NUMERICAL SIMULATION OF AN ULTRASONIC FLOWMETER FOR MEASUREMENT OF TWO-PHASE GAS-LIQUID STRATIFIED FLOW

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Abstract. This paper presents a numerical model of an ultrasonic flowmeter suitable to measure the liquid flow rate in two-phase gas-liquid stratified flows in horizontal pipes. The technique involves the use of two ultrasonic transducers placed externally at the same generatrix located at the bottom of the pipe. An ultrasonic beam emitted from the first transducer travels through the liquid phase and is reflected by the gas-liquid interface towards the second transducer. Then the procedure is reversed, and the second transducer emitts an ultrasonic beam that is received by the first transducer, after reflection at the interface. The mean velocity of the flow along the ultrasonic path is obtained by measuring the time lapsed during the process. The relationship that exists between the mean flow velocity in the liquid phase cross section and the measured mean velocity along the ultrasonic path is called as the hydraulic correction factor. Thus, knowing the hydraulic correction factor and the measured mean velocity along the ultrasonic path, the flow rate of the liquid phase can be accurately determined. In this work, the hydraulic correction factor has been obtained numerically by using a CFD code based on the Reynolds averaged Navier-Stokes equations (RANS) with the k- ω closure model. For two-phase stratified flow, numerical results for the hydraulic correction factor are presented as a function of the liquid phase Reynolds number and the gas-liquid interface position. In order to validate the procedure proposed herein, our numerical predictions are compared with well known correlations for the hydraulic correction factor in single-phase flow available in the literature.

Keywords hydraulic factor, ultrasonic flowmeter, two-phase flow, stratified flow, numerical model, finite element

1. INTRODUCTION

Nowadays ultrasonic flowmeters are widely used in industry to measure the single-phase flow of gases and liquids. Recent advances in acoustics and electronics have prompted the design and construction of ultrasonic flowmeters with high degree of accuracy. However, as industry moves forward, more requirements for flowmeters capable of measuring dynamic flows with stringent accuracy are expected.

In nuclear power plants, flowmeters must be capable of accurately measuring the flow rate of cooling fluid moving through piping systems. Primary flowmeters, such as orifice plates and venturis, are two types of flowmeters usually employed to measure the cooling flow relied upon for controlling plant thermal power output. However, the actual inplant experience has shown that cooling flow measurement is the least reliable parameter due to difficulties in flowmeter maintenance. Typically, in a nuclear power plant safety analysis, the requirement for uncertainty of the cooling flow measurement is less than 2%, which is in general degraded in the orifice and venturi instrumentation, increasing their maintenance costs. Nowadays ultrasonic flowmeters permit improving nuclear power plant monitoring through accurate and reliable cooling flow measurement. The main advantages of ultrasonic flowmeters in comparison with primary flowmeters are their accuracy, stability, repeatability, time response and resolution. They are based on a non-interfering flow measurement technique, having low costs of operation and maintenance. Their applications at nuclear power systems have been demonstrated recently by various nuclear industries such as ABB, AECL and EDF, as reported by French et al. (2000) and Harvel and Chang (2004).

In recent years special attention has been given to the use of numerical modeling as a means to achieving the development of ultrasonic flowmeters capable of satisfying the demands for minimization of measurement error. The application of refined mathematical models and numerical methods along the flowmeter design process can help meeting these requirements. Holm (1995) demonstrated that a hydraulic factor may be deduced for distorted flows for ultrasonic flowmeters using CFD. The hydraulic factor was calculated as the ratio of the mean flow velocity projected onto the sound beam path and the measured mean velocity along the path. Holm (1995) founded a $\pm 1\%$ agreement between the CFD and the hydraulic factor determined experimentally over Reynolds number ranges of 2×10^3 to 1.2×10^4 . Hilgenstock et al. (1996) used a commercial CFD program to simulate numerically the single-phase ultrasonic flowmeters under non-developed gas or liquid flow in pipe bends, using the standard k- ε turbulence model when the flow was considered to be turbulent. By calculating a hydraulic factor for both the fully developed and developing flow,

