DETERMINATION OF THE STRESS JUMP COEFFICIENT FOR TURBULENT FLOW OVER A POROUS BED

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Abstract. *This work analyses a turbulent flow at the vicinity of an interface between a porous and a clear region in order to fit the shear stress jump boundary condition which, in turn, are used in the macroscopic approach (one-phase). The porous media is composed of rods of different shapes, such as: cylindrical, square and longitudinal, elliptic shapes. After obtaining the results, here called "distributed results", an integration process considering the volume average concepts is made to transfer the results from the microscopic domain (two-phases) to a macroscopic domain (one-phase) where the results are them called integrated results. From the fitted curves for the punctual values of velocity, the shear stress jump, β, coefficient was estimated.*

Keywords*: clear fluid, numerical method, porous medium, stress jump coefficient, turbulent flow.*

1. INTRODUCTION

The first work to consider the interface boundary condition was presented by Beavers and Joseph (1967) that detected through experiments the non null axial velocity at the interface; Neale and Nader (1974) presented the continuity velocity condition and the gradient velocity condition at the interface, through the introduction of the Brinkman term in the momentum equation in the porous side; Vafai and Thiyagaraja (1987) studied analytically the flow and heat transfer for three types of interfaces; interface between two different types of porous medium, interface separating a porous medium of a clear region and interface between a porous medium and an impermeable medium.

Ochoa-Tapia and Whitaker (1995a-b) presented a hybrid interface condition in which the shear stress jump at the interface region is assumed. In this study, the shear stress jump is inversely proportional to the permeability of the porous medium. Kuznetsov (1996-98) solved analytically the laminar flow in the channel partially filled with porous medium, taking in account the shear stress jump condition at the interface between porous medium and clear fluid (see Ochoa-Tapia and Whitaker (1995a)). Ochoa-Tapia and Whitaker (1998) proposed the shear stress jump condition where the inertia effects are important. The authors presented too the hybrid interface condition for heat transfer, in which to introduce the jump condition that takes in account a possible flux excess at the interface. Kuwahara et al. (1998) resolved the internal fluid flow in the infinite porous medium formed by square rods with a periodic arrangement, using the low Reynolds turbulence model. The author showed that for $Re_H>10^4$ the fluid flow is turbulent and in this condition the Darcy-Forchheimer model presents good results.

Goyeau et al. (2002) and Goyeau et al. (2003), studied the momentum transport at the interface between the clear fluid and the porous medium, comparing the one model equations set with the two model equations set (two domains), that is, using distinct equations for describe the fluid flow in the clear fluid of the fluid flow in the porous medium. Homogeneous and inhomogeneous porous regions were investigated using the shear stress jump condition proposed by Ochoa-Tapia and Whitaker (1995a-b). For the homogeneous porous layer, both the models well are fitted. However, for non-homogeneous porous medium, the model of the one model equations set showed better description of the variation of the momentum. It was due the non-homogeneous layer closed the interface.

The work of Valencia-López et al. (2003) considered an adjustable coefficient jump condition for mass transfer at the interface between a clear fluid and a porous medium.

Kuznetsov and Xiong (2003) investigated the fluid flow in a duct partially filled with porous material, considering laminar flow in the porous region (low permeability) and turbulent flow in the clear region. Kuznetsov and Becker (2004) constructed a mathematical model that taking in account the effect of the rough of the interface between a porous medium and a clear fluid. The authors observed that this approach results in the Nusselt values greater than those found for smooth interface. In the works of Kuznetsov and Xiong (2003) and Kuznetsov and Becker (2004) was used the jump condition developed by Ochoa-Tapia and Whitaker (1995a).

In the works of Breugem et al. (2004-2005) and Breugem and Boersma (2005) used Direct Numerical Simulation (DNS) to evaluate the closing of the drag term in the Navier-Stokes equations and two models of momentum transfer for laminar flow on a porous wall. The authors concluded that the model of Ochoa-Tapia and Whitaker (1995b) presented a good agreement with the Direct Numerical Simulation, however, the value for the parameter stress jump that was unknown would have to be specified in the begin, constituting a disadvantage. Moreover, the results shown that the calculation of the permeability also depends on the Darcy velocity gradient, implying in a change to develop new models for the variation of the permeability in the interface region.

