Law of the wall formulations for separating flow over rough surfaces

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Abstract. *The concepts of displacement in origin and effective roughness length are used to propose new law of the wall formulations that account for the adverse pressure gradient, the surface roughness and the separation effects. It is observed that the newly deduced laws of the wall do reduce to the canonical boundary layer structure far away from a separation point and to Stratford's solution at the separation. Experimental and numerical results are used to validate the theoretical predictions. Experiments on the flow over a steep, rough hill are performed in a water channel, with the aid of laser-Doppler anemometry. The same flow condition is numerically simulated with the help of an eddy-viscosity model. The original and detailed measurements of the flow structure near the separation and reattachment points, as well as into the recirculation region allow a thorough comparison with the numerical results. These simulations then provide all the necessary quantities needed to validate the wall formulations. Results show that the behaviour of the separating flow over rough surface is well captured by the law of the wall formulations.*

Keywords: *Turbulence, Separation, Scaling laws, Rough hill, Numerical simulation*

1. Introduction

The topography of the Earth and its surface roughness, in addition to the stability conditions of the atmosphere constitutes the three main effects that exert major influence on the behaviour of the atmospheric boundary layer. Typically, in hilly terrain, the flow acceleration on the hilltop, the separation on the lee side and the recirculation region in valleys invariably changes the dynamics of the lower atmosphere. In particular, under specific meteorological conditions, the emitted pollutants may become trapped in recirculation regions, directly affecting residential and industrial areas located on those sites.

The momentum exchange and the transfer of scalars between the Earth's surface and the atmosphere are significantly dependent on the characteristics of the topography and on the features of the surface roughness. However, numerical models for global weather forecast are not able to explicitly account for those boundary conditions, since these scales are smaller than the finest typical grid spacing. Given the importance of these effects on the flow dynamics, the alternative is to implicitly describe its influence as parameterizations of the flow behaviour. Consequently, for high Reynolds number flows, the use of law of the wall formulations is mandatory for the correct specification of the near-wall conditions.

The purpose of this work is then to investigate the asymptotic structure of a separating boundary layer over a rough surface. In particular, the main objective of the present manuscript is to propose a new law of the wall formulation that, based on the concepts of the displacement in origin and the effective roughness length, manage to account for the adverse pressure gradient and the surface roughness effects. The flows of interest are those that separate due to the adverse pressure gradient or on smoothly varying rough surfaces.

For flows over a smooth surface, and provided separation is present, any appropriate near wall flow scaling will depend basically on the following parameters: viscosity, local wall shear stress and local wall pressure gradient. In fact, flows subject to large pressure gradients are observed to experience a large wake velocity deficit. Under this condition, the classical matching arguments of the asymptotic theory of Millikan (1939) break down, implying that the canonical two-layered asymptotic structure of the boundary layer does not hold anymore. In particular, close to a separation point, the friction velocity u_τ $(=\sqrt{\tau_w/\rho})$ tends to zero and becomes an inappropriate scaling parameter. This forces into the problem a new scaling parameter based on the local pressure gradient, $u_{p\nu}$ $(= ((\nu/\rho)(\partial p/\partial x))^{1/3})$, that is used to accommodate a new multi-layered structure (Sychev and Sychev (1980), Durbin and Belcher (1992)). This new scaling velocity was first introduced by Stratford (1959), who showed that a power-law velocity profile exists at a separation point. In further studies, several authors have attempted to embed the classical two-layered structure and Stratford's local solution into a single theoretical framework. Typical examples are the works of Mellor (1966), of Afzal (1983), of Nakayama and Koyama (1984), of Melnik (1989) and of Cruz and Silva Freire (1998, 2002). These authors have basically used asymptotic arguments to construct different scaling laws that reduce to the relevant laws at the appropriate limiting cases.

For flows over a rough surface elevation, the relevant scaling parameters are quite different. The roughness elements completely remove the viscous layer, yielding a very complex flow pattern whose properties must now be scaled against characteristic lengths dictated by the roughness itself. In situations where just a small perturbation to the velocity field occurs so that the flow remains attached, the equations of motion can be linearized to yield a flow structure consisting of two-layers (Jackson and Hunt (1975), Hunt et al. (1988)). The linear theory produces expressions for the relevant scaling parameters and for the speedup. Unfortunately, in situations where perturbations are large enough so that flow separation occurs, no theory of rough wall separation similar to those described in the previous paragraph can be found. In fact, the sensitivity of flow separation to wall roughness is known to be marked even on steep hills. Quantifying the onset and extent of separation, however, on flows over a rough wall and subject to strong adverse pressure gradients has proved to be a very challenging problem.