the authors were able to determine numerically the error in the flow rate measured by the ultrasonic flowmeter. Gol'tsov (1998) has proposed a one-dimensional mathematical model of a single-phase ultrasonic flowmeter suitable to measure the fully developed turbulent gas or liquid flow in circular pipes. The gas or liquid flow was divided in three zones, each one of them having a velocity profile distribution represented by an analytical-empirical expression containing experimental coefficients, which were integrated over the entire flow area and over the diameter of the pipe. A hydraulic factor was determined from the relation between the results of the integration. The model was tested against a standard ultrasonic flowmeter and a maximum error of $\pm 0.3\%$ was established by the authors between the hydraulic factors determined respectively by the model and the experiments. Letton (2003) has proposed a commercial patent of an ultrasonic flowmeter scheme to measure accurately a two-phase gas-liquid stratified flow, using three pairs of ultrasonic transducers working simultaneously. Letton's ultrasonic flowmeter is designed to determining the composition and velocities for both phases of the stratified flow. Henry et al. (2006) have developed a methodology to be applied for commercial single-phase Coriolis flowmeters in order to correct the measurements under a two-phase flow conditions.

In this work we present a numerical methodology to determine the hydraulic correction factor for a two-phase stratified ultrasonic flowmeter. The ultrasonic flowmeter is the transit-time type where two ultrasonic transducers are placed outside a horizontal pipe wall, alternately transmitting and receiving an ultrasonic beam that is approximated by a straight line. Based on this configuration an expression for the hydraulic factor is deduced as a relation between the mean velocity of the liquid along the ultrasonic path and the mean velocity of the liquid on the pipe cross-section occupied by the liquid. The two-phase stratified flow is modeled by solving the Reynolds averaged Navier-Stokes equations with the k- ω turbulence model and considering the stratified flow as a fully developed gas-liquid flow with a smooth horizontal interface without interfacial waves. The model solution is achieved by recasting the mathematical equations in a variational formulation which are solved by the Newton-Raphson root-finding scheme and the finite element method giving the velocity profile distribution for both phases. Next a numerical integration of the liquid velocity profile is performed along the ultrasonic path and through the pipe liquid cross-section resulting in, respectively, the ultrasonic path mean liquid velocity and the area mean liquid velocity. Then the two-phase hydraulic factor is determined as a relation between these two mean velocities. The numerical solution is tested in single-phase flow conditions over a wide range of Reynolds number. For the two-phase stratified flow, numerical results are presented for the hydraulic correction factor as a function of the Reynolds number for the liquid phase and the gasliquid interface position.

2. DESCRIPTION OF THE ULTRASONIC TECHNIQUE

When an ultrasonic beam propagates in a moving liquid it is convected in the flow direction and retarded in the counter-flow direction. Fig. (1) shows a gas-liquid stratified flow inside a horizontal pipe where a pair of ultrasonic transducers, T_1 and T_2 , were placed outside pipe wall on the liquid side. Each transducer alternatively sends and receives an ultrasonic beam traveling through the liquid flow, which is reflected by the gas-liquid interface. The difference in the transit-time between the pair of the transducers can be measured and is used to calculate the mean velocity of liquid along the ultrasonic path s. Provided that one knows the relationship between the mean liquid velocity along the path and the mean velocity in the liquid cross section, the technique can be used to determine the liquid flowrate (Lynnworth, 1979).