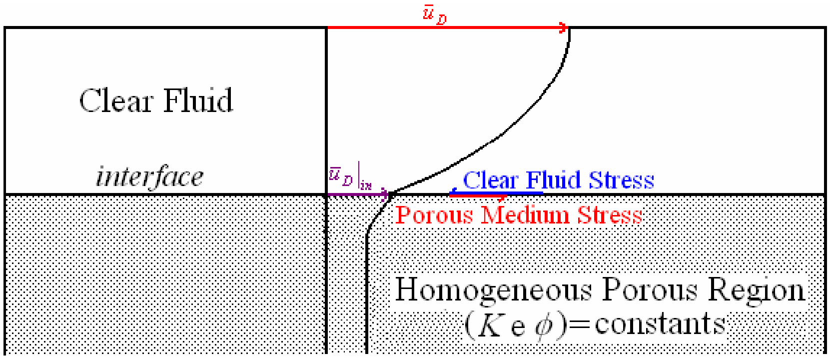
Pedras and De-Lemos (2000-2001a) applied the Double Decomposition Concept in the k-ε turbulence model developing the macroscopic equations of the turbulent flow for porous media. In the work presented by Pedras and De-Lemos (2001*b*), the authors applied the volumetric average in the microscopic equations for kinetic energy, k and its dissipation rate, ε. As consequence of this process of average some terms had been add in the equations of k and ε. These terms add had been adjusted for fluid flow in porous media formed by cylindrical rods with arrangement spatially periodic. Pedras and De-Lemos (2001c) and Pedras and De-Lemos (2003), adjusted the terms obtained in Pedras and De-Lemos (2001*b*) for porous media formed by longitudinal and transverse elliptical rods, respectively. The Double Decomposition Concept was extended for heat transfer in Rocamora and De-Lemos (2000), for natural convection in cavities with porous material in Braga and De-Lemos (2000, 2004-2005) and De-Lemos and Braga (2003), for the mass transport in De-Lemos and Mesquita (2003) and for the double diffusion in a porous structure in De-Lemos and Tofaneli (2004). Recently, Saito and De-Lemos (2005) obtained a heat transfer coefficient for closing of the macroscopic model of two equations of energy.

In the works presented in Rocamora and De-Lemos (2000) and Assato et al. (2005) a continuous function for stress field across the interface was used. Silva and De-Lemos (2003-b) and De-Lemos and Silva (2003) extended the methodology presented in Pedras and De-Lemos (2000-2001a-c) and Rocamora and De-Lemos (2000) to taking account the shear stress jump at interface between the clear fluid and porous media, using the interface condition proposed by Ochoa-Tapia and Whitaker (1995a), which makes use of an adjustable coefficient. Silva and De-Lemos (2003c) extended the interface boundary condition presented in Ochoa-Tapia and Whitaker (1995a) for turbulent fluid flow. In Silva and De-Lemos (2007) the authors developed a method for determination of the stress jump coefficient for laminar flow. In this work is proposed a methodology to determination of the shear stress jump coefficient for turbulent fluid flow.

