INFLUENCE OF THE SCATTERING ON THERMAL TENSIONS IN SEMITRANSPARENT CERAMIC MATERIALS FOR THE CONDUCTION-RADIATION COUPLING

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Abstract. *The ceramic material has been playing a greatly relevant role on the technological industrial development. Due to its physical characteristics, it can be applied on extreme temperature conditions. However, its fragile and low thermal conductivity make it highly susceptible to fissures on its surface or inner parts because of the high tension that results from the heat transfer. That is why the choice of the material is very important to sustain the structural integrity, especially on application that involves high temperatures. For a long time, this choice has been made only on a qualitative basis, through thermal shock tests. The tensions that appear on a ceramic material due to the high temperature gradients is quantitatively determined through the solution of an one-dimensional problem of transient heat transfer for the conduction-radiation coupling, on a semi-transparent environment with anisotropic scattering and constant thermal characteristics. On this article the Generalized Integral Transform Technique (GITT) and the Galerkin Method are used to solve the necessary energy and the radioactive transfer equations simultaneously. The final results allow the analysis of the influence that the conduction-radiation parameter and the scattering type cause on the thermal tensions during the heat of the body.*

Keywords*: Thermal Tensions, Ceramic materials, Anisotropic scattering, GITT, Galerkin Method*

1. INTRODUCTION

When the temperature of a material varies, its internal energy varies as well, manifesting important properties for the engineering projects such as thermal dilatation, heat capacity, conductivity, thermal diffusion and the thermal shock resistance parameter.

The ceramic material, due to its properties such as low conductivity and high fusion temperature can not be replaced in some industrial application, mainly when high temperature is required. Due to its low conductivity, temperature differences in a short length cause the adjacent fibers dilatations and contractions generating thermal tensions that may cause fissures on the surface or in the interior of the body and that may as a result compromise the quality of the product. The knowledge of the temperature field allows knowing the intensity of the tensions and consequently the thermal shock resistance parameter. Diniz et al (2005a) and Diniz et al (2006a) analyzed the thermal tensions on a ceramic body taking into consideration the scattering of the isotropic thermal radiation on the body during the cooling and heat respectively. On the present article it is considered the anisotropic scattering for different functions of phase.

In order to determine the transient thermal field on semitransparent materials, such as ceramics, it is required to solve simultaneously the energy and radiation transference equations (ETR). The former, which has a not linear nature, subjected to the boundary conditions involving the radiation and convection heat exchange between the body and the medium is solved using the GITT methodology presented by Cotta (1993). The Galerkin method is used to compute the radioactive part of the problem which is characterized by a non linear integral differential equation subjected to the semitransparent boundary conditions that emit and reflect the thermal radiation diffusely.

Unlike Diniz et al (2005b) and Diniz et al (2006b) who used an expansion on a potency series, the integral form of the radioactive transference equation is transformed in a set of algebra equations to find the expansion associated to the representation for the radiation on Legendre polynomials on the spatial variant.

Once the expansion is determined, practical interest values for the engineering such as radioactive intensity, radioactive heat liquid flow, incident radiation and the divergent of the hear flow by radiation are obtained in any part of the medium.

2. PHYSICAL PROBLEM AND MATHEMATICAL MODELLING

The physical problem consists of a semitransparent medium, plane and parallel that emits, absorbs and scatters anysotropically the incident radiation. A body with initial temperature, T_i, is put in an isothermal environment with uniform temperature, T_f and submitted to external sources of radiation. Fig. 1 shows in a simplified manner the coordinate system, the heat transfer terms that occurs in each boundary surface and the radiative properties on the internal and external surfaces of the body.

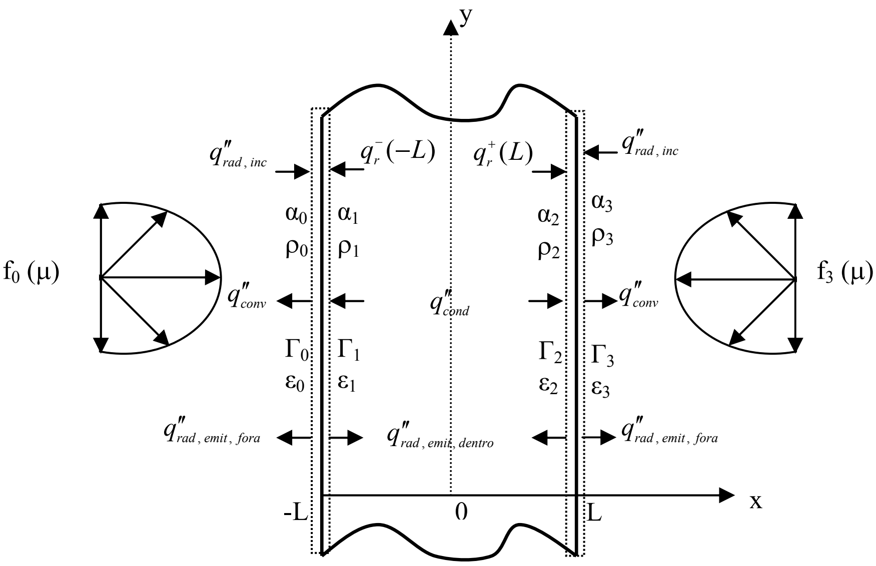

Figure 1. Physical and geometrical representation of the problem.