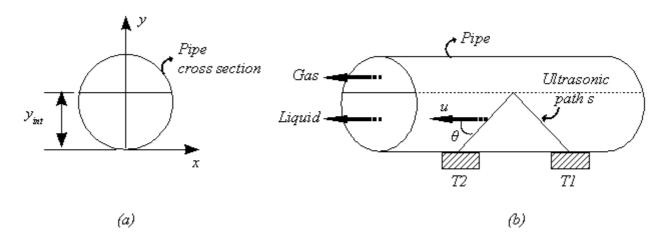


Figure 1. Schematic of a two-phase ultrasonic flowmeter

If the ultrasonic beam is considered as a ray at a fixed angle θ across the liquid flow of velocity profile u, in a short time dt it travels a distance ds from transducer T1 to transducer T2 and

$$\frac{ds}{dt} = c + u\cos\theta \tag{1}$$

where c is the stationary sound speed in the liquid and $ucos\theta$ is the component of liquid velocity in the s direction, as shown in Fig. (1b). Splitting the path s into sI and s2 according to the transducer which is in contact with the pipe, namely sI is from TI to the interface and s2 is from the interface to T2, it follows that

$$dt = \frac{dy}{(c + u\cos\theta)\operatorname{sen}\theta} \tag{2}$$

for the path sI since the s direction is related to the co-ordinate y by $\frac{dy}{ds} = sen\theta$. For the path s2 the time interval consists of the

$$dt = \frac{-dy}{(c + u\cos\theta)\sin\theta} \tag{3}$$

since $\frac{dy}{ds} = -sen\theta$ in that case. Integrating Eqs. (2) and (3) along the paths s1 and s2 we obtain the following expressions for the transit times:

$$\int_{t_{i}}^{t_{i}^{2}} dt = \int_{0}^{y_{int}} \frac{dy}{(c + u\cos\theta)\sin\theta}$$

$$\tag{4}$$

$$\int_{t2}^{t3} dt = \int_{y_{int}}^{0} \frac{-dy}{(c + u\cos\theta)\sin\theta}$$
 (5)

where t2 - t1 is the transit time along s1, t3 - t2 is the transit time along s2 and y_{int} is the interface position. Thus, the total transit time Δt_{I-3} from the T1 to the T2 transducer is

$$\int_{t_{l}}^{t_{2}} dt + \int_{t_{2}}^{t_{3}} dt = \Delta t_{l-3} = 2 \int_{0}^{y_{int}} \frac{dy}{(c + u\cos\theta)sen\theta}$$
 (6)

Next, the ultrasonic beam travels back from the T2 to the T1 transducer, being retarded by the liquid flow. The transit time for the backward ultrasonic emission is obtained with a similar reasoning. However, instead of the Eq. (1) we have that

$$\frac{ds}{dt} = c - u\cos\theta \tag{7}$$

and the transit time for the backward ultrasonic emission is Δt_{3-5} , given by

$$\int_{t_3}^{t_4} dt + \int_{t_4}^{t_5} dt = \Delta t_{3-5} = 2 \int_{0}^{y_{int}} \frac{dy}{(c - u\cos\theta)sen\theta}$$
 (8)

Note that t4 - t3 is the transit time along s2 and t5 - t4 is the transit time along s1. The next step is to subtract Eq. (6) from Eq. (8):

$$\Delta t_{3-5} - \Delta t_{I-3} = \frac{2}{\operatorname{sen}\theta} \int_{0}^{y_{int}} \left[\frac{1}{(c - u\cos\theta)} - \frac{1}{(c + u\cos\theta)} \right] dy \tag{9}$$

In most practical applications we have c >> u, which implies that $\Delta t_{I-3} \cong \Delta t_{3-5}$. For the geometrical configuration shown in Fig. (1) it follows that c is closely approximated by

$$c = \frac{2y_{int}}{sen\,\theta\Delta t_m} \tag{10}$$

where $\Delta t_m = \frac{\Delta t_{l-3} + \Delta t_{3-5}}{2}$. Now, with the condition c >> u and introducing Eq. (10), Eq. (9) reduces to

$$\frac{\Delta t_{3-5} - \Delta t_{l-3}}{\Delta t_m^2} \frac{y_{int}}{sen\,\theta\cos\theta} = u_{line} \tag{11}$$

where u_{line} is the mean liquid velocity along the y co-ordinate,

$$u_{line} = \frac{1}{y_{int}} \int_{0}^{y_{int}} u dy \tag{12}$$

Thus, it can be seen that the ultrasonic flowmeter performs a measurement along the ultrasonic path s that is related to the co-ordinate y through the angle θ . It can be seen also that the sound speed in the liquid does not play any role in Eq. (11) which makes the method insensitive to sound speed variations with liquid pressure and temperature.

