A NUMERICAL ANALYSIS OF THERMAL ENERGY STORAGE PACKED BEDS

Carlos E.L. Nóbrega, nobrega@pobox.com
Sérgio L. Braga, slbrga@mec.puc-rio.br
Centro Federal de Educação Tecnológica Celso Suckow da Fonseca- CEFET-Rio. Av. Maracanã, 229 , DPMC, ZC.20271-110
Pontifícia Universidade Católica do Rio de Janeiro- PUC-Rio, R. Marquês de S. Vicente,225, DEM, ZC. 22453-900.

Abstract. *The role of thermal energy storage in the HVAC industry has been continuously increasing over the last years, powered by the adoption of "peak" and "off-peak" power consumption periods (and rates) by electrical utilities. As a result, many works have been devoted to modeling and predicting the transient response of both sensible and latent heat storage units, over a variety of geometry. Phase change units have some advantages over sensible heat units, due to its higher storage capacity (per unit volume) and lower average chiller operation, which impacts the system performance as a whole. Accordingly, the present work is dedicated to the development and solution of a simple mathematical model for the heat transfer phenomena within a latent heat storage unit, consisting of a tank containing phase change material (PCM) enclosed within spherical capsules. Several assumptions are made with a view to maintain the model as simple as possible, without disregarding a careful reflection about its accuracy and impact on the reliability of the solution. In particular, simulation is carried out for small values of the Stefan number (Ste), which allows the solidification to be predicted by a quasi-stationary model. The model results in a system consisting of a partial differential parabolic equation, which describes the temperature field along the channel, and an integral equation, which is to represent the phase change evolution within a spherical capsule. The model is solved is solved using a fully implicit finite-volume discretization technique. To obtain more general results, all the physical variables are arranged in non-dimensional groups, the influence of which on the response of the storage unit is evaluated.*

Keywords*: thermal storage, phase change, packed beds*

1. INTRODUCTION

Thermal energy storage's role on the struggle for a more rational use of energy resources has increased over the last decades. This could be explained by the wide variety of applications in which it can be found, such as in solar energy, air-conditioning and cogeneration, to name a few. Thermal energy can be stored as both sensible and latent heat. The latter allows a higher storage capacity per unit volume, as well as a storage constant temperature. The use of sensible heat, however, allows constant heat transfer rates, which cannot be accomplished using PCMs due to the variation of the thermal resistance imposed by the solidified or melted layer. There are a number of techniques of latent heat storage, but they can be generally divided into two main categories Dorgan and Elleson (1993):

- "Bulk storage", where the total amount of PCM is fully contained within the same enclosure.
- "Encapsulation", where the total amount of PCM is divided over a number of capsules.

The objective of this work is to investigate why encapsulation has become more popular than bulk storage over the last years, that is, why should one use several (smaller) capsules rather than just one to contain a certain amount of PCM, using more familiar non-dimensional parameters when compared to previous efforts. Accordingly, the transient response of a latent heat storage unit, as described in Figure 1 will be studied. The PCM is contained in spherical capsules, randomly disposed within the storage tank.