The above remarks give cause to the following statement: comprehensive asymptotic theories that have been proposed to describe the flow near a separation point need to be tested against reliable near wall measurements. As a first stage, the phenomena of flows over a smooth hill and the evaluation of the predictions of the law of the wall formulations have been studied by Loureiro et al. (2007a). The benefit of this work was the direct access to a valuable validation parameter: the wall shear stress. This quantity is of essential importance to the understanding of turbulent flow and to the validation of theoretical and numerical procedures, but is also very difficult to evaluate for flows over curved and rough surfaces. Over a smooth surface, however, provided detailed experimental data are obtained, the wall shear stress distribution can be estimated from mean velocity fits across the viscous sublayer and the turbulent logarithmic region, as described in Loureiro et al. (2007b). In particular, results for the entire region of reverse flow can be obtained.

The proposed law of the walls introduced in the present work are extensions of the formulations of Stratford (1959), Mellor (1966), Nakayama and Koyama (1984) and Cruz and Silva Freire (1998, 2002), that account for the roughness effects through the application of the concepts of the error in origin and roughness length. It is observed that the proposed formulations do reduce to the canonical boundary layer structure far away from a separation point and to Stratford's solution at separation. Experimental and numerical results are used to validate the theoretical predictions. The experiments on the flow over a steep, rough hill have been performed in a water channel, with the aid of laser-Doppler anemometry. The same flow condition has then been numerically simulated with the help of an eddy-viscosity model. The original and detailed measurements of the flow structure near the separation and reattachment points, as well as into the recirculation region allowed a thorough comparison with the numerical results. These simulations then provide all the necessary variables needed to validate wall formulations. Results show that the behaviour of the separating flow over rough surface is well captured by the law of the wall formulations.

2. Laws of the wall formulations

Before we proceed to the description of the proposed formulations, some comments about the surface roughness effects seems know in order.

The near wall flow behaviour is intimately related to the wall characteristics. For a smooth surface, a Cartesian coordinate system is easily set, and the boundary conditions are well established, e.g. the no-slip condition is valid directly at the wall, $y = 0$. On the other hand, for a surface of stochastic characteristics, the uncertainty in estimating the point where the no-slip condition should apply is high. Indeed, the velocity can be assumed as zero at any height from the bottom ($y = 0$) to the top ($y = K$) of the roughness elements, where K is the characteristic height of the elements. This discussion justifies the importance of the parameter called "error in origin" (ε). This length scale represents a vertical shift of the coordinate system to a point where the classical relations turn to be valid.

To account for the presence of roughness in the formulation, it is necessary to collect its different geometric features in one sole parameter that characterises the surface. Indeed, this was one if the main purposes of the work carried out by Nikuradse (1933), that established the concept of sand-grain roughness, herein denoted by y_s. Thus, ε and y_s are the two main parameters used in the present work. A more through discussion about these scales can be found in Malhi(1996) and Schlichting (1979).

For engineering applications, the classical law of the wall for rough surfaces is usually written as (Scholz (1925)):

$$u/u_\tau = (1/\varkappa)\ln\left[(y+\varepsilon)/y_s\right] + B, \quad (1)$$

where $\varkappa = 0.4$, $B = 8.5$ (value obtained by Nikuradse in 1933) and y_s is the sand-grain roughness.

Alternatively, the meteorological literature uses the log law written as follows:

$$u/u_\tau = (1/\varkappa) \ln\left[(y-d)/y_0\right], \tag{2}$$

where y_0 represents the effective roughness length and d is the displacement in origin.

The parameter ε (or d, as denoted in meteorology), may though not appear explicitly on the equations. Since the main effect of d is to produce a vertical shift in the coordinate system, its influence can be accounted for as a constant value added to the y axis. For this reason, and so as to simplify the equations, the use of d will be declined in the next section of this work.

The newly proposed formulations are described below. As the main purpose of this work is to evaluate the predictions of these formulations by comparison with experimental and numerical results, just the main parts of the original derivations of the laws will be presented here. For further descriptions, the reader is referred to the original resources.

2.1 Law of Stratford (1959) for rough surfaces

Stratford was one of the pioneers on the study of separating boundary layers, and proposed a simple equation that is valid in a restricted neighbourhood of the separation point. Its simplicity, though, help us understanding some important issues.

Following Stratford, the x-momentum equation near a separation point is balanced solely by the pressure and turbulent terms:

$$u = 2\varkappa^{-1}\left(\rho^{-1}\partial_x P\right)^{1/2} y^{1/2}. \tag{3}$$

To make non-dimensional the above equation, we introduce two new scaling parameters:

$$u/u_{ps} = 2\varkappa^{-1}\left((y_s/\rho)\partial_x P\right)^{1/2}(1/u_{ps})(y^{1/2}/y_s^{1/2}). \tag{4}$$

Eq. (4) can be recast as

$$u^+ = 2\varkappa^{-1}\left((1/u_{ps}^2)(y_s/\rho)\partial_x P\right)^{1/2} y^{+1/2}. \tag{5}$$

Taking as definition of u_{ps} the equation:

$$u_{ps} = \left((y_s/\rho)\partial_x P\right)^{1/2}, \tag{6}$$

it follows that the formulation of Stratford for rough surfaces can be written in non-dimensional form as:

$$u^+ = 2\varkappa^{-1}\, y^{+1/2}, \tag{7}$$

where $u^+ = u/u_{ps}$ and $y^+ = y/y_s$.

