DETERMINATION OF ELASTICS CONSTANTS OF COMPOSITE BEAMS USING NATURAL FREQUENCIES

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Abstract. *The aim of the work reported here is to provide a single non-destructive test to determine the dynamic elastic properties of an orthotropic material using free vibrations of a rectangular beam in completely free boundary conditions. Specimens of these materials possess specific mechanical resonant frequencies that are determined by the elastic modulus, mass, and geometry of the test specimen. Dynamic Young's modulus is determined using the resonant frequency in the flexural mode of vibration. The dynamic shear modulus, is found using torsional resonant vibrations. Impact modal tests are employed to obtain these flexural and torsional vibration frequencies of the free-free suported beam sample. The accuracy was verified making comparisons with results from finite element calculations. The application of the method is demonstrated in one example for which measurements were made.*

Keywords*: elastic constants, non-destructive, natural frequencies, frequency measurements, composite beams.*

1. INTRODUCTION

Fiber-reinforced composites have been gaining wide use in various engineering applications. This kind of material offers high specific strength and stiffness, corrosion and fatigue resistance together with low coefficient of thermal expansion. Today, the determination of elastic properties of composite materials is important for optimum design, quality control and damage detection.

Conventional static test methods used for such characterization suffer from severe problems such as boundary effects and the presence of nonuniform stress/strain fields. Moreover, in these methods, strains are measured locally, rendering the measurements vulnerable to inhomogenities. Particularly for composites this is a drawback and test results for such materials often exhibit large scatter.

The resonant beam technique is a simple, effective and nondestructive method to measure the dynamic properties of elastic materials. Standard tests like ASTM E 1876-01 are available to characterize the Young's modulus using flexural mode of vibration and shear modulus by torsional resonant vibrations. This test method is specifically appropriate for determining the modulus of materials that are elastic, homogeneous and isotropic. But it can be applicable to composite and inhomogeneous materials only with careful consideration of the effect of inhomogenities and anisotropy. The character of the reinforcement and inhomogenities in the beams will have a direct effect on the elastic properties. These effects must be considered in interpreting the test results.

The advantages of vibrations tests are derived from their excellent repeatability and accuracy along with their simplicity to run and set up. The increasing popularity of dynamic characterization of composite beams, plates and shells can be observed in Ayorinde (1995), Lai and Ip (1996) and Ip *et al.*(1998).

The objective of this paper is to present and justify a simple and practically useful method to determinate the dynamic properties of composite materials based on the resonant beam technique.

2. BEAM MODEL

The determination of both, the Young's modulus from the flexural modes of vibration and the shear modulus from the torsional modes use relations developed by Pickett (1945). The analytical equations presented in this paper are limited to specimens with regular geometry (rectangular parallelepiped) and vibrating freely, with no significant restraint or impediment. Deviations from the specified geometry and boundary condition change the resonant frequencies and introduce significant errors in the calculation of the elastic properties.

2.1. Flexural modes of vibration

The following equations were used to calculate Young's modulus from the flexural resonant frequencies:

$$E = 0.9465(mf_f^2/b)(L^3/t^3)T_1 \tag{1}$$

$$T_1 = 1 + 6.585(1 + 0.0752\mu + 0.8109\mu^2)(t/L)^2 - 0.868(t/L)^4 - \left[\frac{8.340(1 + 0.2023\mu + 2.173\mu^2)(t/L)^4}{1 + 6.338(1 + 0.1408\mu + 1.536\mu^2)(t/L)^2}\right] \qquad (2)$$

where

E = Young's modulus, Pa,
m = mass of the bar, kg,
b = width of the bar, m,
L = length of the bar, m,
t = thickness of the bar, m,
f_f = fundamental resonant frequency of bar in flexure, Hz,
T_1 = correction factor for fundamental flexure mode to account for finite thickness of bar, Poisson's ratio (μ), and so forth.

2.2. Torsional mode of vibration

The shear modulus can be calculated from the torsional resonant frequency using the following equation:

$$G = \frac{4Lmf_t^2}{bt}[B/(1 + A)] \qquad (3)$$

$$B = \left[\frac{b/t + t/b}{4(t/b) - 2.52(t/b)^2 + 0.21(t/b)^6}\right] \qquad (4)$$

$$A = \frac{[0.5062 - 0.8776(b/t) + 0.3504(b/t)^2 - 0.0078(b/t)^3]}{[12.03(b/t) + 9.892(b/t)^2]} \qquad (5)$$

where:

G = shear modulus, Pa,
m = mass of the bar, kg,
b = width of the bar, m,
L = length of the bar, m,
t = thickness of the bar, m,
f_t = fundamental resonant frequency of bar in torsion, Hz,
A = an empirical correction factor dependent on the width-to-thickness ratio of the test specimen (Spinner and Valore, 1958).