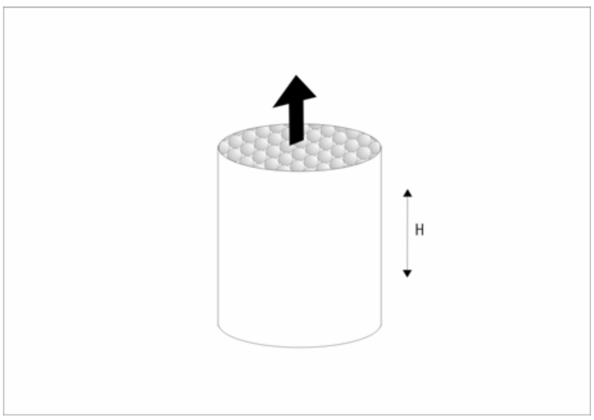

Figure 1: Storage Tank

Prusa, Maxwell and Timmer (1999) developed a model for thermal energy storage using rectangular containers which takes into account the unavoidable sensible heat which accompanies latent heat storage, since there is always a difference between storage and inlet transport fluid temperatures. Arnold (1990, 1991) developed a numerical code to predict the charge and the discharge of latent heat storage within spherical capsules. A calorimeter was used to measure the evolution of the overall thermal resistance as a function of the solidification front progress, resulting in a semi-empirical model able to predict some actual features of latent thermal storage, such as subcooling and PCM fracture due to difference on solid/liquid specific volumes. Gong and Mujundar (1997) proposed the use of several PCMs in a system consisting of multiple shell-and-tube thermal storage units. Regarding cylindrical capsules (tubes), Alexiades and Solomon (1993) proposed a quite simple model which disregards PCM temperature distribution, as well as sensible heat storage within the PCM and the transport fluid. Farid and Kanzawa (1989) proposed a model which takes under consideration both PCM sensible heat and temperature distribution during phase change, and suggested the use of several PCMs with different melting temperatures to make it possible to achieve more uniform heat transfer rates throughout the tank. Although several works have been recently devoted to the study of encapsulated PCMs , such as Vyshak and Jilan (2007) and Hendra et al.(2005), a simple comparison between bulk and fragmented PCMs performances seems to have been disregarded by previous efforts.

2. MODEL DEVELOPMENT

The mathematical model is based on some simplifying assumptions:

1) The tank is represented by a control volume with one inlet and one outlet.
2) The tank is initially "hot", filled with both PCM and transport fluid at phase change temperature T_m.
3) All walls are perfectly insulated.
4) All vertical walls are impermeable.
5) The thermal capacitance and resistance of the tubes walls are negligible.
6) The sensible heat stored within the PCM is small when compared to the latent heat.
7) The spherical capsules bed is represented by a porous media, through which the transport fluid flows. The heat transfer is represented by a heat generation rate per unit volume, which is distributed along the flow direction

All the assumptions are realistic within the scope of the present study. The first four assumptions, for instance, are common to all model and experimental devices. The fifth is as good as smaller are the thickness and thermal conductivity of the capsules, which is also a common feature of actual devices. The sixth is reasonable for small values of the Stefan number, which is frequently the case in latent heat units. This assumption allows the use of a simple and yet effective phase-change model. The seventh assumption enables us to represent this two-dimensional domain by two separate domains, which are coupled through the heat generation rate.

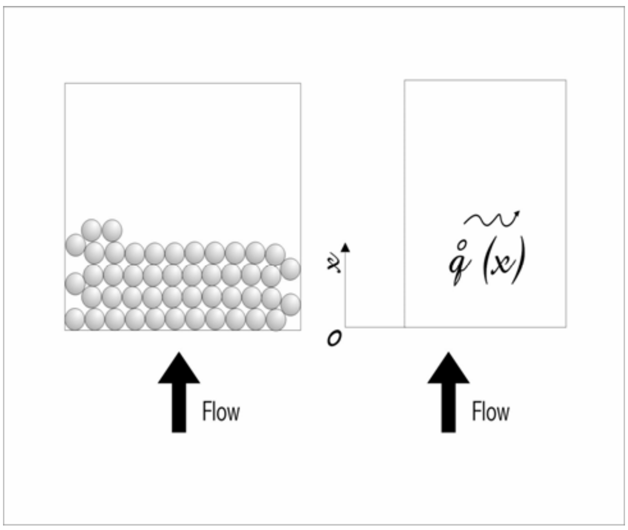

Figure 2: A comparison between physical and numerical domains

As depicted in Figure 2, the flow cross-section in the real tank is wider than in the model. This happens because when one assumes a uniform flow field throughout the tank, a representative area can be found only on a volume rate basis:

$$\overline{A} = \frac{\left(V_T - NS \frac{4\pi}{3} R_0^2 \right)}{H} \tag{1}$$

$$u = \frac{\dot{V}}{A_{AVE}} \tag{2}$$

This enables us to write the one-dimensional transport equation, from which the transport fluid temperature field will be obtained:

$$\rho\ Cp \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \right) = K \frac{\partial^2 T}{\partial x^2} + \dot{q} \tag{3}$$

where $\dot{q}$ is a heat generation rate (per unit volume) which is locally distributed according to

$$\dot{q} = N_s \frac{\left(T_m - T \right)}{R_{th}} \frac{1}{H A_{AVE}} \tag{4}$$

which represents the heat transferred from the PCM to the transport fluid. The overall thermal resistance R_t combines in series the thermal resistances offered by convection through the capsules surface and conduction through the progressively increasing layer, as represent by Figure 3.