On the other hand, in order to measure the liquid flow rate, it is necessary to know the mean velocity over the area occupied by the liquid, namely

$$u_{area} = \frac{1}{\Omega_L} \int_{\Omega_L} u d\Omega \tag{13}$$

where Ω_L is the fraction of the pipe cross section occupied by the liquid. The conversion of the velocity u_{line} into the velocity u_{area} is obtained by a correction factor called the hydraulic factor, commonly represented by K_h and defined as

$$K_h = \frac{u_{line}}{u_{area}} \tag{14}$$

Finally a full volume liquid flow expression can be obtained as follows

$$Q_L = \Omega_L \frac{u_{line}}{K_h} \tag{15}$$

with Q_L is the volumetric liquid flow. As the liquid velocity profiles u_{line} and u_{area} , the hydraulic factor depends on the laminar or turbulent regime of the flow.

2.1 Single-phase K_h expressions

In a particular situation where the pipe in Fig. (1) is full of gas or liquid, a single-phase flow occurs and a constant value of $K_h = 1.33$ is easy to determine analytically for a laminar flow with parabolic velocity profile. For fully developed turbulent flow occurring in smooth pipes, an analytical expression can be derived for K_h from the universal velocity distribution law (Lynnworth, 1979; Schlichting, 1979), as

$$K_b = 1.119 - 0.011(logRe)$$
 (16)

where Re is the Reynolds number. Commercially available single-phase ultrasonic flowmeters use empirical correlations for K_h , similar to Eq. (16), which can be applied to rough pipes provided the flow is fully developed. There are two correlations of this type (Gol'tsov, 1998), as follows:

$$K_{h} = \left\{ 0.889 + 0.0091 log \left[Re + 0.0001 (log Re)^{2} \right] \right\}^{-1}$$
(17)

$$K_b = 1.125 - 0.0115 log(0.94 Re)$$
 (18)

The most modern ultrasonic flowmeters design has made use of computational fluid dynamics (CFD) to overcome the problem of determining the hydraulic factor for fully developed and developing turbulent flow. Depending on the required accuracy, different K_h 's should be used as u, Re, or profile varies (Lynnworth, 1979).

Next sections will be devoted to explain how to find the mean liquid velocities u_{line} and u_{area} numerically in order to calculate a two-phase hydraulic factor on the basis of Eqs. (1) – (14).

3. GAS-LIQUID STRATIFIED ULTRASONIC FLOWMETER MODEL

3.1 Two-phase stratified model

In this section the two-phase stratified flow is modeled as it has been proposed by De Sampaio *et al.* (2006). We consider here a fully developed gas-liquid two-phase stratified flow in a horizontal pipe with the interface between the phases as a flat plane. The Reynolds averaged Navier-Stokes equations with the k- ω turbulence model describe the flow in both phases:

$$\nabla \cdot \left(A_i \, \nabla u \right) - \frac{d \, p}{d \, z} = 0 \tag{19}$$

$$\nabla \cdot (B_i \nabla \kappa) - \beta_2 \rho_i \kappa \omega + S_i = 0 \tag{20}$$

$$\nabla \cdot (C_i \nabla \omega) - \beta_1 \rho_i \omega^2 + \frac{\alpha_1 \omega}{\kappa} S_i = 0 \tag{21}$$

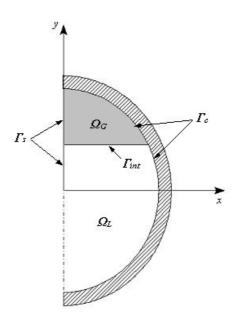


Figure 2. Schematic representation of the domains for the two-phase stratified model