where ρ_i (i = 0, 1, 2, 3) e ε_i (i = 1, 2) represent respectively the reflectivity and emission of the boundary surface. The functions $f_i(\mu)$ (i = 1, 2), represent the intensity of the external radiation that act upon the body.

The energy equation for simultaneous heat transfer by conduction and radiation in a participant medium, without internal heat generation is written in the non dimensional form by:

$$\frac{\partial \Theta(\tau,\xi)}{\partial \xi} = \frac{\partial}{\partial \tau}\left[\alpha^* \frac{\partial \Theta(\tau,\xi)}{\partial \tau}\right] - \frac{1}{N}\frac{\partial Q^R(\tau;\xi)}{\partial \tau}, \quad -\tau_0 < \tau < \tau_0, \quad \xi > 0 \tag{1}$$

According to Siegel (1998) and Sadooghi (2005), for transparent boundary surfaces it can be written that:

$$K^* \frac{\partial \Theta(\tau,\xi)}{\partial \tau} - Bi\,\Theta(\tau,\xi) = 0, \quad \tau = -\tau_0, \quad \xi > 0 \tag{2}$$

$$K^* \frac{\partial \Theta(\tau,\xi)}{\partial \tau} + Bi\,\Theta(\tau,\xi) = 0, \quad \tau = \tau_0, \quad \xi > 0 \tag{3}$$

with inlet condition:

$$\Theta(\tau,\xi) = -1, \quad -\tau_0 \le \tau \le \tau_0, \quad \xi = 0 \tag{4}$$

and the radiation heat flow divergent in the dimensionless form is given as:

$$\frac{\partial Q^R(\tau;\xi)}{\partial \tau} = (1-\omega)\left\{\left[\Theta^*(\tau,\xi)\right]^4 - G_n^*(\tau,\mu)\right\} \tag{5}$$

where:

$$\Theta^*(\tau,\xi) = \left(\frac{T_f - T_i}{T_f}\right)\cdot \Theta(\tau,\xi) + 1 \tag{6}$$

where K* and α* represent the conductivity and thermal diffusivity of the body. Bi is the Biot number, Θ is the dimensionless transient temperature, τ represents the medium optical thickness, ω is the number of the simple scattering, ξ is a timing dimensionless variant, N is the factor of the conduction and radiation coupling and μ is the direction of the radiation.

In the dimensionless solution of the problem, it was used the following groups and the dimensionless parameters:

$$\tau = \beta x \qquad \xi = \frac{K_0 \beta^2 t}{\hat{\rho} C_P} \qquad \Theta(\tau,\xi) = \frac{T(x,t) - T_f}{T_f - T_i} \qquad I(\tau,\mu) = \frac{n^2 \overline{\sigma} T_r^4}{\pi} \cdot \psi(\tau,\mu) \tag{7}$$

$$Q^R(\tau;\xi) = \frac{q^r(x;t)}{4 n_i^2 \overline{\sigma} T_f^4} \qquad G^*(\tau,\mu) = \frac{G(\tau,\mu)}{4 n^2 \overline{\sigma} T_r^4} \qquad F_i(\mu) = \frac{f_i(\mu)}{4 n_i^2 \overline{\sigma} T_f^4} \qquad N = \frac{K_0 \beta (T_f - T_i)}{4 n_i^2 \overline{\sigma} T_f^4} \tag{8}$$

where $\hat{\rho}$ and C_P are the density and specific heat for the medium, β is the coefficient of extinction , n the coefficient of refraction, $\overline{\sigma}$ the Stefan-Boltzmann constant and ψ(τ, μ) represents the dimensionless radioactive intensity.

The conductivity and diffusivity that appear in Eq. (1-3) are respectively expressed in function of the dimensionless temperature, as proposed by Nishikawa et al (1995) as:

$$K^*(\Theta) = 1 + A\Theta(\tau,\xi) \tag{9}$$

$$\alpha^*(\Theta) = 1 + B\Theta(\tau,\xi) \tag{10}$$

where A and B are constants.

