INVERSE PROBLEM FOR ESTIMATION OF APPARENT THERMAL DIFFUSIVITY

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Abstract. *Due to the relevant applications in the food and pharmaceutical industries, there is a growing demand for the formulation and solution of inverse heat transfer problems. In this work an inverse heat conduction problem is solved using optimization methods to estimate apparent thermal diffusivity of foods at different drying temperatures. Temperature and moisture versus time were obtained numerically at the center of the food using the 1D Fourier equation with drying temperatures in the range between 20 to 70°C. The solution of the partial differential equation is made with a finite difference method coupled to an optimization technique of Differential Evolution used in inverse method. The estimation of apparent thermal diffusivity is based on transient temperature measurements taken by a thermocouple on the inner surface of the food on which the heat transfer occurs, the experimental results had been obtained of literature. This study approximates the thermal diffusivity as a function polynomial with unknown coefficients, which can be expressed in terms of food moisture content. The inverse problem is solved as an optimization problem in which a squared residue functional is minimized with the method already cited. Statistical analysis shows no significant differences between reported and estimated curves. The determination of thermophysical properties from an inverse method is an attractive technique obtaining the apparent thermal diffusivity with acceptable accuracy for the model investigated however, the convective effect and shrinkage assumptions in the model provides greater reliability on the calculated thermal diffusivity.*

Keywords: *Inverse problem, thermal diffusivity, Differential Evolution, food, finite difference method.*

1. INTRODUCTION

There are numerous methods to measure the thermal diffusivity proposed in the specialized literature. Nevertheless, most of them need relatively complex instrumentation or experimental assemblies and demand an expertise of the thermal phenomena. Several papers present such methods and results of thermal diffusivity of different foods, some are cited as follow. Choi and Okos (1983a, b) propose a line heat-source thermal conductivity probe with auxiliary thermocouple to determine thermal conductivity and thermal diffusivity, simultaneously. Sweat (1986) recommends determination of food thermal diffusivities from experimentally obtained values for thermal conductivity, specific heat and mass density. Shyamal *et al.* (1994) study the thermal properties of the wheat, specifically the specific heat, thermal conductivity and thermal diffusivity. These researchers conclude that thermal diffusivity decrease linearly with moisture content. Tao *et al.* (1994) analyse the thermal properties of two varieties of rice bran. Carciofi *et al.* (2002) estimate the effective thermal diffusivity of mortadella using data of the cooking process. Bouillereaux *et al.* (2003) determine the thermal properties of the gelatin gel during thawing using artificial neural networks. Kubásek *et al.* (2006) estimate thermal diffusivity of the olive oil during treatment high-pressure and Baïri *et al*. (2007) determinate thermal diffusivity of foods using 1D Fourier cylindrical solution.

Growing interest has recently been evidenced in the analysis and solution of inverse problems of heat transfer, it can be cited the works of Mendonça *et al*. (2005) and Simpson and Cortés (2004) using the inverse method to estimate thermophysical properties of foods. Many papers in the literature involving inverse problems use deterministic methods, based on gradient information, to minimize the objective function (Khachf *et al.*, 2002). Although such methods can lead to local rather than global minima, their main advantage lies in their good convergence rate.

New optimization methodologies are being used to solve inverse problems, particularly stochastic approaches, which usually supply a good solution or until the global optimum; however, the computational time they require generally exceeds that of deterministic methods (Wood, 1996; Suram *et al.*, 2005). Other techniques based on artificial intelligence field, such as genetic algorithms and artificial neural networks, have been used for the solution of inverse problems (Mikki *et al.*, 1999; Bouillereaux *et al.*, 2003; Ayhan *et al.*, 2004; Sablani *et al.*, 2004).

This paper presents a simple procedure to estimate apparent thermal diffusivity of banana variable with the moisture content from a range of numerical/experimental temperatures, using Differential Evolution as optimization technique for obtain parameters of piecewise function through of inverse method. The problem considered here is relevant in food processing operations, such as the analysis of transient heat transfer during the drying, cooling or freezing of fruits and vegetables in continuous systems, which requires knowledge of the thermal properties of foods. Many references about fundamentals of banana drying were found in the literature. Furthermore, few references concern about thermophysical properties during drying, as Pérez (1998), Lima (1999) and Queiroz and Nebra (2001). However the two first works use other numerical methodology to obtain the apparent thermal diffusivity while the last work to solve the mass transfer determining diffusion and convective coefficients.