The above equation shows that the two important parameters of the problem are y_s and u_{ps} = $[(y_s/\rho)(\partial_x P)]^{1/2}$.

2.2 Law of the wall of Nakayama and Koyama (1984) for rough surfaces

Nakayama e Koyama (1984) obtained a law of the wall for adverse pressure gradient flows by conducting a one-dimensional analysis on the turbulent kinetic energy equation with assumptions of local similarity. Considering the two possible limiting cases of (i) a constant stress layer far away from the separation region and of (ii) a zero wall stress layer when the boundary layer detaches from the surface, the authors propose a turbulent kinetic energy equation that upon integration yields:

$$u^+ = (1/\varkappa^*)\left[3(\zeta-\zeta_s) + \ln\left[((\zeta_s+1)(\zeta-1))/((\zeta_s-1)(\zeta+1))\right]\right], \tag{8}$$

where

$$\zeta = \left((1+2\tau^+)/3\right)^{1/2}, \tag{9}$$

and

$$u^+ = u/(\tau_w/\rho)^{1/2}, \qquad \tau^+ = 1+\alpha y^+, \tag{10}$$

$$\alpha = (\nu/\sqrt{\tau_w/\rho})(\partial_x P/\tau_w), \tag{11}$$

$$y^+ = (\tau_w/\rho)^{1/2} y/\nu, \tag{12}$$

$$\varkappa^*(\alpha) = \frac{\varkappa + (3/2)^{1/2}\varkappa_0\alpha}{1+\alpha} = \frac{0.4+0.6\alpha}{1+\alpha}, \tag{13}$$

where t_s corresponds to a slip value.

The law of the wall of Nakayama and Koyama becomes valid for rough surfaces if we include in the formulation the two characteristic scales. This modification can be done by redefinition of the variables

$$y^+ = y/y_s, \qquad \alpha = (1/u_\tau^2)((y_s/\rho)\partial_x P) = u_{ps}^2/u_\tau^2, \tag{14}$$

where y_s is the sand-grain roughness and must be estimated from the experimental results.

Please note that when $\alpha \to 0$, Eq. (8) becomes:

$$u^+|_{\alpha\to 0} = (1/\varkappa)\ln(y^+/y^+_{slip}),$$

where, $\varkappa$ = 0.4, $u^+ = u/u_\tau$, $y^+ = y/y_s$, $y^+_{slip} = e^{-\varkappa\beta}$ and β = 8.5. Thus, we notice this limiting form is actually equivalent to the Eq. (1) of Nikuradse.

It is observed that for the other limiting case, $\alpha \to \infty$, Eq. (8) leads to the Stratford equation for rough surface, Eq. (7). The extension of the formulation of Nakayama and Koyama for flows over rough surfaces is then a straightforward process, performed on the basis of the procedure carried out for the Stratford law.

2.3 Law of the wall of Mellor (1966) for rough surfaces

Based on dimensional arguments, Mellor (1966) has investigated the effect of pressure gradients on the behaviour of turbulent boundary layers without restriction to equilibrium conditions. When a large external pressure gradient is applied to a boundary layer, no portion of the defect profile overlaps the logarithmic law. In fact, as previously suggested by Coles (1956) and by Stratford (1959), very near a separation point the logarithmic part of the velocity profile ceases to exist. However, if Millikan's (1939) arguments are recast and a new pressure gradient parameter is included in the analysis, an equation can be derived that satisfies the required limiting forms as a separation point is approached. Mellor (1966) wrote this equation as:

$$u = u_v + (2/\varkappa)\left[(\tau^2 + Py)^{1/2} - u_\tau\right] + (u_\tau/\varkappa)\ln\left[4(u_\tau^3/(\nu P))((\tau^2 + Py)^{1/2} - u_\tau)((\tau^2 + Py)^{1/2} + u_\tau)\right], \tag{15}$$

where u_v is a constant that is a function of the pressure gradient and $P = (1/\rho)\partial_x P$. Based on this equation, Mellor writes two different expressions that govern the flow on the two limiting cases of the classical near boundary layer flow and of the flow near to a separation point. The choice for the appropriate expression depends on whether the parameter $\alpha = (\nu P)/u_\tau^3$ is small or large.