The term $G_n^*(\tau,\mu)$ that appears in Eq. (5), according to Özişik (1973) is the dimensionless incident radiation, defined by:

$$G_n^*(\tau) = \frac{1}{2}\left[\int_0^1 P_n(\mu)\psi^+(\tau,\mu)d\mu + (-1)^n \int_0^1 P_n(\mu)\psi^-(\tau,-\mu)d\mu\right] \tag{11}$$

where $P_n(\mu)$ is the order n Legendre polynomial and argument μ.

The Eq. (11) is solved by the Galerkin Method and ψ (τ, μ) must satisfy the dimensionless radiative transfer equation below, considering the anisotropic scattering of radiation:

$$\mu \frac{\partial \psi(\tau,\mu)}{\partial \tau} + \psi(\tau,\mu) = \mathrm{S}(\tau) + \frac{\omega}{2}\int_{-1}^{1} p(\mu,\mu')\psi(\tau,\mu')\mathrm{d}\mu', \quad -\tau_0 < \tau < \tau_0, \quad -1 \le \mu \le 1 \tag{12}$$

$$\psi^+(-\tau_0,\mu) = (1-\rho_0)\cdot \mathrm{F}_1(\mu) + \varepsilon_1 \Theta_1^4 + 2\rho_1 \int_0^1 \psi^-(-\tau_0,-\mu')\mu'\mathrm{d}\mu', \quad \mu > 0 \tag{13}$$

$$\psi^-(\tau_0,-\mu) = (1-\rho_3)\cdot \mathrm{F}_2(\mu) + \varepsilon_2 \Theta_2^4 + 2\rho_2 \int_0^1 \psi^+(\tau_0,\mu')\mu'\mathrm{d}\mu', \quad \mu > 0 \tag{14}$$

The function of phase, p (μ, μ'), represents the probability in which the incident radiation in the direction μ will be scattered in the direction μ' (Özişik, 1973). The functions $F_i(\mu)$ (i = 1, 2), represent the dimensionless external radiation intensity on the body.

3. PROBLEM SOLUTION

The energy and radiative transfer equations are coupled due to the temperature of the body. Therefore, the resolution process must occur simultaneously. The temperature distribution in the medium is obtained from the energy equation which involves the radiative heat flow divergent and that is determined by the solution of the radiative transfer equation, which can not be solved without the knowledge of temperature field.

3.1. Auxiliary Eigenvalue Problem

Following the Generalized Integral Transform Technique methodology, the first step consists in choosing the auxiliary eigenvalue problem that will be used in the integral transformation of the equation system to be solved. As the proposed model is described by a second-order differential equation system, the auxiliary problem is related to the classical Sturm-Liouville problem (Mikhailov e Özişik, 1984, Özişik, 1980 e Cotta, 1993):

$$\frac{d^2\phi_i(\tau)}{d\tau^2}+\gamma_i^2\phi_i(\tau)=0, \quad -\tau_0<\tau<\tau_0 \tag{15}$$

$$\frac{d\phi_i(\tau)}{d\tau}-Bi\phi_i(\tau)=0, \quad \tau=-\tau_0 \tag{16}$$

$$\frac{d\phi_i(\tau)}{d\tau}+Bi\phi_i(\tau)=0, \quad \tau=\tau_0 \tag{17}$$

The problem described by the Eq. (15-17) is solved, and the eigenfunctions, eigenvalues and norms are obtained respectively by:

$$\phi_i(\tau)=Cos(\gamma_i\tau) \tag{18}$$

$$\gamma_i tg(\gamma_i\tau_0)=Bi \tag{19}$$

$$N_i(\gamma_i)=\frac{\tau_0\left[\gamma_i^2+Bi^2\right]+Bi}{\gamma_i^2+Bi^2} \tag{20}$$

3.2. Determination of the Inverse and Transformed Formulas

The second step in the use of the GITT as a proposed resolution tool is the definition of two formulas: transformation and inversion. According to the eigenfunctions orthogonal properties:

$$\overline{\Theta}_i(\xi)=\frac{1}{N_i^{1/2}}\int_{-\tau_0}^{\tau_0}\psi_i(\tau)\Theta(\tau,\xi)d\tau \qquad \text{Transformed} \tag{21}$$

$$\Theta(\tau,\xi)=\sum_{i=1}^{\infty}\frac{\psi_i(\tau)\overline{\Theta}_i(\xi)}{N_i^{1/2}} \qquad \text{Inverse} \tag{22}$$