Thus the objective of this work was to study heat transfer aspects of the banana during drying process and use transient temperatures to estimate the apparent thermal diffusivity variable with the moisture content. A second objective was to explore, analyse and validate a new transient measurement methodology using inverse method for determination of apparent thermal diffusivity in the range between 20 to 70°C for the drying temperature.

2. MATERIALS AND METHODS

2.1 Heat Transfer Equation

The method used to estimate the apparent thermal diffusivity was based on the conduction heat transfer equation, where the apparent thermal diffusivity was a parameter to be estimated. To simplify the problem the following hypotheses were considered:

(i) The banana is represented in the geometric form of an infinite cylinder of length L (m) and radius R (m) defined between [0; R], where $R << L$ (see Fig. 1a); thus, the longitudinal heat and moisture transfer were neglected and the axial symmetry was considered.
(ii) The thermal diffusivity was considered variable with moisture content during drying.
(iii) The banana is considered homogeneous.

One of the boundaries is in contact with the surrounding air thus resulting in a convective boundary condition for both the temperature and the moisture content as illustrated in Fig. 1b. The linear system of equations proposed with associated initial and boundary conditions for the modelling of such physical problem involving the energy conservation equation, based on Fourier's law and mass transfer equation described by Fick's unidirectional diffusion equation (Crank, 1975) as follows:

$$\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\alpha\frac{\partial T}{\partial r}\right), \tag{1}$$

$$\frac{\partial X}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD_{ef}\frac{\partial X}{\partial r}\right), \tag{2}$$

where α (m^2/s) is the thermal diffusivity, T (°C) is the internal temperature, D_{ef} (m^2/s) is the effective mass diffusivity, X (kg$_w$/kg$_{dm}$) is the moisture content (dry basis), r (m) is the transfer direction and t (s) is the time.

As initial condition, it was considered that initial temperature and moisture of the food are uniform, Eqs. (3) and (4). Null flux (symmetry) conditions were considered at the banana geometric center, Eqs. (5) and (7). The convective effect of moisture and heat transfer at surface, Eqs. (6) and (8), was considered.

Initial conditions:

$T(r,0) = T^0, \forall r,$ (3)

$X(r,0) = X^0, \forall r,$ (4)

Boundary conditions:

$$\begin{cases} \left(\dfrac{\partial T}{\partial r}\right)_{r=0} = 0, & (5)\\ -k\left(\dfrac{\partial T}{\partial r}\right)_{r=R} = h(T_R - T_e) + \rho_s \Delta r \dfrac{\partial \overline{X}}{\partial t}\left[h_{fg} + c_v(T_R - T_e)\right], & (6) \end{cases}$$

$$\left(\frac{\partial X}{\partial r}\right)_{r=0} = 0, \quad (7)$$

$$-D_{ef}\left(\frac{\partial X}{\partial r}\right)_{r=R} = h_m(X_R - X_e), \quad (8)$$

where k (W/m^{o}C) is the thermal conductivity of the fruit, h (W/m^{2o}C) is the heat transfer convective coefficient, Δr is the spatial mesh step, ρ_s = 1970 (kg/m^3) is the dry solid density, h_{fg} (J/kg) represents the latent heat of vaporization of water obtained by air dry conditions, c_v (J/kg.K) is the specific heat of vapour of water, $\overline{X} = \frac{1}{R}\int_0^R X(r,t)dr$ (kg$_w$/kg$_{dm}$) is the average moisture content in the section and h_m (m/s) is the mass transfer convective coefficient. The values of h_m and D_{ef} used in this work were obtained of the work of Queiroz and Nebra (2001) that used the same experimental data of this work.