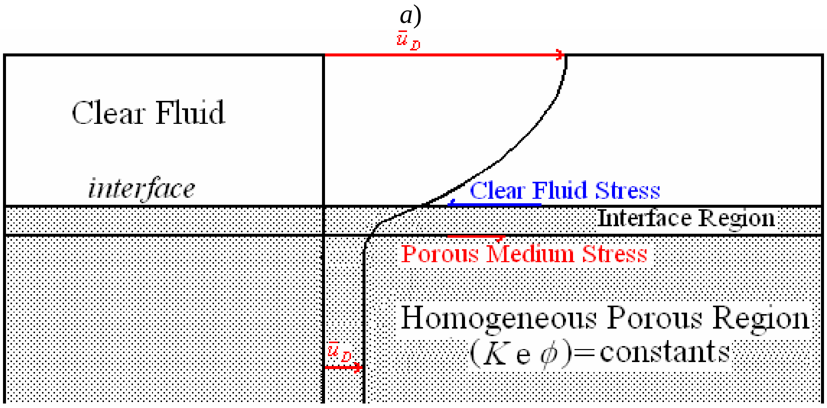
2. STRESS JUMP BOUNDARY CONTIDION

In regions with porous structures, as shown in the Figure 1*a*, the porous layer can be divided in two regions, a region where the properties of the porous medium (porosity, ϕ, and permeability, K) are constant and a region where these properties vary, called interface region, this region appears when the concept of the volumetric average is used [see Silva and De-Lemos (2006*a*-b)]. The application of the volumetric average concept in all calculation domain taking to continuum transition of the properties of the fluid flow of the porous layer for clear layer. However, due mathematical difficulties for treat with an only set of transport equations in domain contends porous structure and clear fluid, one uses distinct equations to treat the fluid flow in the porous medium and the clear fluid. The Figure 1*b* shown a channel partially filled with homogeneous porous material, without the interface region. The absence of this region results in the discontinuity of the values of diffusive flux (shear stress) above and below of the interface (Figure 1*b*). As the purpose to adjust this discontinuity; Silva and De-Lemos (2003c), considered a shear stress jump condition at the interface for turbulent flow, to the effect adjust the discontinuity caused for the absence of the interface region. However, this condition was constructed in such a way that presents a coefficient that needs to be determined, for the closing of the contour condition.

$$\underbrace{\left(\mu_{eff} + \mu_{t_\phi}\right)\frac{\partial \overline{u}_D}{\partial y}\bigg|_{\text{Porous Medium}}}_{\text{Stress at the Porous Medium}} - \underbrace{\left(\mu + \mu_t\right)\frac{\partial \overline{u}_D}{\partial y}\bigg|_{\text{Clear Fluid}}}_{\text{Stress at the Clear Fluid}} = \overbrace{\beta}^{\text{Ajustable Coefficient}} \underbrace{\frac{\left(\mu + \mu_t\big|_{in}\right)}{\sqrt{K}}\overline{u}_D\big|_{in}}_{\text{Jump Stress Term}} \quad (1)$$



a)



b)

Figure 1: Fluid flow in a channel partially filled with porous material, *a*) continuity in the diffusion flux, *b*) discontinuity in the diffusive flux.

3. STRESS JUMP COEFFICIENT

In the Figure 2*a* is shown the sketch of channel with porous material. The values of the velocities and its derivatives, in the porous medium and the clear fluid, are taking as presented. These values were obtained of the integrated results, characterizing continuity of the properties of the fluid flow across transversal section of the channel. The Figure 2*b* present the channel partially filled with porous material, in which the interface region is collapsed in a line, here called interface. This procedure generates a discontinuity or jump in the shear stress field at interface between the clear fluid and the porous medium.

The work of Silva and De-Lemos (2003c) had as objective to adjust the shear stress jump at interface between the clear fluid and the porous media through of equation:

$$\left(\mu_{eff} + \mu_{t_\phi}\right)\frac{\partial \bar{u}_D}{\partial y}\bigg|_{\text{Porous Medium}} - \left(\mu + \mu_t\right)\frac{\partial \bar{u}_D}{\partial y}\bigg|_{\text{Clear Fluid}} = \beta \frac{\left(\mu + \mu_t|_{in}\right)}{\sqrt{K}}\bar{u}_D\big|_{in} \quad (2)$$

where μ_{eff} is the effective viscosity, $\mu_{eff} = \mu/\phi$, μ_{t_ϕ} is the macroscopic turbulent viscosity, μ is the dynamic viscosity and $\mu_t|_{in}$ is the interfacial viscosity $\left[\mu_t|_{in} = \left(\mu_{t_\phi} + \mu_t\right)/2\right]$, K is the permeability, β is the stress jump coefficient and $\bar{u}_D|_{in}$ is the interfacial average velocity $\left[\bar{u}_D|_{in} = \left(\bar{u}_D|_{PM} + \bar{u}_D|_{CF}\right)/2\right]$.

For facilitate the manipulation of the jump condition presented in the equation (2), we rewrite this equation of the following form:

$$\left(\tau^*_{PM} - \tau^*_{CF}\right) = \beta \frac{\left(\mu + \mu_t|_{in}\right)}{\sqrt{K}}\bar{u}_D\big|_{in} \quad (3)$$

where;

$$\tau^*_{PM} = \left(\mu_{eff} + \mu_{t_\phi}\right)\frac{\partial \bar{u}_D}{\partial y}\bigg|_{\text{Porous Medium}} , \tag{4}$$

$$\tau^*_{CF} = \left(\mu + \mu_t\right)\frac{\partial \bar{u}_D}{\partial y}\bigg|_{\text{Clear Fluid}} \tag{5}$$

where, τ^*, is the modified shear stress. (laminar shear stress + turbulent shear stress).
Approaching the expressions (4) and (5) for finite differences:

$$\tau_{PM} \approx \left(\frac{\mu + \phi\mu_{t_\phi}}{\phi}\right)\frac{\Delta \bar{u}_D}{\Delta y}\bigg|_{\text{Porous Medium}} \tag{6}$$