$$R_{th} = \frac{1}{4\pi K_{pcm}} \left[\frac{1}{R_m(x)} - \frac{1}{R_0} \right] + \frac{1}{4\pi h R_0} \tag{5}$$

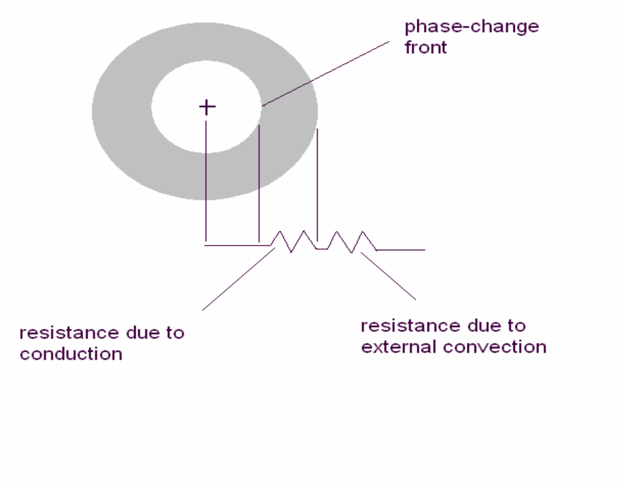

Figure 3: Schematic of the heat transfer process within a capsule

The position of the solidification front R_m will be found by using a quasi-stationary model of change of phase (Alexiades and Solomon (1993)). Accordingly,

$$\frac{k_{pcm}}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial T_{pcm}}{\partial R}\right) = 0 \qquad R_0 > R > R_m \quad , \quad t > 0 \tag{6}$$

Subjected to the following boundary conditions:

$$T_{pcm}\left(R_m, t\right) = T_m \tag{7a}$$

$$K_{pcm}\left.\frac{\partial T_{pcm}}{\partial R}\right|_{R_0} = h\left[T(t) - T_{pcm}(R_o, t)\right] \quad t > 0 \tag{7b}$$

to obtain

$$T_{pcm}(R,t) = T_m + \left[(T - T_m)\frac{1 - {R_m}/{R_0}}{1 - \left(1 - \frac{k_m}{hR_0}\right)\frac{R_m}{R_0}}\right] \qquad R_0 \le R \le R_m \quad t > 0 \tag{8}$$

Knowing the temperature profile, it is possible to apply a differential balance on the solidification front

$$\rho_{pcm}\,\gamma\,\frac{\partial R_m}{\partial t} = -K_{pcm}\left.\frac{\partial T_{pcm}}{\partial R}\right|_{R_m} \qquad t > 0 \tag{9}$$

in which γ is the latent heat of the phase change material

$$R_m \quad (0) \; = \; R_0 \text{ (initial position of the solidification front)} \tag{10}$$

resulting in

$$2\left(1 - \frac{2K_s}{hR_o}\right)\left(\frac{R_m}{R_0}\right)^3 - 3\left(\frac{R_m}{R_0}\right)^2 + 1 + \frac{2K_m}{hR_0} = \frac{6K_m}{\rho_{pcm}\,\gamma R_0^2}\int_0^t \left(T - T_m\right)dt \tag{11}$$

Since one cannot obtain an explicit form for R_{fs}/R_0, it is necessary to solve Equation (11) simultaneously with Equation (3). In order to conduct a more concise analysis, it is advisable to rewrite Eqs. (11) and (3) in non-dimensional forms. Defining the following non-dimensional parameters,

$$\tau = {tu}/{H} \tag{12}$$

$$x^* = {x}/{H} \tag{13}$$

$$R^* = {R_m}/{R_0} \tag{14}$$

$$k^* = {k_{pcm}}/{k} \tag{15}$$

$$Pe = {Hu}/{\alpha_{pcm}} \tag{16}$$

and replacing them in Eq.(3)

$$\frac{\partial\theta}{\partial\tau}+\frac{\partial\theta}{\partial x^*} = \frac{1}{Pe}\frac{\partial^2\theta}{\partial x^{*2}} - 4\pi\frac{k^*}{Pe}\frac{Ns}{v_f}\frac{\left(H/R_0\right)}{\left(R_T/R_0\right)^2}\frac{1}{(\frac{1}{Bi}+\frac{1}{R^*}-1)} \tag{17}$$