3.3. Integral Transformation of the Partial Differential Equations

The operator $\frac{1}{N_i^{1/2}}\int_{-\tau_0}^{\tau_0}\phi_i(\tau)d\tau$ is applied in Eq. (1) in order to obtain the integral transformation of the energy equation:

$$\frac{d\overline{\Theta}_i(\xi)}{d\xi}=D_i-E_i-\sum_{j=1}^{\infty}\left[A_{ij}+B\sum_{k=1}^{\infty}W_{ijk}\overline{\Theta}_k(\xi)\right]\overline{\Theta}_j(\xi)-\frac{1}{N}\overline{G}_i[\overline{\Theta}_i(\xi)] \tag{23}$$

where:

$$\overline{G}_i[\overline{\Theta}_i(\xi)]=\frac{(1-\omega)}{N_i^{1/2}}\int_{-\tau_0}^{\tau_0}\phi_i(\tau)\left\{\left[\Theta^*(\tau,\xi)\right]^4-G_n^*(\tau,\mu)\right\}d\tau \tag{24}$$

$$D_i=-\frac{\phi_i(\tau_0)}{N_i^{1/2}}\frac{\alpha^*(\tau_0)}{K^*(\tau_0)}Bi\Theta(\tau_0,\xi) \tag{25}$$

$$E_i=\frac{\phi_i(-\tau_0)}{N_i^{1/2}}\frac{\alpha^*(-\tau_0)}{K^*(-\tau_0)}Bi\Theta(-\tau_0,\xi) \tag{26}$$

$$A_{ij} = \frac{1}{(N_i N_j)^{1/2}} \int_{-\tau_0}^{\tau_0} \frac{\partial \phi_i(\tau)}{\partial \tau} \frac{\partial \phi_j(\tau)}{\partial \tau} d\tau \tag{27}$$

$$W_{ijk} = \frac{1}{(N_i N_j N_k)^{1/2}} \int_{-\tau_0}^{\tau_0} \frac{\partial \phi_i(\tau)}{\partial \tau} \frac{\partial \phi_j(\tau)}{\partial \tau} \phi_k(\tau) d\tau \tag{28}$$

The term $G_n^*(\tau,\mu)$ that appears in Eq. (24) has formal resolution by:

$$G_n^*(\tau) = Y_n(\tau) + \omega \sum_{m=0}^{M} \int_{-\tau_0}^{\tau_0} K_{mn}(\tau,\tau') \cdot G_m^*(\tau') d\tau' \tag{29}$$

Where it was defined that:

$$\begin{aligned} K_{mn}(\tau,\tau') = a_m \Bigg\{ & \frac{1}{2} \int_0^1 \frac{1}{\mu} P_m(\mu) P_n(\mu) \cdot r \cdot e^{-|\tau-\tau'|/\mu} d\mu + \rho_1 \beta^* J_n(\tau_0+\tau) \left[(-1)^m J_m(\tau_0+\tau') + \alpha_2 J_m(\tau_0-\tau') \right] + \\ & \rho_2 \beta^* J_n(\tau_0-\tau) \left[\alpha_1 E_2(\tau') + E_2(\tau_0-\tau') \right] (-1)^n \Bigg\} \end{aligned} \tag{30}$$

$$\begin{aligned} Y_n(\tau) = \frac{1}{2} \Bigg\{ & 2 \int_{-\tau_0}^{\tau_0} K_{0n}(\tau,\tau') S(\tau') d\tau' + \beta^* \varepsilon_1 \Theta^4(\tau_0,\xi) \left[J_n(\tau_0+\tau) + (-1)^n \alpha_2 J_n(\tau_0-\tau) \right] + \\ & \beta^* \varepsilon_2 \Theta^4(-\tau_0,\xi) \left[\alpha_1 J_n(\tau_0+\tau) + (-1)^n J_n(\tau_0-\tau) \right] + (1-\rho_0) \int_0^1 P_n(\mu) F_1(\mu) e^{-(\tau_0+\tau)/\mu} d\mu + \\ & 2\rho_2 \beta^* \left[\alpha_1 J_n(\tau_0+\tau) + (-1)^n J_n(\tau_0-\tau) \right] (1-\rho_0) \int_0^1 F_1(\mu) e^{-2\tau_0/\mu} \mu d\mu + \\ & 2\rho_1 \beta^* \left[J_n(\tau_0+\tau) + (-1)^n \alpha_2 J_n(\tau_0-\tau) \right] (1-\rho_3) \int_0^1 F_2(\mu) e^{-2\tau_0/\mu} \mu d\mu + \\ & (-1)^n (1-\rho_3) \int_0^1 P_n(\mu) F_2(\mu) e^{-(\tau_0-\tau)/\mu} d\mu \Bigg\} \end{aligned} \tag{31}$$

$$J_n(y) = \int_0^1 P_n(\mu) \cdot e^{-y/\mu} d\mu \qquad E_n(z) = \int_0^1 \eta^{n-2} \cdot e^{-z/\eta} d\eta \tag{32}$$

$$\alpha_1 = 2\rho_1 E_3(2\tau_0) \qquad \alpha_2 = 2\rho_2 E_3(2\tau_0) \qquad \beta^* = \frac{1}{1-\alpha_1\alpha_2} \tag{33}$$

The dimensionless incident radiation is represented in terms of Legendre polynomials on the optical variable, τ according to Cengel (1984), being:

$$G_n^*(\tau) = \sum_{k=0}^{K} c_{nk} P_k\left(\frac{\tau}{\tau_0}\right), \quad n = 0, 1, 2, \ldots, N \quad e \quad k = 0, 1, 2, \ldots, K \tag{34}$$

where c_{nk} are the expansion coefficients to be determined. Once the coefficients are known, the radiation intensities, the incident radiation, radioactive thermal heat and the divergent of radioactive heat flow are determined on any point from the medium by its definitions.