3. EXPERIMENTAL PROCEDURE

3.1. Apparatus

The elastic constants of the material are determined through the measurement of the fundamental resonant frequency of a vibrating free-free beam. The major problem is to configure a completely free beam. A test apparatus suitable for this experiment is shown in Fig. 1.

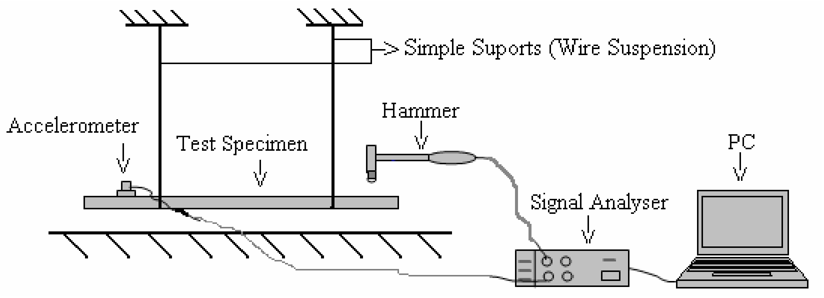

Figure 1. Apparatus for dynamic properties measurement.

The excitation of the test specimen is accomplished by an impulse hammer with a force transducer and the response of the beam is measured by a light accelerometer, providing experimental measurement data with minimum interference on the specimen vibration. A frequency response was obtained by a digital signal analyzer and the resonant frequency read from the spectrum. The positions of the support were changed for each resonant mode tested.

3.2. Procedure

3.2.1 Fundamental flexural resonant frequency

The supports were placed at the first mode nodal points of the specimen, which correspond to 0.224*L* from each end, where *L* is the length of the beam. The response was measured by an accelerometer placed close to a nodal point, in order to be able to generate a clear vibration signal and to cause as little interference as possible.

An impact hammer was used to apply a point force excitation at the specimen center line, to avoid torsional modes (Fig. 2). At least ten readings were taken to average out random errors.

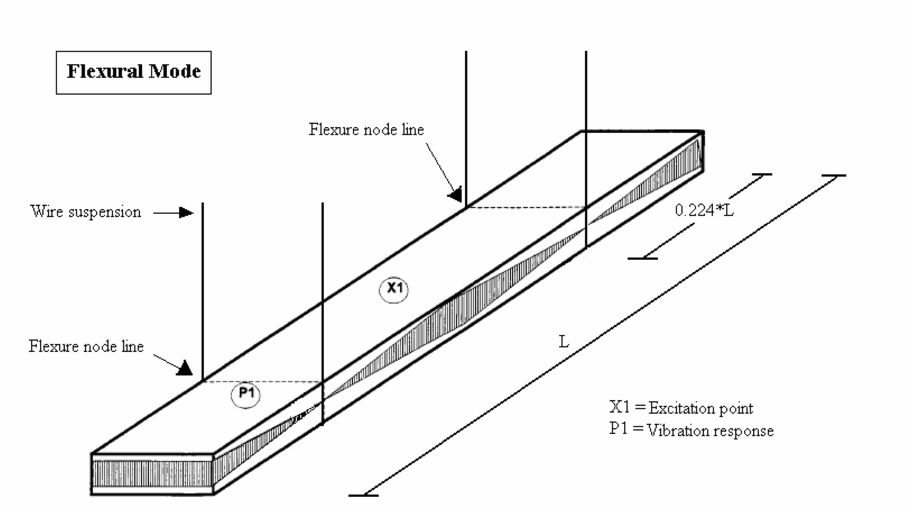

Figure 2. Supports, excitation and vibration response of flexural modes measurements.

3.2.2 Fundamental torsional resonant frequency

Simple supports were placed at midpoint of length and width of the specimen. An accelerometer was placed also close to the support for the same reason mentioned before, as well as the transient excitation point. These points are indicated in Fig. 3. At least ten transfer functions readings were taken to average out random errors.