initial condition: $\theta(x*,\ 0) = 0$ (18a)

boundary conditions

$$\theta(0,\ \tau) = 1.0 \quad \tau > 0 \tag{18b}$$

$$at\, x^* = 1, \quad \frac{\partial\theta}{\partial x^*} = 0 \tag{18c}$$

Also, defining the modified Stefan number

$$Ste_{mo} = \frac{\rho \quad C_p (T_m - T_{in})}{\rho_{pcm}\gamma} \tag{19}$$

and introducing the non-dimensional variables into Eq. (11) one would obtain

$$2\left(1-\frac{1}{Bi}\right)R^{*3} - 3R^{*2} + 1 + \frac{2}{Bi} = 6\frac{k^*}{\rho^*}\frac{Ste_{mo}}{Pe}\left(\frac{H}{R_0}\right)^2\int_0^\tau \theta d\tau \tag{20}$$

The solution is then obtained by solving simultaneously Eqs. (17) and (20). The domain relative to Eq. (17) is divided into a finite number of volumes by using the upwind scheme to represent advection and the fully implicit scheme for the transient term (Patankar, (1980)). The resulting tri-diagonal matrix is then solved for each time step, and after the temperature field ($\theta(x^*)$) is obtained, the radii ratio field $R^*(x^*)$ is calculated from Eq. (20). Since the value of the integral in Eq.(20) is available from simple numeric integration, this equation turns into a simple algebraic cubic equation for R^*, which is solved by Newton's method. Accordingly, a discretization scheme is only required by Eq,(17), which is presented below for an interior nodal point "i". All the geometric constants in Eq.(17) have been lumped into a non-dimensional parameter V^*.

$$\left(\theta_i - \theta_i^{t-1}\right)\frac{\delta x^*}{\delta t^*} + \left(\theta_i - \theta_{i-1}\right) = \frac{1}{Pe}\left(\frac{\theta_{i+1}-\theta_i}{\delta x^*} - \frac{\theta_i - \theta_{i-1}}{\delta x^*}\right) - \frac{4\pi k^* V^* \theta_i \delta x^*}{Pe\left(\frac{1}{Bi}+\left(\frac{1}{R^*}-1\right)\right)} \tag{21}$$

3. RESULTS

Since the mathematical model adopted disregards the sensible heat capacity of the PCM, the storage unit is considered to be initially at the phase change temperature. Accordingly, the phase-change front shields the capsule core (which remains at T_m throughout the process) and all the heat flux is consumed by the phase change progress. The time required to charge the unit as a function of the number of spheres N_s is illustrated in Figure 4. It can be seen that increasing the number of spheres will shorten the process only up to a certain point, and that excessive fragmentation of the storage media will result in decreasingly storage rates. The existence of an optimum degree of fragmentation can be explained by verifying the limits of the thermal resistance and the total heat transfer area as R_0 is continuously reduced. The total area increases as R_0 approaches zero, since the capsule bed can be taken as a porous media. However, R^* always varies between 1 and 0, which causes the thermal resistance to vary from its initial value to infinite. The continuous reduction of R_0 will result in smaller average values of R^*, and thus higher average values for the thermal resistance R_{th}. Accordingly, the increasing degree of fragmentation of a given volume of PCM represents a trade-off between the total heat transfer area and the thermal resistance within an individual capsule.