$$\tau_{CF} \approx \left(\mu + \mu_t\right)\frac{\Delta \bar{u}_D}{\Delta y}\bigg|_{\text{Clear Fluid}} \tag{7}$$

where,

$$\mu_{t_\phi} = \rho c_\mu f_\mu \frac{\left[\langle k\rangle^i\right]^2}{\langle \varepsilon\rangle^i} \tag{8}$$

where $c_\mu = 0.09$ is an empirical constant of the turbulence model, $f_\mu = exp\left[-3.4/\left(1 + Re_t/50\right)^2\right]$ is a damping function and $Re_t = \rho\left[\langle k\rangle^v\right]^2 / \mu\langle\varepsilon\rangle^v$ is the turbulent Reynolds number.

Dimensionlezing the equation (3) for the average kinetic energy of the fluid flow, $\rho\bar{U}^2_D$, where, $\bar{U}_D$, is the average Darcy velocity in the transversal section of the channel:

$$\frac{\left(\tau^*_{PM} - \tau^*_{CF}\right)}{\rho\bar{U}^2_D} = \frac{\left(\mu + \mu_t|_{in}\right)}{\rho\bar{U}^2_D}\frac{\beta}{\sqrt{K}}\bar{u}_D\big|_{in} \tag{9}$$

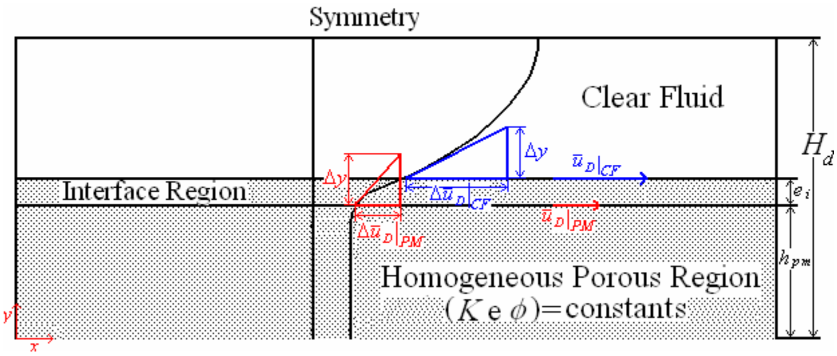
Rearranging the equation (9), obtained:

$$\left|\frac{\left(\tau^*_{PM} - \tau^*_{CF}\right)}{\rho\bar{U}^2_D}\right| = \beta\left|\frac{\left(\mu + \mu_t|_{in}\right)}{\rho\bar{U}_D\sqrt{K}}\frac{\bar{u}_D|_{in}}{\bar{U}_D}\right| = \beta\left|\frac{\bar{u}_D|_{in}}{Re^*_K\,\bar{U}_D}\right| \tag{10}$$

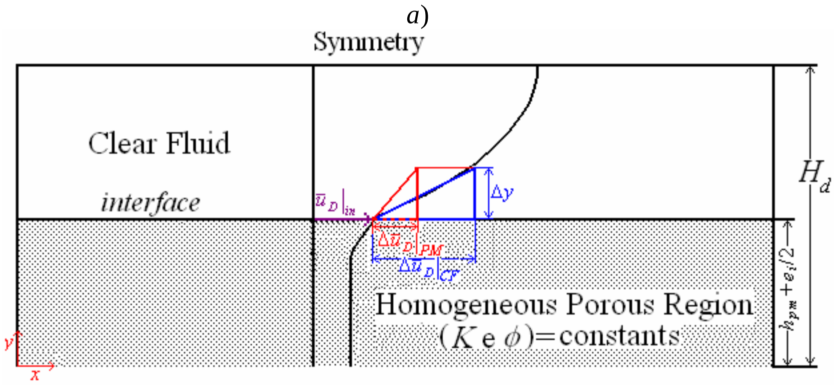
where

$$Re^*_K = \frac{\rho\bar{U}_D\sqrt{K}}{\left(\mu + \mu_t|_{in}\right)} \tag{11}$$

Substituting the numerical values of the expressions (6), (7), in the equation (10), together with the values of permeability, K, and porosity, ϕ, presented in Tables 1 and 2, is possible to estimate the value of the shear stress jump coefficient, β.



a)



b)

Figure 2: Fluid flow in a channel partially occupied by a porous material: a) with interface region, b) without interface region.