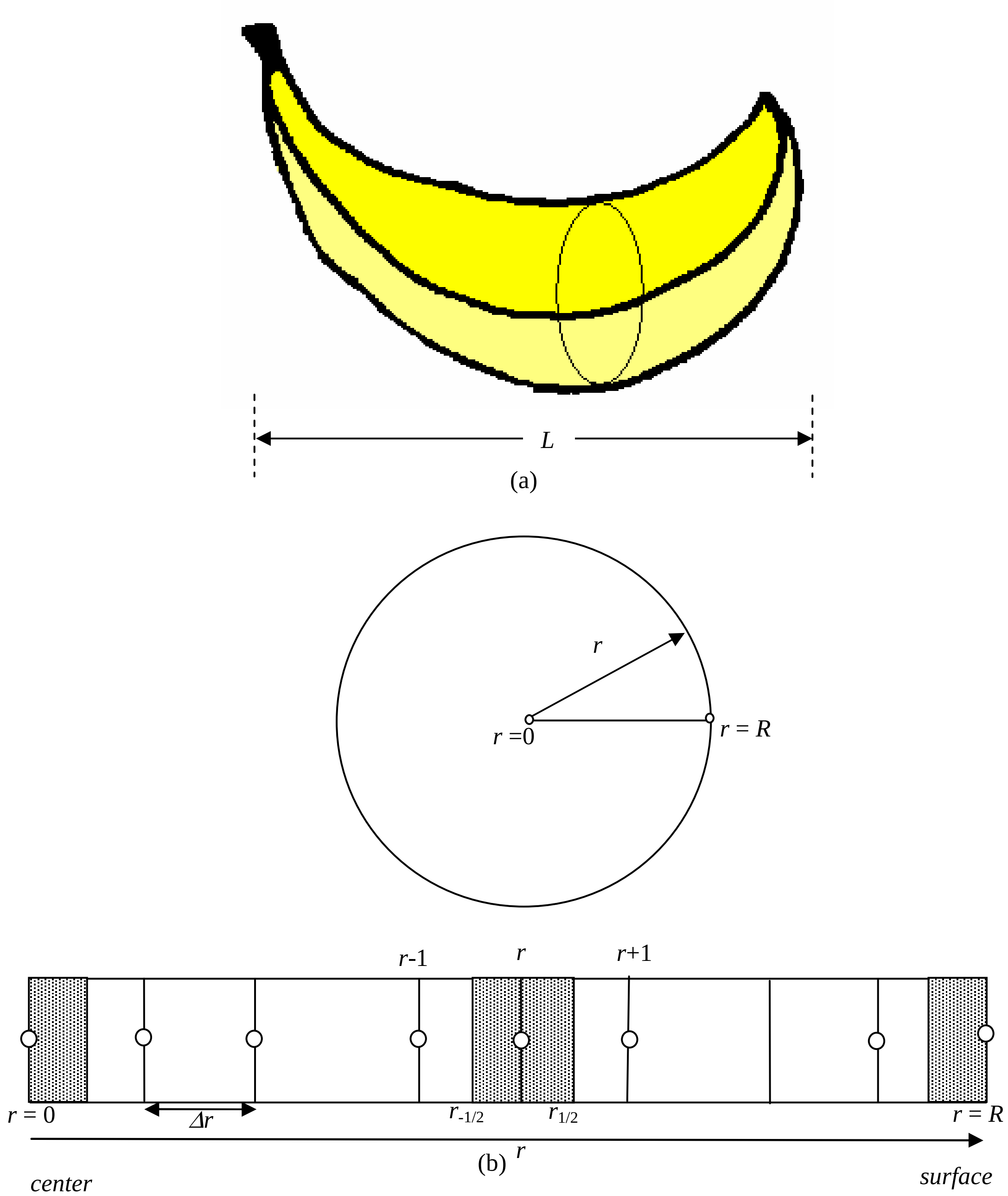

Figure 1 – Computational domain (banana).

Due to the characteristics of the mathematical problem (one-dimension and homogeneous material), the simpler finite difference technique (Smith, 1985) can be used rather than the finite element method or finite volume method for the solution of these partial differential equations. In this work, an explicit scheme was selected. Using this numerical scheme, the Eq. (1) can be described and approximated in the following terms,

$$\left(\frac{T_r^{t+\Delta t}-T_r^t}{\Delta t}\right)=\frac{\alpha}{2r\Delta r^2}\left[r_{1/2}\left(T_{r+1}^t-T_r^t+T_{r+1}^{t+\Delta t}-T_r^{t+\Delta t}\right)-r_{-1/2}\left(T_r^t-T_{r-1}^t+T_r^{t+\Delta t}-T_{r-1}^{t+\Delta t}\right)\right] \tag{9}$$

The Eq. (1) at r = 0 can be replaced by $\frac{\partial T}{\partial t}=2\frac{\partial}{\partial r}\left(\alpha\frac{\partial T}{\partial r}\right)$, thus at the food center (Fig. 1b) symmetric condition was considered where $T_{r+1}=T_{r-1}$,

$$\left(\frac{T_r^{t+\Delta t}-T_r^t}{\Delta t}\right)=\frac{2\alpha}{\Delta r^2}\left[T_{r+1}^t-T_r^t+T_{r+1}^{t+\Delta t}-T_r^{t+\Delta t}\right] \tag{10}$$