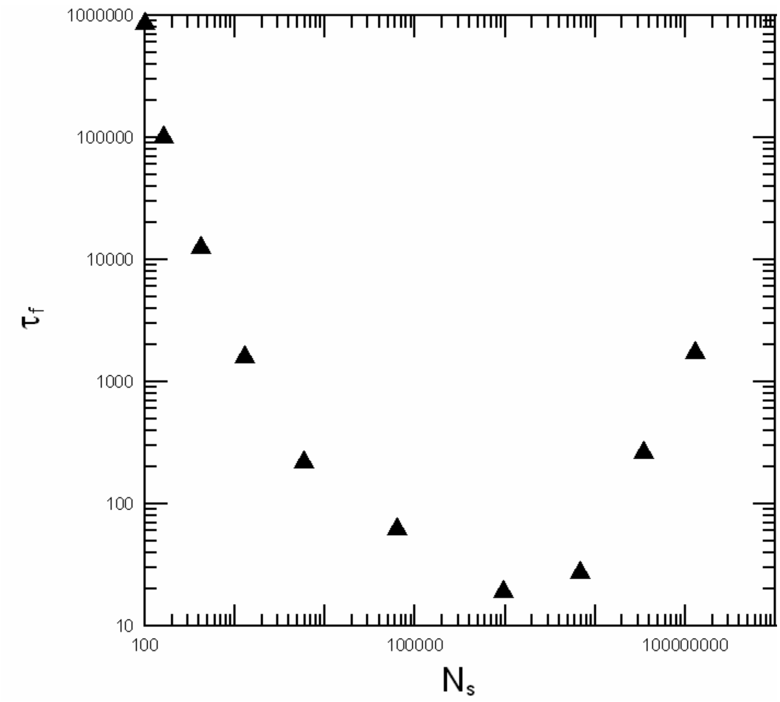

Figure 4: Influence of the number of capsules over the storage process

The values depicted in figure 4 were obtained by varying the number of capsules simultaneously with the values of Bi, so that the solely effect would be the increase in the number of capsules. Figures 5 and 6 depict the sole influence of Bi over the R^* distribution throughout the tank. It can be seen that for a small value of Bi (Figure 4), the increase in the number of capsule will drastically shorten the storage process, whereas for a high value of Bi (Figure 5), the time required to charge the unit is insensitive to the degree of fragmentation. For a high Bi value, the thermal resistance offered by the conduction is higher than that offered by convection, so that an increase in the total heat transfer area has little significance. Conversely, for a small value of Bi, the component relative to convection prevails, such that an increase in the number of capsules will significantly shorten the storage process.

