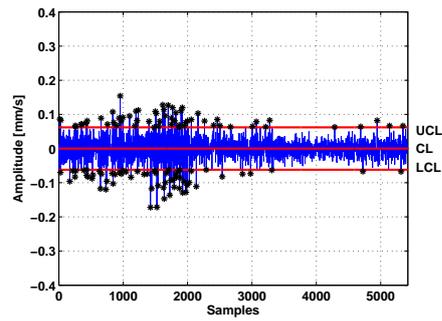
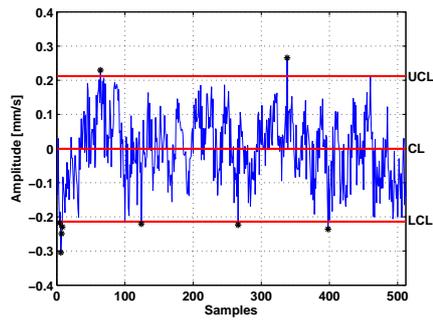
by RMS with 4 grams became from the permissible state to tolerable and with 8 grams from tolerable to impermissible. Thus, the ADAR model represents a more sensitive method able to detect structural changes before to the conventional methods. This advantageous is relative directly to the dynamic characteristics contained in run-up tests.

Table 1. Results of the methods used to identify the unbalance - healthy or damaged conditions with the RMS level.

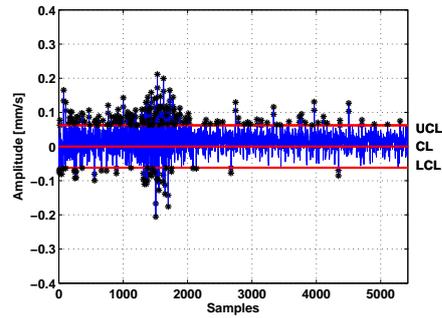
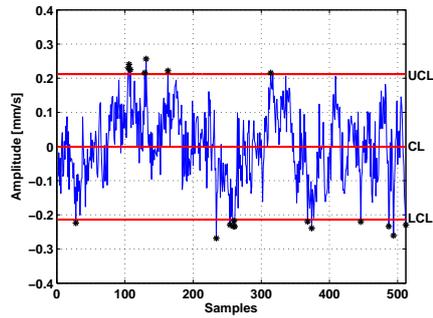
| Unbalance mass | RMS Level | Diagnosis by NBR 10082 |
|---------------------|-----------|------------------------|
| Test false-positive | 1.05 mm/s | Permissible |
| 1 grams | 1.09 mm/s | Permissible |
| 2 grams | 1.25 mm/s | Permissible |
| 4 grams | 1.44 mm/s | Permissible |
| 8 grams | 2.15 mm/s | Tolerable |
| 16 grams | 4.90 mm/s | Impermissible |

Table 2. Results of the methods used to identify the unbalance - healthy or damaged conditions with AR and ADAR models.

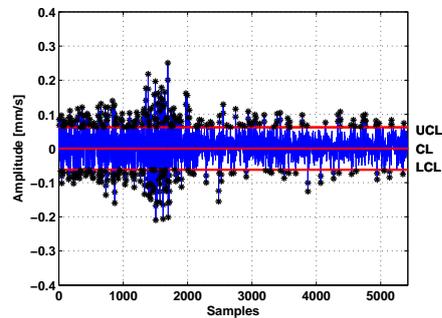
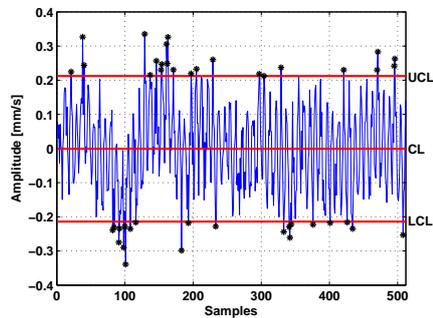
| Unbalance mass | AR outliers(limit of 25) | Diagnosis by AR | ADAR outliers (limit of 271) | Diagnosis by ADAR |
|---------------------|--------------------------|-----------------|------------------------------|-------------------|
| Test false-positive | 2 | Healthy | 90 | Healthy |
| 1 grams | 9 | Healthy | 142 | Healthy |
| 2 grams | 19 | Healthy | 204 | Healthy |
| 4 grams | 44 | Damage | 286 | Damage |
| 8 grams | 55 | Damage | 722 | Damage |
| 16 grams | 246 | Damage | 1402 | Damage |



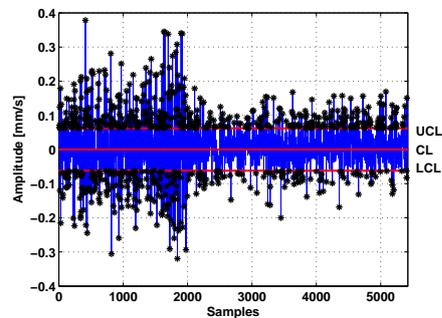
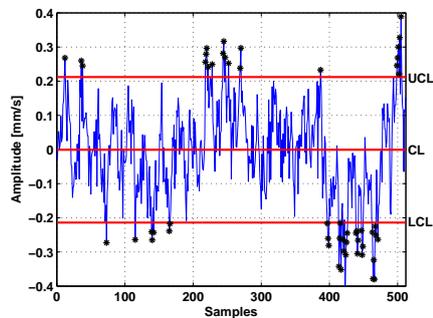
(a) AR prediction error with 1.09 mm/s of RMS and (b) ADAR prediction error with 137 outliers and unbalance mass of 1 grams.



(c) AR prediction error with 1.25 mm/s of RMS and (d) ADAR Prediction error with 204 outliers with unbalance mass of 2grams.



(e) AR prediction error with 1.44 mm/s of RMS and (f) ADAR prediction error with 286 outliers with unbalance mass of 4 grams.



(g) AR prediction error with 2.15 mm/s of RMS and (h) ADAR prediction error with 722 outliers and unbalance mass of 8 grams.

Figure 13. Diagnosis by statistical process control through AR and ADAR error prediction.

6. FINAL REMARKS

This paper proposed a new approach for damage detection in rotating machines by using digital resampling techniques, AR models and statistical process control. This approach was called by the authors as auto-regressive model in the angle

domain (ADAR). In order to illustrate the ADAR approach and the steps associated, several experimental tests with different levels of unbalance mass added in a rotor were performed. The results presented the advantages of the ADAR by comparing with classical AR model in the time domains and with RMS analysis with NBR 10082. Further study is necessary by using experimental data from industrial rotating machines in order to improve and test in real work conditions.

7. ACKNOWLEDGEMENTS

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9. Responsibility notice

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