4. RESULTS AND DISCUSSION

The Tables 1 and 2 present the values of porosity, ϕ, permeability, K, obtained using the calculation cell (Silva and De-Lemos (2006*a*-b), the numbers of Darcy, $Da = K \big/ H_d^2$, and Reynolds, Re_{H_d} , based in the channel height, H_d, the modified Reynolds number, $Re_K^* = \left(\rho \bar{U}_D \sqrt{K}\right) \big/ \left(\mu + \mu_t|_{in}\right)$, based in the characteristic length of the porous media, $\sqrt{K}$, the modified stress values at the porous medium, τ_{PM}^* , at the clear fluid, τ_{CF}^* , and the interfacial Darcy velocity, $\bar{u}_D|_{in}$, obtained of the integrated results, as presented in the Figure 2*b*. Observed an increase in the interfacial velocity, $\bar{u}_D|_{in}$, and in the modified stress at the porous media, τ_{PM}^* , with the increase of the modified Reynolds number based on $\sqrt{K}$.

In the Figure 3 the angular coefficient value, β=0.6535, present in the equation (10), was obtained through the fitted of curve for the minimum squared method in the data showed in Tables 1 e 2. Notice that the R^2=0.97 parameter indicates a good correlation of the data with the fitted curve. Moreover, it observes that to occur an increase of the difference of stress with the increase of the jump stress term, $\left|\dfrac{\bar{u}_D|_{in}}{Re_K^* \, \bar{U}_D}\right|$, implying in an increase of the momentum exchange between the fluid layers above and below at interface. On the other hand, the increase of the porosity, ϕ, it takes the reduction of the stress difference, causing a reduction of the momentum exchange at the interface. This behavior was expected, a time that, the increase of the value of the porosity, it makes with that the porous medium tend to clear fluid ($\phi \rightarrow 1$), doing with that the difference between the value of the stress at the porous medium and at the clear fluid tend to zero, that is, $\tau_{PM}^* \rightarrow \tau_{CF}^*$.

Table 1: Fluid flow characteristics at interface region, for ϕ=0.7382.

ϕ	H_d [m]	$K \times 10^{-6}$ [m^2]	$Da \times 10^{-4}$	$Re_{H_d} \times 10^4$	Re_K^*	$\tau_{PM}^* \times 10^{-1}$ [N/m^2]	τ_{CF}^* [N/m^2]	$\mu_t\|_{in} \times 10^{-2}$ [Ns/m^2]	$\bar{u}_D\|_{in} \times 10^{-1}$ [m/s]
0.7382	0.075	6.3902	11.360	1.2013	20.285	4.5482	1.8770	1.9018	2.2547
		6.5508	11.646	0.78387	8.6538	4.1006	1.4301	3.0001	1.3127
		10.000	17.778	0.95979	12.115	5.3295	1.8268	3.2500	1.6836
	0.095	6.3902	7.0806	2.6400	23.686	8.9635	4.4502	2.8744	3.0541
		6.5508	7.2585	1.6340	8.8534	7.5144	3.3860	4.8871	1.6194
		10.000	11.080	1.9341	11.740	10.678	4.0391	5.3999	2.0976
	0.115	6.3902	4.8319	4.5675	28.117	13.056	6.9897	3.4813	3.7325
		6.5508	4.9533	2.6940	8.9939	11.080	5.4737	6.5862	1.8158
		10.000	7.5614	3.2223	12.398	16.292	6.4898	7.0682	2.4423