The Galerkin method applied to Eq. (29) with $G_n^*(\tau,\mu)$ given by Eq. (34) will result:

$$G_n^*(\tau)=\frac{1}{2}\Bigg\{(1-\rho_0)\int_0^1 F_1(\mu)\cdot e^{-(\tau_0+\tau)/\mu}d\mu+\left[\varepsilon_1\Theta^4(\tau_0,\xi)+2\rho_1 K_1\right]E_2(\tau_0+\tau)+(1-\rho_3)\int_0^1 F_2(\mu)\cdot e^{-(\tau_0-\tau)/\mu}d\mu+$$
$$\left[\varepsilon_2\Theta^4(-\tau_0,\xi)+2\rho_2 K_2\right]E_2(\tau_0-\tau)+\int_{-\tau_0}^{\tau_0}S(\tau')E_1(|\tau-\tau'|)d\tau'+\omega\sum_{n=0}^{N}\sum_{k=0}^{K}a_n c_{nk}Z_{nk}(\tau)\Bigg\} \tag{35}$$

with the expansion coefficient, c_{nk} obtained from the following matrix system:

$$\left[b_{mnkl}\right]\{c_{mk}\}=\{d_{nl}\} \tag{36}$$

and:

$$K_1=\beta^*\Bigg\{\alpha_2(1-\rho_0)\int_0^1 F_1(\mu)e^{-2\tau_0/\mu}\mu d\mu+(1-\rho_3)\int_0^1 F_2(\mu)e^{-2\tau_0/\mu}\mu d\mu+\left[\alpha_2\varepsilon_1\Theta_1^4+\varepsilon_2\Theta_2^4\right]E_3(2\tau_0)+$$
$$\int_{-\tau_0}^{\tau_0}S(\tau)\left[E_2(\tau_0+\tau)+\alpha_2E_2(\tau_0-\tau)\right]d\tau+\omega\sum_{n=0}^{N}\sum_{k=0}^{K}a_n c_{nk}T_{nk}\left[(-1)^n+\alpha_2(-1)^k\right]\Bigg\} \tag{37}$$

$$K_2=\beta^*\Bigg\{(1-\rho_0)\int_0^1 F_1(\mu)e^{-2\tau_0/\mu}\mu d\mu+\alpha_1(1-\rho_3)\int_0^1 F_2(\mu)e^{-2\tau_0/\mu}\mu d\mu+\left[\varepsilon_1\Theta_1^4+\alpha_1\varepsilon_2\Theta_2^4\right]E_3(2\tau_0)+$$
$$\int_{-\tau_0}^{\tau_0}S(\tau)\left[\alpha_1E_2(\tau_0+\tau)+E_2(\tau_0-\tau)\right]d\tau+\omega\sum_{n=0}^{N}\sum_{k=0}^{K}a_n c_{nk}T_{nk}\left[\alpha_1(-1)^n+(-1)^k\right]\Bigg\} \tag{38}$$

$$Z_{nk}(\tau)=\int_0^1\frac{1}{\mu}P_n(\mu)\left[\int_{-\tau_0}^{\tau}P_k\left(\frac{\tau'}{\tau_0}\right)e^{-(\tau-\tau')/\mu}d\tau'+(-1)^n\int_{-\tau_0}^{\tau}P_k\left(\frac{\tau'}{\tau_0}\right)e^{-(\tau-\tau')/\mu}d\tau'\right]d\mu$$
$$\sum_{\nu=0}^{k}\frac{1}{\tau_0^{\nu}}\Bigg\{\left[(-1)^n+(-1)^{\nu}\right](2\nu-1)!!C_{k-\nu}^{\nu+1/2}\left(\frac{\tau}{\tau_0}\right)S_{n,\nu}-$$
$$\frac{(k+\nu)!}{2^{\nu}\nu!(k-\nu)!}\left[(-1)^kS_{n,\nu}^*(\tau_0+\tau)+(-1)^nS_{n,\nu}^*(\tau_0-\tau)\right]\Bigg\} \tag{39}$$

$$b_{mnkl}=\frac{2\tau_0}{2k+1}\delta_{mn}^{kl}-\omega a_m\left\{\frac{1}{2}T_{mnkl}+\beta^*T_{nl}T_{mk}\left[(-1)^m\rho_1+(-1)^{n+l+k}\rho_2+\alpha_2\rho_1\left\langle(-1)^k+(-1)^{m+n+l}\right\rangle\right]\right\} \tag{40}$$