By doing $u^+ = u/u_\tau$ and $y^+ = yu_\tau/\nu$, and recalling that $\alpha = \nu P/u_\tau^3$, the Eq. (15) can be recast as:

$$u^+ = u_v^+ + (2/\varkappa)\left[(1 + \alpha y^+)^{1/2} - 1\right] + (1/\varkappa)\ln\left[4/\alpha((1 + \alpha y^+)^{1/2} - 1)((1 + \alpha y^+)^{1/2} + 1)\right], \tag{16}$$

an equation that is appropriate just to smooth surfaces and small values of α. The values of the constant $u_v^+ = B^+(\alpha)$ is given by Table 1 in the work of Mellor (1966).

In order to account for the roughness effects, Eq. (16) can be used if we apply the following transformation in Eq. (15):

$$u^+ = u/u_\tau, \qquad y^+ = y/y_s, \qquad \alpha = u_{ps}^2/u_\tau^2, \tag{17}$$

which is indeed the same variables' transform used for the law of the wall of Nakayama and Koyama (1984). The sand-grain roughness y_s should be taken from the experimental data as well.

The above equation is valid to $\alpha \to 0$, i.e. for finite values of u_τ and $P \to 0$, since it leads us to:

$$u^+|_{\alpha\to 0} = (1/\varkappa)\ln y^+ + u_v^+. \tag{18}$$

Table 1. Correction in the constant B^+ of Mellor (1966).

α	-0.01	0	0.02	0.05	0.10	0.20
B^+	8.52	8.50	8.54	8.66	8.86	9.23

Please note the corrected values of the constant u_v^+ in Eq. (16). The modified u_v^+ assures the equivalence of Eq. (16) to the Eq. (1) of Nikuradse. As $\alpha \to 0$, it follows that $u_v^+ \to 8.5$. Thus, the original values can be transformed to account for the roughness effects. The result is presented in Table (1).

According to Mellor (1966), for large values of α, the equation below should be used:

$$u^* = u_v^* + (2/\varkappa)\left[\xi^{1/2} - \alpha^{-1/3}\right] + (1/(\varkappa\alpha^{1/3}))\ln\left[(4/\alpha)(\xi^{1/2} - \alpha^{-1/3})(\xi^{1/2} + \alpha^{-1/3})\right], \tag{19}$$

where $\xi = (\alpha^{-2/3} + y^*)$, $y^* = yu_{p\nu}/\nu$, $u^* = u/u_{p\nu}$, $u_{p\nu} = [(\nu/\rho)dP/dx]^{1/3}$ and $u_v^* = B^*(\alpha) = B^+/\alpha^{1/3}$.

The Eq. (19) can be directly applied for rough surfaces if we use the transformation $u_{p\nu} = u_{ps}$, $\alpha = u_{ps}^2/u_\tau^2$, which lead us, in the limiting case of $\alpha \to \infty$, to the equation of Stratford for rough surfaces, Eq. (7). Note that u_v^* should be estimated from corrected values of B^+.

2.4 Law of the wall of Cruz and Silva Freire (1998, 2002) for rough surfaces

Introducing a new scaling parameter, Cruz and Silva Freire (2002) proposed the law of the wall for a separating flow to be written as

$$u = \gamma\, 2\varkappa^{-1}\sqrt{u_\tau^2 + (1/\rho)(\partial_x P)_w\, y} + \gamma\, u_\tau \varkappa^{-1}\ln(y/L_c), \tag{20}$$

where $\gamma = \tau_w/\left|\tau_w\right|$ is used to indicate the flow direction and

$$L_c = \left(\sqrt{(\tau_w/\rho)^2 + 2(\nu/\rho)(\partial_x P)_w u_R} - (\tau_w/\rho)\right)(\rho^{-1}(\partial_x P)_w)^{-1}, \tag{21}$$

where u_R is a characteristic velocity scale that can be obtained as the highest real root of

$$u_R^3 - (\tau_w/\rho)\, u_R - (\nu/\rho)(\partial_x P)_w = 0. \tag{22}$$

Equation (21) can be used indistinctly in all flow regions, including regions of reverse flow. In the limiting cases $y(\partial_x P)_w << \tau_w$, $\tau_w = 0$ and $y(\partial_x P)_w >> \tau_w$ the reference length scale, L_c, reduces respectively to

$$L_c = \nu/u_\tau, \qquad L_c = \sqrt{2}\,\nu/u_{p\nu} \qquad \text{and} \qquad L_c = 2\left|\tau_w/(\partial_x P)_w\right|. \tag{23}$$