The above equations are defined within each domain showed in the Fig. 2. The terms in Eqs. (19) – (21) are $A_i = \mu_i + \mu_{ti}$, $B_i = \mu_i + \sigma_2 \mu_{ti}$, $C_i = \mu_i + \sigma_1 \mu_{ti}$, $S_i = A_i \nabla u \cdot \nabla u$, and $\mu_{ti} = \alpha_2 \rho_i \kappa / \omega$ where k is the kinetic energy, ω is the energy dissipation, μ_{ti} is the eddy viscosity and α_1 , α_2 , β_1 , β_2 , σ_1 , σ_2 are the k- ω model parameters. dp/dz is the pressure loss along the co-ordinate z (perpendicular to the paper sheet), and u is the flow velocity. The subscripts l and l define, respectively, the liquid and gas phases. The boundary and interfacial conditions are defined on the

symmetry boundary Γ_s where $\nabla u \cdot \mathbf{n} = 0$, $\nabla \kappa \cdot \mathbf{n} = 0$ and $\nabla \omega \cdot \mathbf{n} = 0$. On the pipe boundary Γ_c u = 0, $\kappa = 0$ and $\omega = \overline{\omega}_{ci}$ with $\overline{\omega}_{ci} = \frac{2 \mu_i}{\beta_0 \ \rho_i \ Y_p^2}$ as implemented by Segal (2006) where $\beta_0 = 0.072$ is a model constant and Y_p is the

distance of the closest grid point to the pipe wall. At the interface Γ_{int} the conditions were set up by $\sum_{i=1,2} A_i \nabla u \cdot \mathbf{n}_i = 0$,

 $\kappa = 0$ and $\omega = 10^6 u_0/d$, where d is the inner pipe diameter, $u_0 = Q_L / \left(\frac{\pi d^2}{4}\right)$ and Q_L is the liquid flow rate. More detailed information about the model can be found in De Sampaio *et al.* (2006).

Recasting the problem described above in a variational form as

$$\sum_{i=I,2} \int_{\Omega_i} A_i \nabla \phi \cdot \nabla u \ d\Omega_i = -\sum_{i=I,2} \int_{\Omega_i} \phi \frac{dp}{dz} \ d\Omega_i$$
 (22)

$$\sum_{i=1,2} \int_{\Omega_i} \varphi \, \beta_2 \, \rho_i \, \omega \kappa \, d\Omega_i + \sum_{i=1,2} \int_{\Omega_i} B_i \, \nabla \varphi \cdot \nabla \kappa \, d\Omega_i = \sum_{i=1,2} \int_{\Omega_i} \varphi \, A_i \, \nabla u \cdot \nabla u \, d\Omega_i$$
(23)

$$\sum_{i=1,2} \int_{\Omega_i} \varphi \, \beta_i \, \rho_i \, \omega^2 \, d\Omega_i + \sum_{i=1,2} \int_{\Omega_i} C_i \, \nabla \varphi \cdot \nabla \omega \, d\Omega_i = \sum_{i=1,2} \int_{\Omega_i} \varphi \, \alpha_i \frac{\omega}{\kappa} A_i \, \nabla u \cdot \nabla u \, d\Omega_i$$
(24)

$$\begin{split} & \text{finding} & u \in V_u = \left\{u \in H_I(\Omega), u = 0 \text{ on } \Gamma_c\right\}, & \kappa \in V_\kappa = \left\{\kappa \in H_I(\Omega_L \cup \Omega_G), \kappa = 0 \text{ on } \Gamma_c, \kappa = 0 \text{ on } \Gamma_{\text{int}}\right\}, \\ & \omega \in V_\omega = \left\{\omega \in H_I(\Omega_L \cup \Omega_G), \omega = \overline{\omega}_{\text{ci}} \text{ on } \Gamma_{\text{ci}}, \ \omega = \overline{\omega}_{\text{int}} \text{ on } \Gamma_{\text{int}}\right\} & \text{for any } \phi \in V_\phi = \left\{\phi \in H_I(\Omega), \phi = 0 \text{ on } \Gamma_c\right\} & \text{and} \\ & \varphi \in V_\varphi = \left\{\varphi \in H_I(\Omega_L \cup \Omega_G), \varphi = 0 \text{ on } \Gamma_c, \varphi = 0 \text{ on } \Gamma_{\text{int}}\right\} & \text{and imposing the conditions } Q_G = 2\int_{\Omega_G} u \, d\Omega & \text{and} \\ & Q_L = 2\int_{\Omega_G} u \, d\Omega & \text{, the problem is closed.} \end{split}$$