$$2d_{nl}=2\int_{-\tau_0}^{\tau_0}\int_{-\tau_0}^{\tau_0}S(\tau')K_{on}(\tau,\tau')P_l\left(\frac{\tau}{\tau_0}\right)d\tau d\tau'+(1-\rho_0)R_{nl}+(-1)^n(1-\rho_3)R_{nl}^*+$$
$$\left\{\varepsilon_1\Theta^4(-\tau_0,\xi)\left[1+(-1)^{l+n}\alpha_2\right]+\varepsilon_2\Theta^4(\tau_0,\xi)\left[\alpha_1+(-1)^{l+n}\right]\right\}\beta T_{nl}+$$
$$2\rho_2\beta T_{nl}\left[\alpha_1+(-1)^{l+n}\right](1-\rho_0)\int_0^1F_1(\mu)e^{-2\tau_0/\mu}\mu d\mu+$$
$$2\rho_1\beta T_{nl}\left[1+(-1)^{l+n}\alpha_2\right](1-\rho_3)\int_0^1F_2(\mu)e^{-2\tau_0/\mu}\mu d\mu \tag{41}$$

$$T_{ij}=\int_{-\tau_0}^{\tau_0}\int_0^1P_i(\mu)P_j\left(\frac{\tau}{\tau_0}\right)e^{-(\tau_0+\tau)/\mu}d\mu d\tau=\sum_{\nu=0}^{j}\frac{(j+\nu)!}{(2\tau_0)^{\nu}\nu!(j-\nu)!}\left[(-1)^{j+\nu}S_{i,\nu+1}-S_{i,\nu+1}^*(2\tau_0)\right] \tag{42}$$

$$T_{mnkl} = \int_0^1 \int_{-\tau_0}^{\tau_0} \int_{-\tau_0}^{\tau_0} \frac{1}{\mu} P_m(\mu) P_n(\mu)\, r\, e^{-|\tau-\tau'|/\mu} P_k\left(\frac{\tau'}{\tau_0}\right) P_l\left(\frac{\tau'}{\tau_0}\right) d\tau' d\tau d\mu =$$

$$\left[(-1)^{n+m} + (-1)^{k+l}\right] \sum_{\nu=0}^{k} \frac{1}{(2\tau_0)^{\nu}} \left\{ \tau_0 \left[1 + (-1)^{l+k+\nu}\right] \sum_{i=0}^{[l/2]} h_i (l-2i)! \right.$$
$$\times \sum_{j=0}^{k-\nu} \frac{(-1)^j (\nu+k+j)!}{2^j (\nu+j)!(k-\nu-j)!(l-2i+j+1)!} \sum_{s=0}^{[n/2]} h_s S_{m,\nu+n-2s} - \frac{(k+\nu)!}{\nu!(k-\nu)!}$$
$$\left. \times \sum_{i=0}^{l} \frac{(l+i)!}{(2\tau_0)^i\, i!(l-i)!} \sum_{s=0}^{[n/2]} h_s \times \left[(-1)^i S_{m,\nu+i+n-2s+1} - (-1)^l S^*_{m,\nu+i+n-2s+1}(2\tau_0)\right] \right\} \tag{43}$$

$$S_{n,i} = \int_0^1 \mu^i P_n(\mu) d\mu = \frac{i(i-1)(i-2)(i-3)\cdots(i+2-n)}{(i+n+1)(i+n-1)(i+n-3)\cdots(i+3-n)} \tag{44}$$

$$S^*_{n,i}(y) = \int_0^1 \mu^i e^{-y/\mu} P_n(\mu) d\mu = \sum_{j=0}^{[n/2]} h_j E_{n+i-2j+2}(y) \tag{45}$$

$$h_i = \frac{(-1)^i (2n-2i)!}{2^n i!(n-i)!(n-2i)!} \quad e \quad [n/2] = \begin{cases} n/2 & para \;\; n \;\; par \\ (n-1)/2 & para \;\; n \;\; ímpar \end{cases} \tag{46}$$

$$\delta^{kl}_{mn} = \begin{cases} 1 & para \quad m=n \quad e \quad k=l \\ 0 & caso\ contário \end{cases} \tag{47}$$

$$\begin{aligned} C^{m+1/2}_{n-m}(x) &= \text{Gegenbauer Polynomials} \\ &= \frac{1}{\Gamma(m+1/2)} \sum_{i=0}^{(n-m)/2} \frac{(-1)^i \Gamma(n+1/2-i)}{i!(n-m-2i)!} (2x)^{n-m-2i} \end{aligned} \tag{48}$$