The extension of this formulation to account for the roughness effects can be performed through a modification on the characteristic length scale as:

$$L_c = \left(\sqrt{(\tau_w/\rho)^2 + 2(y_s/\rho)(\partial_x P)_w u_R^2} - (\tau_w/\rho)\right)(\rho^{-1}(\partial_x P)_w)^{-1}, \tag{24}$$

where u_R is know obtained as the positive root of

$$u_R^2 - \gamma\, u_\tau u_R - (y_s/\rho)(\partial_x P)_w = 0. \tag{25}$$

Thus, applying the following transformation of variables

$$u^+ = u/u_{ps}, \qquad u_\tau^+ = u_\tau/u_{ps}, \qquad y^+ = y/y_s, \qquad \hat{y} = y/L_c, \tag{26}$$

we obtain the law of the wall for a rough-wall turbulent boundary layer subjected to an adverse pressure gradient and separation:

$$u^+ = 2\gamma\,\varkappa^{-1}\sqrt{B{u_\tau^+}^2 + y^+} + \gamma\, u_\tau^+\varkappa^{-1}\ln\hat{y}, \tag{27}$$

where $B = 2.89$, in order to satisfy the equation of Nikuradse.

Indeed, Eq. (27) satisfies both limiting conditions. For $(\partial_x P)_w \to \infty$, we find that

$$u^+ = 2\varkappa^{-1}y^{+1/2}, \tag{28}$$

thus leading to the Stratford equation for rough surfaces, Eq. (7).

Alternatively, in the limit when $(\partial_x P)_w \to 0$, Eq. (27) tends to

$$u/u_\tau = 2\varkappa^{-1}\sqrt{B} + \gamma\,\varkappa^{-1}\ln\hat{y}, \tag{29}$$

which is indeed the equation of Nikuradse, Eq. (1).

3. Experimental and numerical validation

The above described new law of the wall formulations will be tested against the data of Monteiro et al. (2006). This work has experimentally investigated the problem a neutrally stratified, fully rough boundary layer flowing over a steep model hill. The rough surface used in this work was essentially two-dimensional. It was comprised by a sequence of square bars equally distributed over the smooth wall of the water channel. The two-dimensional hill was constructed with a Witch of Agnesi shape with a maximum 18.6^o slope. Measurements of longitudinal and vertical components of mean velocity and its turbulent components were carried out with the aid of laser Doppler anemometry. The experimental data allow a thorough description of the near-wall flow, extending from the upstream region, along into the separated zone and to the downstream lee side. Measurements were taken at 11 stations along the test section, and Figure (1) illustrates its spatial distribution.

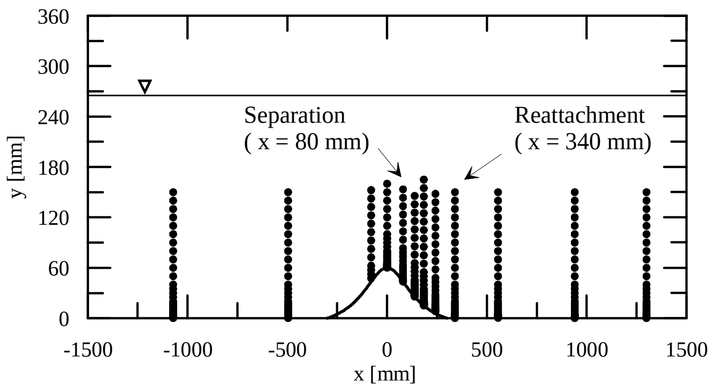

Figure 1. Spatial distribution of the experimental profiles and illustration of the coordinate system.

The general properties of the incoming flow are presented in Table (2). For further details on the experimental campaign, the reader is referred to the original resources.

Table 2. Properties of undisturbed profile.

Boundary layer thickness	δ	100 mm
External velocity	U_δ	0.3133 m/s
Friction velocity	u_τ	0.022537 m/s
Roughness length	y_0	0.396 mm
Reynolds roughness length	$U_\delta y_0/\nu$	123

The experimental data of Monteiro et al. (2006) must be supplemented by data obtained through numerical simulations of the same flow geometries. The main objective of the computational part of this work is to provide some data that have not been reported by the original work, but that are crucial for theory evaluation. A critical example is the pressure distribution at the wall, which could not be measured on the small roughness surface of Monteiro et al (2006). Another quantity that is difficult to be obtained experimentally for rough surface, but instead could be estimated from numerical results is the wall shear stress. A secondary objective of the simulations is to have data with a sufficiently fine domain discretization so as to allow for accurate data interpolation.

The simulations for this flow geometry were conducted with the code ANSYS CFX, release 10. The code solves the Reynolds averaged Navier-Stokes equations (RANS) through a finite-volume formulation coupled with a scheme for the treatment of the convective and diffusive terms simultaneously. Turbulence closure was achieved by choosing the κ-ω of Wilcox and reformulated by Menter (1994). In fact, six different types of turbulence modeling were applied to the smooth

Table 3. Length of separated flow according to numerical predictions. H = hill height.