The solutions for the velocity profile, kinetic energy and energy dissipation are obtained in both phases, by using an iterative process combining two numerical techniques. The first is an external Newton-Raphson method aimed to adjust y_{int} and dp/dz, in order to satisfy the imposed conditions for Q_G and Q_L . The second runs internally and involves the finite element solution of the non-linear problem given by Eqs. (22) - (24), for given values of y_{int} and dp/dz. In Fig. (3) it is shown typical results of velocity profile and kinetic energy distribution.

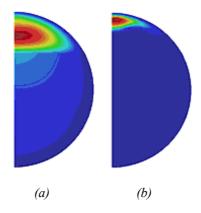


Figure 3. Typical two-phase stratified flow numerical results: (a) velocity and (b) kinetic energy

Next, the solution algorithm performs a numerical integration of the liquid velocity profile, respectively, along the y_{int} as defined by the Eq. (12) and over the area occupied by the liquid phase as defined by the Eq. (13). These results are then used to determine the hydraulic factor according the Eq. (14).

3.2 Single-phase hydraulic factor verification

In this section the numerical results are presented for the single-phase flow. The single-phase flow results are used to verify the performance of the ultrasonic flowmeter modeling by measuring a liquid (or gas) flow inside a horizontal circular smooth pipe. The results were obtained assigning the same fluid properties and flow rates for both phases of the two-phase model. The interface conditions on k and ω are vanished which permits mimicking a single-phase computation leading to a numerical convergence in terms of the dp/dz. Then the hydraulic factor can be computed by a numerical integration of the velocity profile using Eqs. (12) - (13).

Figure (4) presents the values of the hydraulic factor $K_{h\ calc}$ given by the present model, and the values of $K_{h\ correl}$ predicted by empirical correlations from literature for turbulent flow. In Fig. (4) the $K_{h\ calc}$ data are shown as a function of Reynolds number in comparison with the universal law correlation Eq. (16), and two experimental correlations available from industrial ultrasonic flowmeters, Eqs. (17) – (18).

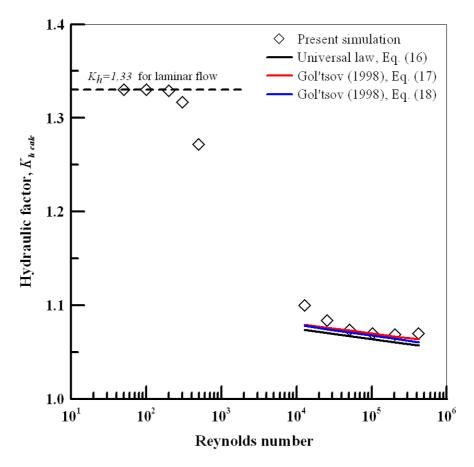


Figure 4. Single-phase hydraulic factor as a function of Reynolds number: comparison of numerical simulation with correlations

The results show that the present numerical model is in good agreement with the correlations for $Re < 10^3$ (laminar flow) and $Re > 10^4$ (turbulent flow).

4. NUMERICAL RESULTS FOR THE TWO-PHASE HYDRAULIC FACTOR

The numerical results presented in this section include the stratified flow range according to the Mandhane et al. (1974) two-phase map as shown in Fig. (5). Based on this map all of the cases simulated in this work are situated in the smooth stratified flow range, which accommodates to a condition of the model that states a smooth horizontal interface between the phases without considering interfacial waves. The gas-liquid stratified flow was simulated as an air-water horizontal flow at atmospheric pressure inside a 2 in. nominal diameter pipe.