3.4. Dimensionless average temperature Calculus

The average temperature, such as in Fig. 1, is calculated according to the distribution of the temperature in the following way:

$$\Theta(\xi)_{med} = \frac{1}{2\tau_0} \int_{-\tau_0}^{\tau_0} \Theta(\tau,\xi) d\tau \tag{49}$$

3.4. Dimensionless Thermal Tension Calculus

The low conductivity of the ceramics cause thermal gradients in very short length from the surface that was submitted to temperature variation. Considering that the temperature shown in Illustration 1 is function only of the direction x, and the thermal tension in the dimensionless form, considering Timoshenko and Goodier (1970) is defined by:

$$\zeta^* = \Theta(\xi)_{med} - \Theta(\tau,\xi) \tag{50}$$

4. RESULTS AND DISCUSSION

The transformed problem was solved by a computational code written in programming language Fortran, using the software Fortran PowerStation 4.0 and implemented in a Pentium personal computer III–750Mhz.

The effect of the scattering upon the thermal tensions and the temperature distribution is analyzed for three different cases: isotropic scattering, forward linear and backward linear anisotropic scattering. The obtained results are shown respectively in Figures 2 and 3. On the analyses, it was considered the following parameters and dimensionless groups: τ_0=1.0, ω=0.5 and N=0.05. The boundary surfaces, transparent, are subjected to a low range of heat transference by convection proportional to Biot = 5 and external isotropic radiation of unit intensity ($F_0 = F_3 = 1$).

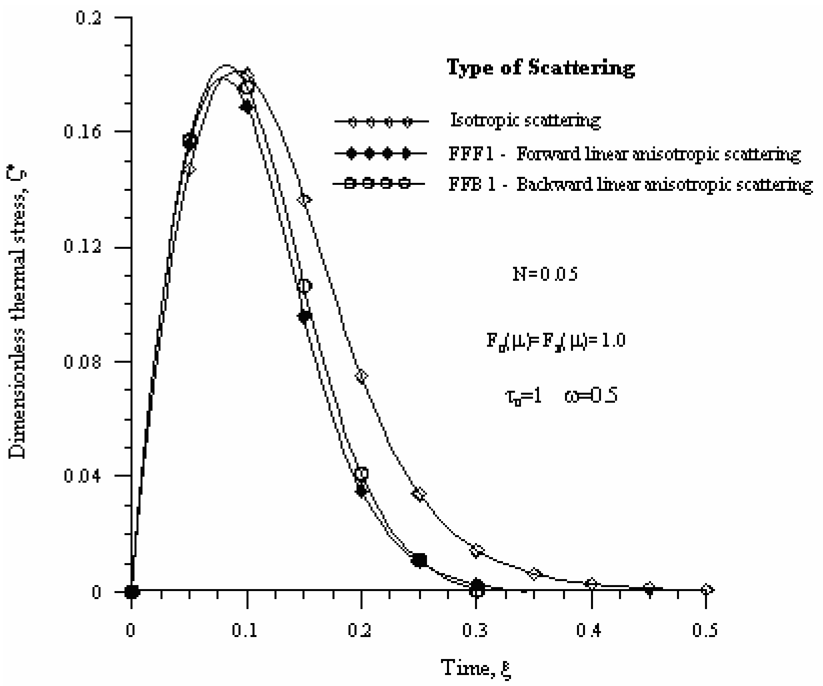

Figure 2- Effect due to the scattering of the thermal tensions in a ceramic body with constant thermal properties and N = 0.05, Biot number = 5 and external radiation $F_0(\mu) = F_3(\mu) = 1.0$.

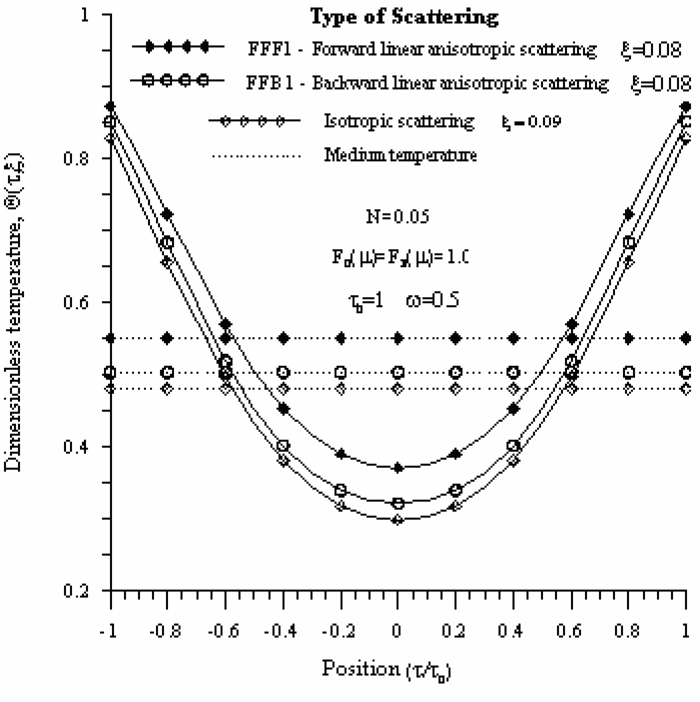

Figure 3: Effect of the scattering in the distribution of temperature in a ceramic body with constant thermal properties such as N = 0.05, Biot = 5 and external radiation sources $F_0(\mu) = F_3(\mu) = 1$.