Work	Separation Point (x/H)	Reattachment Point (x/H)	Length (x/H)
Experiments	1.33	5.67	4.34
Numerical simulation	1.13	4.65	3.52

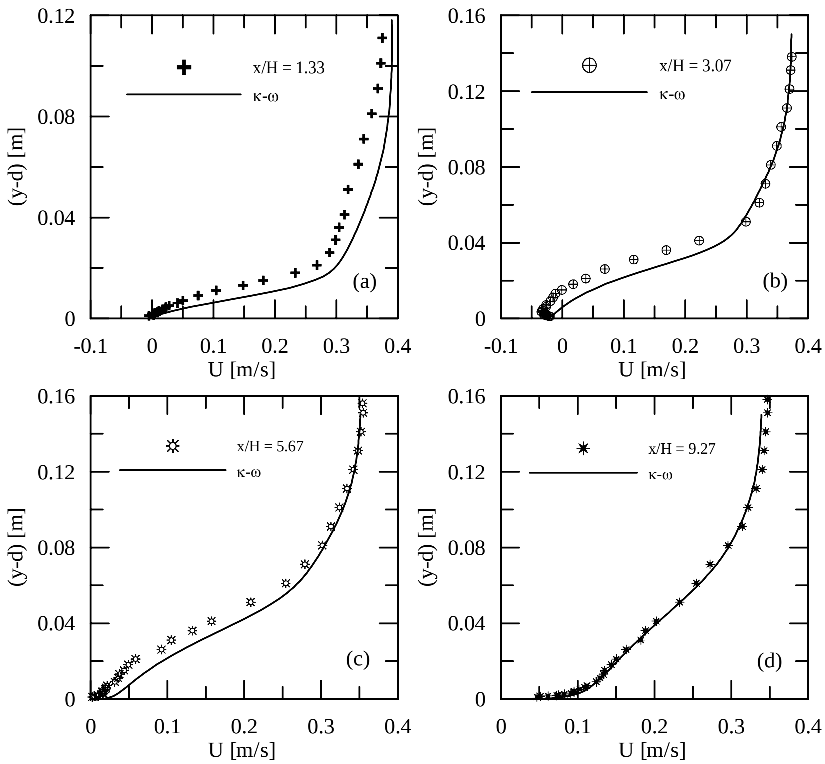

Figure 2. Comparison between experimental and numerical mean longitudinal velocity profiles for stations: (a) x/H = 1.33, (b) x/H = 3.07, (c) x/H = 5.67, (d) x/H = 9.27.

hill problem (Loureiro et al., 2007c). For the prediction of mean velocity profiles and wall shear stress, the κ-ω and the SST-model proved to be the best choices.

The κ-ω based SST model is supposed to give highly accurate predictions of the onset and the extent of separation from a smooth surface under an adverse pressure gradient. The near wall treatment is provided by a method that automatically shifts from a wall function approach to a low-R formulation as mesh size is refined. By resorting to the analytical solutions of the κ-ω formulation in the logarithmic and viscous regions of the flow, a blending procedure can be implemented to specify an algebraic equation for ω that can be used throughout the inner regions of the flow. This equation is then solved together with the fluxes for the momentum and κ (which is artificially kept to zero) equations. Wall functions are replaced by a low Reynolds number formulation once a near wall grid resolution of at least $y^+ < 2$ is achieved.

For rough surface problems, on the other hand, the code used has some limitations in representing the real boundary conditions. The sand-grain roughness parameter, y_s, can be specified for application in the law of the wall formulation, which is much similar to Eq. (1). However, there is no indication that the characteristic roughness length is taken into consideration in the low Reynolds number model. Then, anticipating the numerical results, we can presume that data estimated in recirculation region will not show an agreement with experiments as good as those obtained for the smooth surface. The separated flow is the critical region of the domain because, as $u_\tau \rightarrow 0$, $y^+ \rightarrow 0$ as well, and then the automatic near wall treatment forces the use of the low Reynolds number model.

The computations were performed on a Pentium D, 2.8 GHz, with 2 Gb DDR400 RAM operating in dual channel mode. Grid independence tests showed that a structured mesh with 110,376 elements was refined enough to provide independent results. Boundary conditions were taken directly from the experimental data, including the mean and fluctuating quantities. A comparison between the measured and the computed regions of separated flow is given in Table 3.

Figures 2 present the experimental longitudinal mean velocity profiles compared with the results of the numerical simulation. Please note the experimental error in origin account for in Figures 2. Four stations have been chosen for flow evaluation, namely, (i) station x/H = 1.33, that is nearly the location of the separation point, (ii) station x/H = 3.07, a profile located inside the recirculation bubble, (iii) station x/H = 5.67, which is approximately the reattachment point, and (iv) station x/H = 9.27, located downstream of the separation bubble, where the flow is returning to equilibrium conditions.

As discussed in the previous section, good predictions were not expected for the flow inside the recirculating region. Actually, the numerical code uses in this location the low Reynolds number model, which does account for the rough surface effects. In spite of that, reasonable estimations were obtained for the near-wall flow at station x/H = 1.33, and a fairly good estimation for station x/H = 9.27.