Table 2: Fluid flow characteristics at interface region, for ϕ=0.8429

ϕ	H_d [m]	$K \times 10^{-6}$ [m^2]	$Da \times 10^{-4}$	$Re_{H_d} \times 10^4$	Re_K^*	$\tau_{PM}^* \times 10^{-1}$ [N/m^2]	τ_{CF}^* [N/m^2]	$\mu_t\|_{in} \times 10^{-2}$ [Ns/m^2]	$\bar{u}_D\|_{in} \times 10^{-1}$ [m/s]
0.8429	0.075	4.9293	8.7632	0.49425	3.8826	2.4747	1.2885	3.6794	0.88166
		8.7431	15.543	0.77509	9.7730	4.1704	1.0971	3.0359	1.4434
		9.6097	17.084	0.60463	5.9414	3.9740	1.1190	4.1186	1.0804
		13.624	24.220	0.77438	8.8423	5.1865	1.3552	4.2227	1.3862
	0.095	4.9293	5.4618	1.1609	4.4994	4.7205	2.3436	5.9478	1.1092
		8.7431	9.6876	1.7812	12.222	8.2182	2.6590	4.4494	1.9477
		9.6097	10.648	1.3298	6.6925	7.5086	2.7175	6.4028	1.3512
		13.624	15.096	1.5980	9.1873	10.068	3.0628	6.6782	1.7262
	0.115	4.9293	3.7273	2.0816	5.1693	6.9104	3.9531	7.6972	1.2971
		8.7431	6.6110	3.1759	14.906	15.831	4.3724	5.3943	2.3929
		9.6097	7.2663	2.3145	7.3975	11.551	4.5098	8.3590	1.5742
		13.624	10.302	2.7030	9.9676	15.300	4.9704	8.6296	1.9006

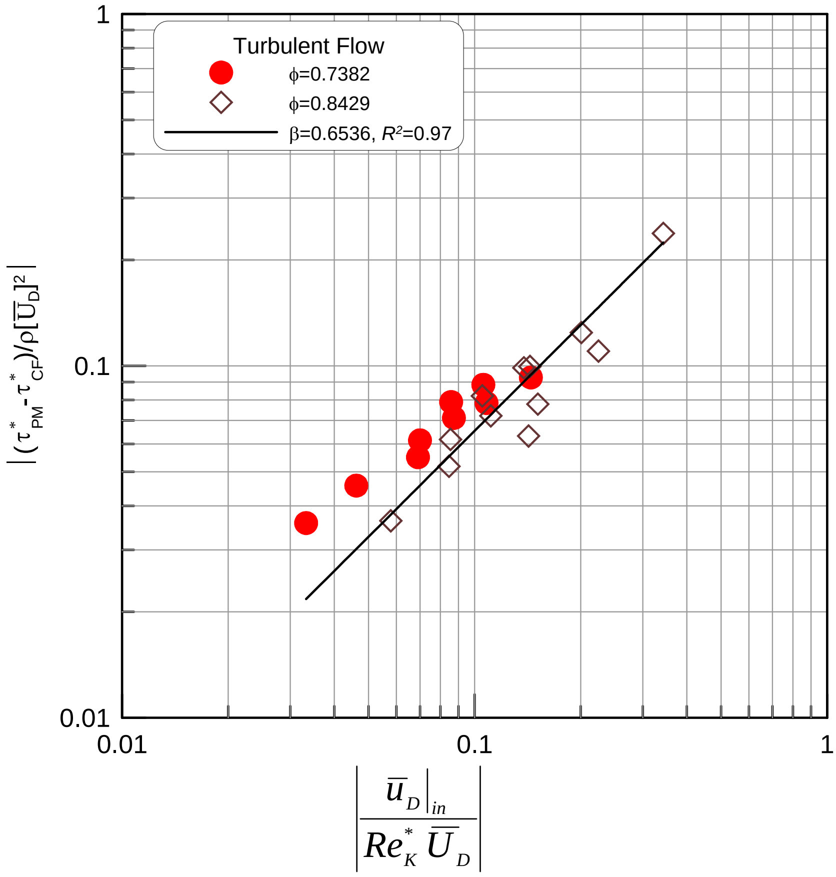


Figure 3: Determination of the stress jump coefficient, β.

5. CONCLUSIONS

This work presents a methodology for determination of the stress jump coefficient, β, for turbulent flow, that accommodates the stress jump caused by absence of the interface region. The values of the properties of the fluid flow were obtained to determine the jump coefficient. The momentum exchange between the fluid layers above and below at the interface, caused for the stress jump is proportional to the stress jump term, $\left|\frac{\bar{u}_D|_{in}}{Re_K^* \bar{U}_D}\right|$, in turn, is function of the Re_K^*, and of the interfacial velocity, $\bar{u}_D|_{in}$, that depends of the Re_K^*. Presently, can be inferred that the stress difference is strong function of the Re_K^*. On the other side, as expected, the increase of the value of the porosity, it makes with that the porous medium tend to clear fluid ($\phi \rightarrow 1$), making with that the difference between the stress values and consequently the momentum exchange, tend to zero, that is, $\tau_{PM}^* \rightarrow \tau_{CF}^*$.

The present methodology show that the value of the jump coefficient, β, that better accommodates the momentum exchange at the interface, for turbulent flow, is β=0.6535.

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