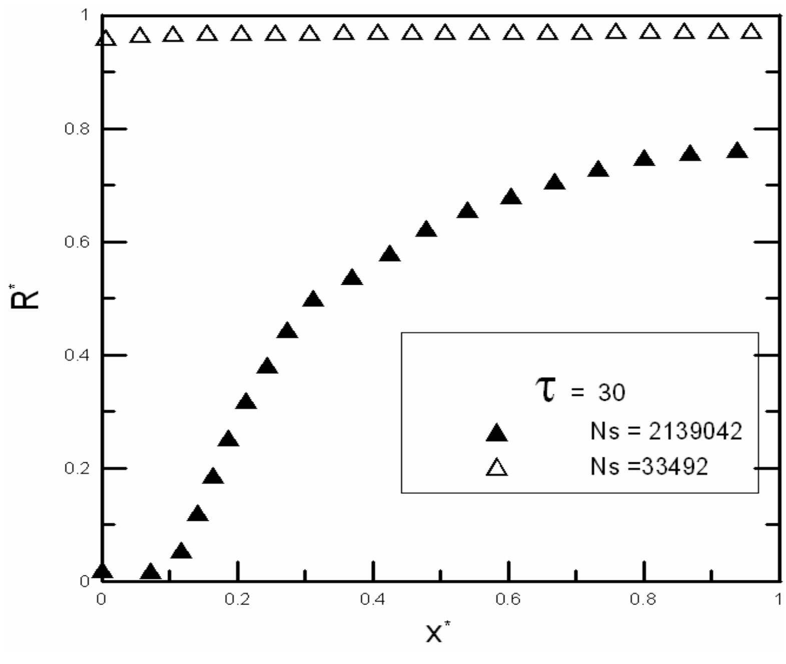

Figure 5: Influence of the number of capsules over the storage process, Bi =0,1

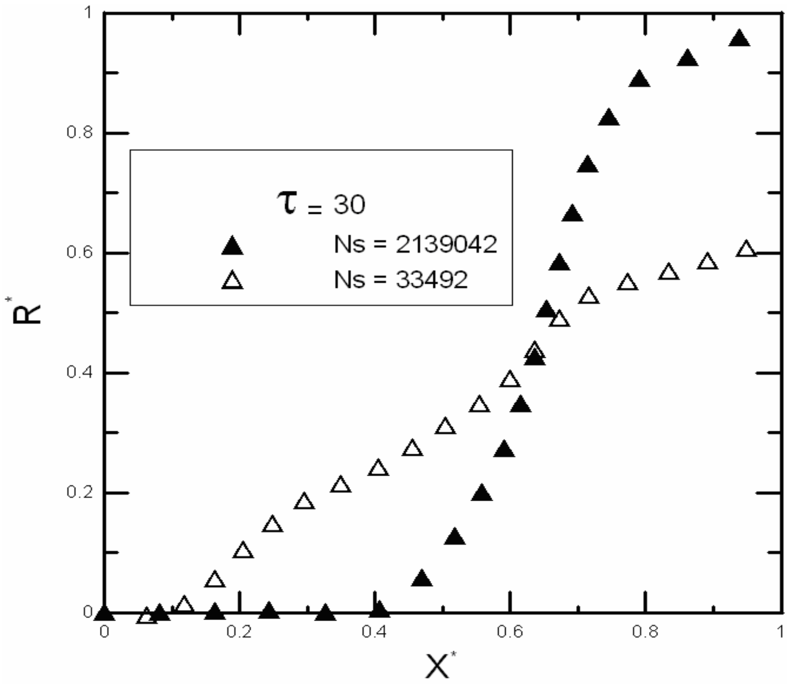

Figure 5:

Figure 6: Influence of the number of capsules over the storage process, Bi =10,0

4. CONCLUSIONS

A simple mathematical model for the thermal storage within phase change packed beds has been developed and solved using a discretization technique. The results show that the current trend in the design of such systems, namely the use of a fragmentation of the storage media in fact shortens the storage process up to a certain limit, beyond which the performance stars to decrease.

5. REFERENCES

Alexiades, V., Solomon, A.D., "Mathematical Modeling of Melting and Freezing Processes", Hemisphere Publishing Corporation, Washington, 1993.

Arnold, D., 1990, "Dynamic simulation of encapsulated ice stores, part I – the model", ASHRAE Transactions, 96(1), pp.1103-1110.

Arnold, D**.,** 1991, "Laboratory Performance of encapsulated ice store", ASHRAE Transactions, 97(2), 1170-1178.

Dorgan, C.E; Elleson, J.S., 1993, "Design Guide for Cool Thermal Storage", ASHRAE, Atlanta, GA

Farid, M. and.Kanzawa, A, 1989, "Thermal performance of a heat storage module using PCM´s with different melting temperatures: Mathematical Modeling", Journal of Solar Energy Engineering, 111, pp.152-157

Gong, Z.X. and Mujundar, A., 1997, "Thermodynamic optimization of the thermal process in energy storage using multiple phase change materials", Applied Thermal Engineering, vol.17, (11), pp. 1067-1083.

Hendra R., Hamdani H, Mahia T.M., Majuski H., 2005, " Thermal and melting heat transfer characteristics in a latent heat storage system using mikro", Applied Thermal Engineering 25(11),pp: 1503-1515.

Patankar, S.V, 1980, "Numerical Heat Transfer and Fluid Flow", Hemisphere Publishing Corporation, Washington.

Prusa, G.M. Maxwell, K.J. Timmer, ASHRAE Report, Project Number 481-RP (1989).

Vyshak NR, Jilan G., 2007, "Numerical analysis of latent heat thermal energy storage system", Energy Conversion and Management, (48), pp. 2161-2168.

NOMENCLATURE

A_{AVE} = Average flow area
Bi =Biot Number
C_p = Transport fluid specific heat
h = Convective heat transfer coefficient

H = Tank Height
HTF = Heat transfer fluid
K = Heat transfer fluid thermal conductivity
K_{pcm} = Phase change material thermal conductivity
K^* = Thermal conductivity ratio
Ns = Number of Spheres
P = Tank length
PCM = Phase change material
Pe = Peclet number
q = Heat generation
R = Radial Position
R_0 = Capsule radius
R_R = Radii ratio
R_{th} = local thermal resistance
Ste_{mo} = Modified Stefan number
t = Time
T = Transport Fluid Temperature Field.
T_{in} = Heat transfer fluid inlet temperature
T_{pcm} = Temperature through the phase change layer
T_m = Temperature of phase change
u = Characteristic velocity of the flow
V_t = Storage Tank Volume
x = Flow direction
x^* = Non-dimensional flow direction

Greek symbols

ρ = Density
γ = PCM latent heat
τ = Non-dimensional time
θ = Non-dimensional temperature

Subscripts

m = melting
pcm = phase change material

6. RESPONSIBILITY NOTICE

The author(s) is (are) the only responsible for the printed material included in this paper.