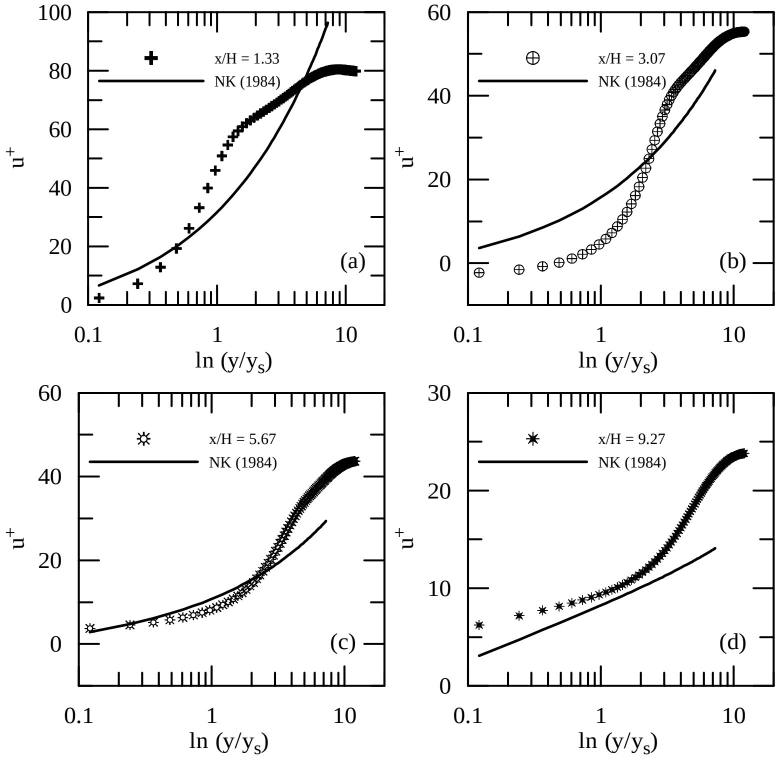

Figure 3. Behaviour of the law of the wall for rough surfaces based on the formulation of Nakayama and Koyama (1984). Stations: (a) $x/H = 1.33$, (b) $x/H = 3.07$, (c) $x/H = 5.67$, (d) $x/H = 9.27$.

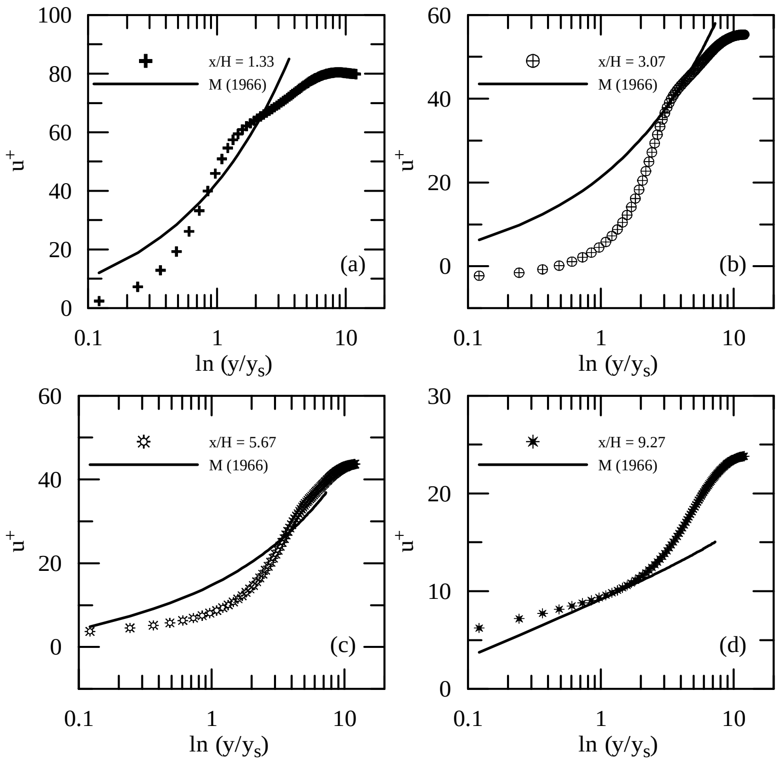

Figure 4. Behaviour of the law of the wall for rough surfaces based on the formulation of Mellor (1966). Stations: (a) $x/H = 1.33$, (b) $x/H = 3.07$, (c) $x/H = 5.67$, (d) $x/H = 9.27$.