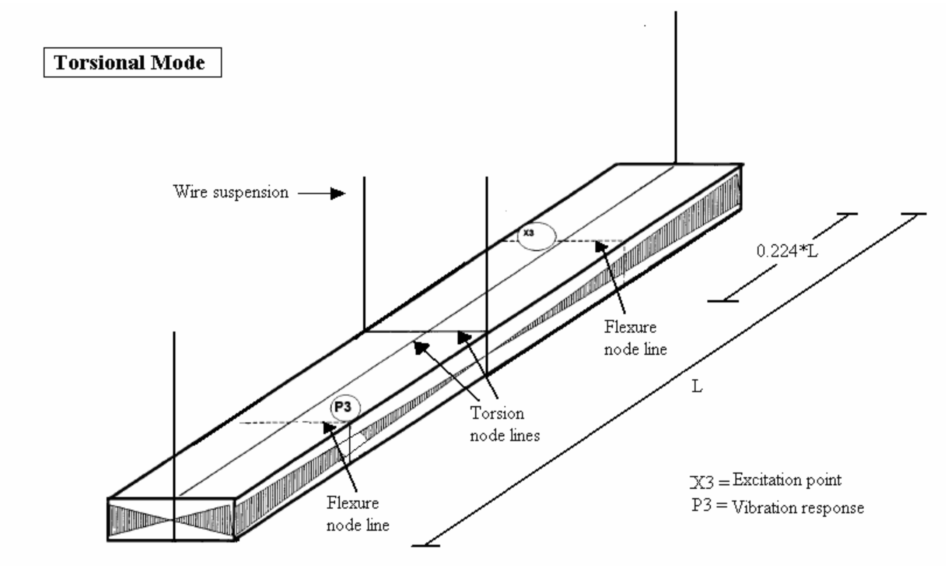

Figure 3. Supports, excitation and vibration response of torsional modes measurements.

4. RESULTS AND DISCUSSIONS

The following axes for the specimen were considered: 1 - axis, along the length; 2 - axis, along the width and 3 - axis, along the thickness. Two specimens with different material orientation were prepared. In the first beam specimen, showed in Fig. 4, the fibers were oriented parallel (0º) to axis 1, and in the second the fibers, showed in the Fig. 5 were oriented perpendicular (90º) to axis 1.

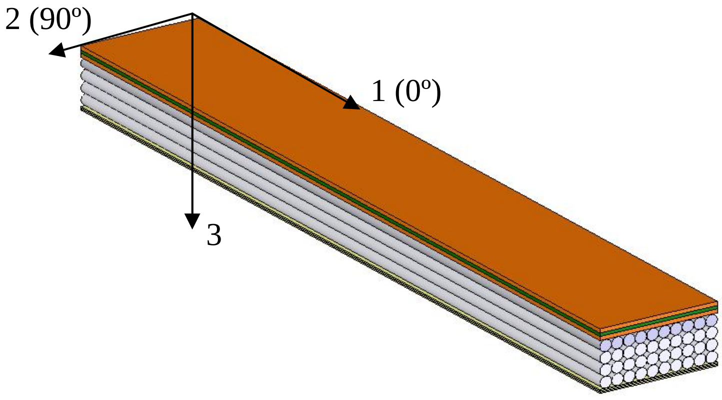

Figure 4. Composite beam with fibers parallel (0º) to axis 1.

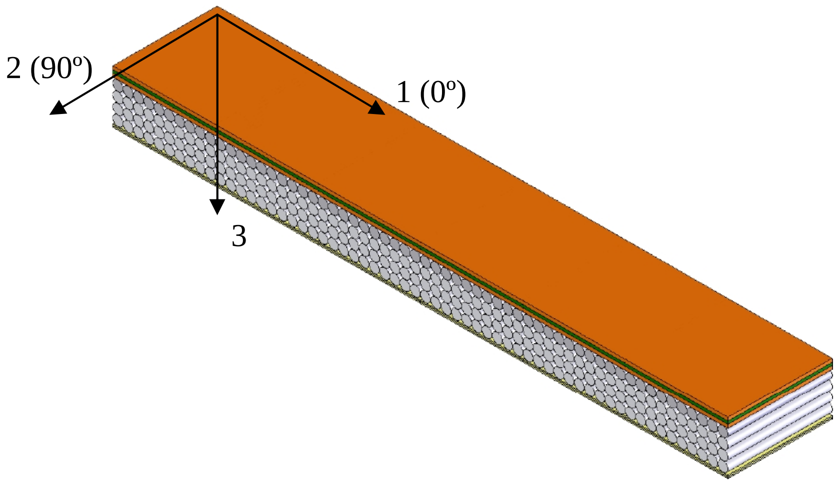

Figure 5. Composite beam with fibers perpendicular (90º) to axis 1.