Fig. (6) presents the numerical results for the two-phase hydraulic factor, $K_{h 2P}$, as a function of the non-dimensional interface position, $y_{posi} = \frac{y_{int}}{d}$, for various liquid Reynolds number Re_{Ls} . The liquid Reynolds number was defined as

 $Re_{Ls} = \frac{\rho_L u_{Ls} d}{\mu_L}$ where the liquid density ρ_L and viscosity μ_L were selected for water at 25 °C, d is the inner diameter of the pipe equal to 51.2 mm and u_{Ls} is the liquid superficial velocity from the stratified flow range shown in Fig. (5).

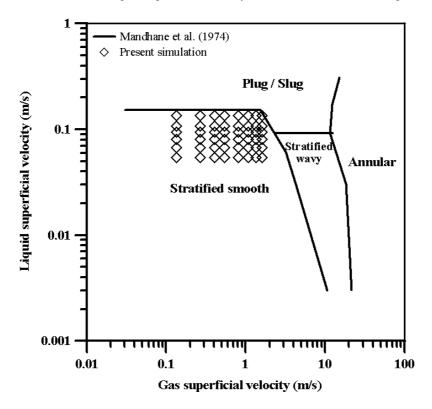


Figure 5. Present simulation range according to the Mandhane et al. (1974) two-phase flow map

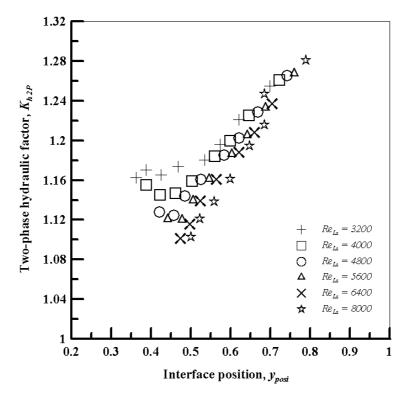


Figure 6. The two-phase hydraulic factor, $K_{h 2P}$, calculated numerically as a function of the interface position, y_{posi}

The same $K_{h\ 2P}$ as a function of Reynolds number, Re_{Ls} , is presented in Fig. (7) for various non-dimensional interface position, y_{posi} . Comparing these figures it can be seen that the two-phase hydraulic factor is strongly dependent on the non-dimensional interface position for the liquid Reynolds number range analyzed. Also their values are higher than the single-phase hydraulic factors for the same Reynolds number range which, *a priori*, indicates that the single-phase hydraulic factor correlations are not applicable to the two-phase gas-liquid stratified flows.

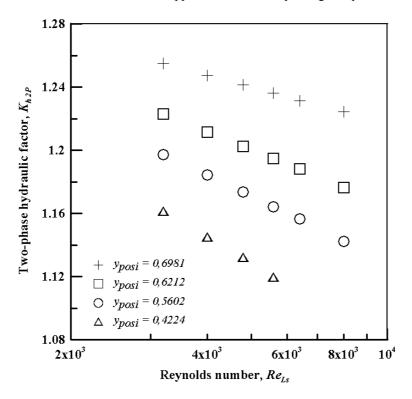


Figure 7. The two-phase hydraulic factor, K_{h2P} , calculated numerically as a function of the Re_{LS}

5. CONCLUSIONS

In this paper we have presented a numerical model for a two-phase gas-liquid stratified flow ultrasonic flowmeter. The model was validated numerically by comparing the results for a single-phase flow with theoretical - empirical correlations for the hydraulic factor as a function of Reynolds number. The results show good agreement with those obtained by correlations for Reynolds number $Re < 10^3$ (laminar flow) and $Re > 10^4$ (turbulent flow).

The two-phase hydraulic factors were determined for smooth stratified flows, and a strong dependence of the hydraulic factor on the interface position has been verified. Also, the two-phase hydraulic factors obtained show that the existing correlations for single-phase flow cannot be applied to two-phase stratified flows in a straightforward way.

An experimental verification of the present model is being carried out. It is expected that the experimental results will give an important background to improve the quality of the numerical model.

6. ACKNOWLEDGEMENTS

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