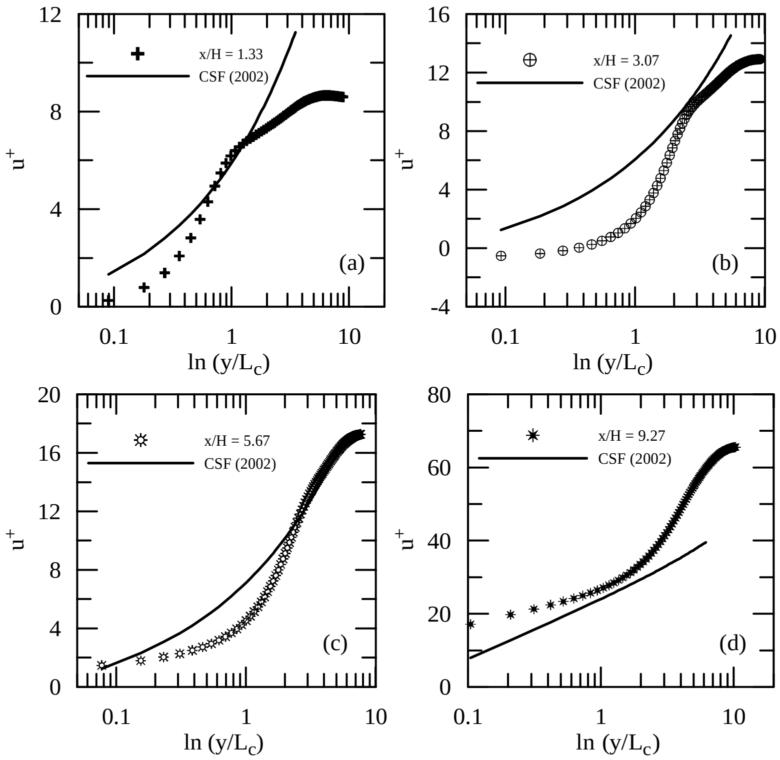

Figure 5. Behaviour of the law of the wall for rough surfaces based on the formulation of Cruz and Silva Freire (2002). Stations: (a) $x/H = 1.33$, (b) $x/H = 3.07$, (c) $x/H = 5.67$, (d) $x/H = 9.27$.

The behaviour of the proposed law of the wall formulations are presented in Figures 3 to 5, where the symbols represent the numerical data and the lines are the estimated law of the walls. To evaluate the formulations based on the works of Nakayama and Koyama (1984) and of Mellor (1966), the coordinate system is normalized by the parameters u_τ and y_s, in accordance to Eqs. (14) and (17). On the other hand, following the transformation of variables used in Eq. (26), Figure 5 show the law of the wall of Cruz and Silva Freire (2002) normalized by the characteristic scales u_{ps} and L_c.

Agreement with experimental and numerical data was found to be fairly good for all the proposed formulations. Indeed, there are no discrepancies in the predictions of the different formulations. The three different laws provide consistent results, in particular, a quite good agreement is obtained for stations $x/H = 5.67$ and 9.27. The formulations also provide similar estimations for stations $x/H = 1.33$ and 3.07, although with a less reasonable agreement with data.

In particular, all estimations for station $x/H = 3.07$, which is located inside the separation region, showed poor agreement with reference data. This fact might be partly attributed to the poor representation of the experimental data provided by the numerical simulation.

4. Final remarks

The present work has conducted a detailed analysis on the description of turbulent boundary layer over a rough surface and subjected to separation, including its region of reverse flow. The concepts of displacement in origin and effective roughness length were used to propose new law of the wall formulations that account for the adverse pressure gradient, the surface roughness and the separation effects. Predictions given by the three different scaling laws were compared with the experimental data of Monteiro et al. (2006). The performance of the formulations was evaluated in four distinct regions: (i) at the separation point, (ii) in the reverse flow region, (iv) at the reattachment point and (v) downstream of the separation bubble. It is observed that the newly deduced laws of the wall do reduce to the canonical boundary layer structure far away from a separation point and to Stratford's solution at separation.

Experimental and numerical results were used to validate the theoretical predictions. Experiments on the flow over a steep, rough hill were performed in a water channel, with the aid of laser-Doppler anemometry. The same flow condition has been numerically simulated with the help of an eddy-viscosity model. The original and detailed measurements of the flow structure near the separation and reattachment points, as well as into the recirculation region allow a thorough comparison with the numerical results. These simulations then provide all the necessary variables needed to validate the wall formulations. Results show that the behaviour of the separating flow over rough surface is well captured by the law of the wall formulations, in particular near to the reattachment point and downstream of the separation bubble. The poor agreement observed for predictions at stations $x/H = 1.33$ and 3.07 might be attributed to the lower ability of the numerical simulations used to evaluate the law of the walls in reproducing the flow behaviour.

In summary, this study provides a broad and independent analysis of the problem, and investigates the ability of the scaling laws to predict the flow behaviour from upstream of the separation region, along to the recirculation bubble and downstream of the reattachment point, accounting for the presence of the surface roughness.

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