The test specimen were made of a unidirectional aluminum wires reinforced with glass/epoxy. Table 1 contains the measured dimensions and masses of the beam specimens.

Table 1. Characteristics of beam specimens.

Orientation	Thickness	Width	Length	Mass
0°	0.013 m	0.024 m	0.233 m	0.148 kg
90°	0.013 m	0.024 m	0.202 m	0.126 kg

For each specimen, the fundamental frequency of the flexural and torsional mode of vibration was determined. The importance of knowing the frequency to as high a degree of precision as possible is seen at once if it is recognized that the elastic modulus is proportional to the square of the resonant frequency, whatever the mode of vibration. Table 2 shows the resonant frequencies and elastic properties determined by Eq. 1 and Eq. 3 for each specimen.

Table 2. Resonant frequencies and elastic properties of beams.

Orientation	f_f (Hz)	f_t (Hz)	E (Mpa)	G (Mpa)
0°	929.722	1756.442	**29820.0**	**2283.0**
90°	346.425	1649.113	**2429.0**	**1561.0**

5. NUMERICAL MODELS

The accuracy of this method may be analyzed by comparing results with those obtained by a finite element model (FEM), using Ansys package. The dimensions and material properties of the two beams listed in Tab.1 and Tab.2 were used in the FEM model. Elements SHELL93 for composite materials were used. The resonant frequency of flexural and torsional mode was determined for each model. Table 3 shows the experimental and numerical frequencies and the percentual difference between them.

Table 3. Numerical and experimental resonant frequencies and respective percentual difference.

Orientation	f_f^{EXP} (Hz)	f_f^{FEM} (Hz)	Difference	f_t^{EXP} (Hz)	f_t^{FEM} (Hz)	Difference
0°	929.72	970.64	4.4%	1756.44	1929.27	9.83%
90°	346.42	353.36	2.0%	1649.11	1890.49	14.5%

The difference between numerical and experimental resonant frequencies is mainly attributed to deviations in the specimen geometry and mass, and restraints at the supports. Furthermore, the material had high damping capacity which caused difficulties in the determination of the resonant frequency. For the beam with 90° of orientation in the aluminum the major error may be due to possible cracks or inhomogeneties.

6. CONCLUSIONS

The present study showed that reliable values of elastic properties of composite beams can be determined from measured natural frequencies of two beams specimens with free boundaries. The good agreement between measured and numerical resonant frequencies presented validates this method. The experimental procedure employed in this work involves standard equipment and was carried out fairly easily. However, the presence of closely coupled modes may somewhat complicate the experiment.

The main potentials of the method are the non-destructive characteristics. The position of support system and excitation allow that specific mode can be excited individually. The properties are measured globally avoiding inhomogeneities problems and boundary effects.

7. REFERENCES

ASTM E 1876-01, 2001, “Dynamic Young’s Modulus, Shear Modulus, and Poisson’s Ratio by Impulse Excitation of Vibration”, American Society for Testing and Materials, Philadelphia, PA.

ANSYS On Line Help for Version 10.0.

Ayorinde, E.O., 1995, “Elastic Constants of Thick Orthotropic Composite Plate”, Journal of Composite Materials, Vol. 29, No. 8, pp. 1025-1039.

Ip, K.H., Tse, P.C. and Lai, T.C., 1998, “Material Characterization for Orthotropic Shells Using Modal Analysis and Rayleigh-Ritz Models”, Composites: Part B, Vol 29, No. 4, pp. 397-409.

Lai, T.C., and Ip, K.H., 1996, “Parameter Estimation of Orthotropic Plates by Bayesian Sensitivity Analysis”, Composite Structures, Vol. 34, No. 1, pp. 29-32.

Pickett, G., 1945, “Equations for Computing Elastic Constants for Flexural and Torsional Resonant Frequencies of Vibration of Prisms and Cylinders”, Proceedings ASTM, Vol. 45, pp. 846-865.

Spinner, S., and Valore, R.C., 1958, “Comparisons Between the Shear Modulus and Torsional Resonance Frequencies for Bars and Rectangular Cross Sections”, Journal of Research, NIST, JNBAA, Vol. 60, pp. 459.

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