In Fig. 2, it is possible to perceive that the scattering cause a strong influence on the thermal tension when external source of radiation act upon the body and it can be put aside if the sources do not exist, as shown in Fig. 4.

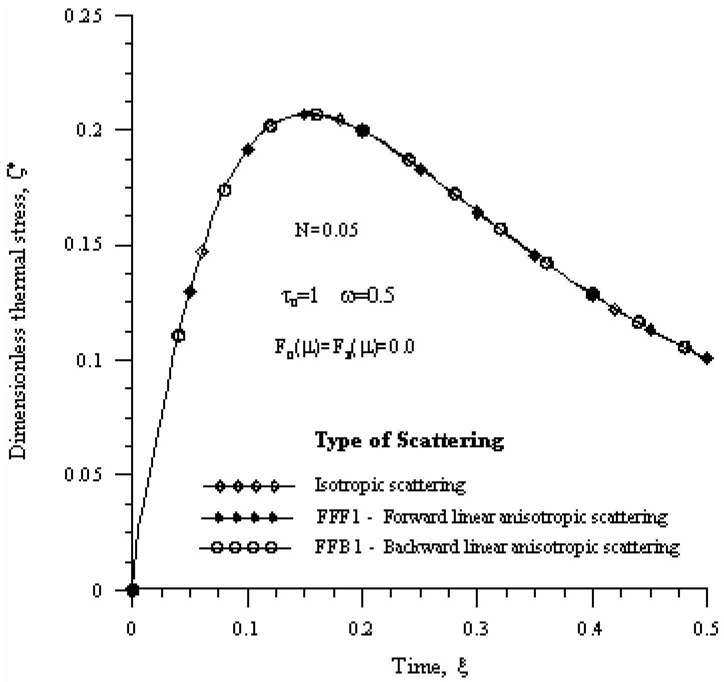

Figure 4: Effect of the scattering in the thermal tensions of a ceramic body with constant thermal properties such as N = 0.05, Biot = 5 and external radiation sources $F_0(\mu) = F_3(\mu) = 0$.

The Figures 3 and 5 show the effect of the scattering on the temperature distribution in a body with maximum thermal tensions. It is possible to perceive, on the presence of external radiation sources, the local temperature, on the forward linear scattering, is bigger than when that occurs linearly backwards. That happens because the diffusion in the former case happens on the heat flow direction.

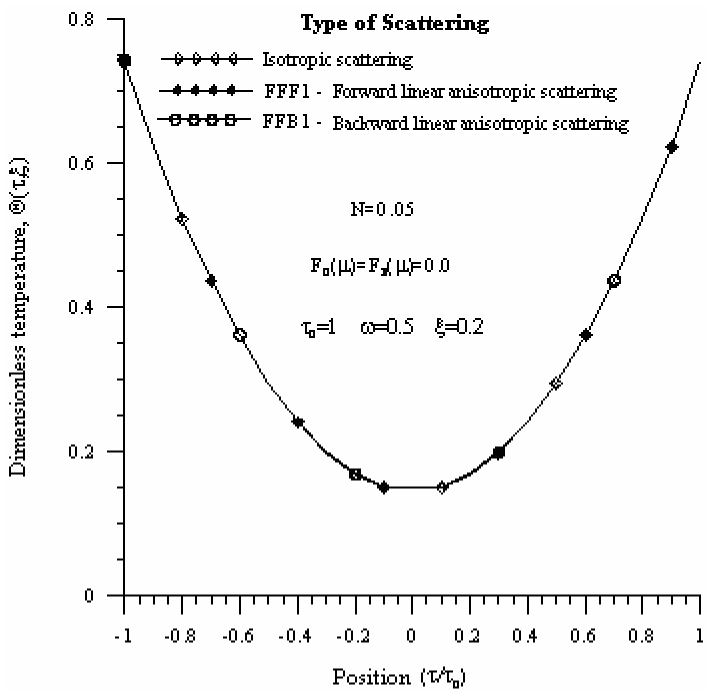

Figure 5: Effect of the scattering in the temperature distribution on a ceramic body with Constant thermal properties such as N = 0.05, Biot = 5 and external radiation sources $F_0 (\mu) = F_3 (\mu) = 0$.

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