To boundary condition at surface, since Eq. (6), can be calculated as follows,

$$T_R^{t+\Delta t}=\frac{\left[T_{R-1}^{t+\Delta t}+\frac{h\Delta r}{k}T_e-\frac{\rho_s\Delta r^2}{k}\left(\frac{\overline{X}^{t+\Delta t}-\overline{X}^t}{\Delta t}\right)(h_{fg}-c_vT_e)\right]}{\left[1+\frac{h\Delta r}{k}+\frac{\rho_s\Delta r^2c_v}{k}\left(\frac{\overline{X}^{t+\Delta t}-\overline{X}^t}{\Delta t}\right)\right]}. \tag{11}$$

Substituting Eq. (11) in Eq. (9) we obtain the temperature $T_{R-1}^{t+\Delta t}$. The discretization of the Eq. (2) is omitted here due to its analogy with Eq. (1).

2.2 Inverse Problem

Knowing the food's geometry and physical properties, as the boundary and initial conditions, enables one to solve Eqs. (1) to (8), thus determining the transient temperature and mass distribution in the food. This type of problem is called a direct problem. If any of these magnitudes or a combination of them is unknown, but experimental data are available on the temperature measured inside and/or on the external surface of the food, one has an inverse problem that allows one to determine the unknown magnitudes, provided those data contain sufficient information.

The interest of the present work is to estimate the apparent thermal diffusivity using experimental data of the temperature obtained experimentally at the center of the banana during a time interval. In this work is desired to minimize the difference between experimental and predicted temperatures. Mathematically it is desired to minimize the objective function,

$$f=\sqrt{\frac{\sum_{j=1}^{n}(\tau_0^j-T_0^j(\alpha))^2}{n}}, \tag{12}$$

where T_0^j (°C) is the temperature of the banana at node central, r = 0, j is the time indicator, calculated numerically by the explicit finite difference method and τ_0^j (°C) is the experimental temperature of the banana at thermocouple central, r = 0, and n is the number of samples.

In most of the techniques developed to solve inverse problems, the numerical model must be able to solve the direct problem with values arbitrated to the magnitudes to be determined. Since the procedures for the solution are usually iterative, the direct problem must be solved several times. Thus, it is desirable to have a precise method for the solution of the direct problem that requires a relatively short computational time. The Differential Evolution method was used as the optimization technique and is described as follows.

2.3 Differential Evolution

Evolutionary algorithms are computer-based problem-solving systems of evolutionary computation area based on principles of evolution theory. The interest in evolutionary algorithms is increasing very fast, their robust and powerful adaptive search mechanisms. Evolutionary algorithms have been used in many problems, dealing with multidimensional and multimodal search. There are a variety of evolutionary models that have been proposed and studied, such as genetic algorithms, evolution strategy, evolutionary programming, genetic programming, and recently differential evolution that are referred as evolutionary algorithms. They share a common conceptual base of simulating the evolution of individual structures via selection and reproduction procedure. The basic idea is to maintain a population of candidate solutions that evolve under selective pressure that favors better solutions (Goldberg, 1989; Bäck *et al*., 1997).

Differential Evolution (DE) is a population-based stochastic function minimizer (or maximizer) relating to evolutionary computation, whose simple yet powerful and straightforward features make it very attractive for numerical optimization. DE uses a rather greedy and less stochastic approach to problem solving than do evolutionary algorithms. DE combines simple arithmetical operators with the classical operators of recombination, mutation and selection to evolve from a randomly generated starting population to a final solution.

DE differs from conventional genetic algorithms in its use of perturbing vectors, which are the difference between two randomly chosen parameter vectors. The DE algorithm was first introduced by Storn and Price (1995), and was successfully applied in the optimization of some well-known non-linear, non-differentiable and non-convex functions by Storn (1997).

The different variants of DE are classified using the following notation: DE/ϕ/β/δ, where ϕ indicates the method for selecting the parent chromosome that will form the base of the mutated vector, β indicates the number of difference vectors used to perturb the base chromosome, and δ indicates the recombination mechanism used to create the offspring population. The *bin* acronym indicates that the recombination is controlled by a series of independent binomial experiments.

The fundamental idea behind DE is a scheme whereby it generates the trial parameter vectors. In each step, the DE mutates vectors by adding weighted, random vector differentials to them. If the cost of the trial vector is better than that of the target, then the target vector is replaced by the trial vector in the next generation. The variant implemented in Matlab (MathWorks) was the DE/*rand*/1/*bin*, which involved the following steps and procedures:

<u>Step 1:</u> *Parameter setup*

The user chooses the parameters of population size, the boundary constraints of optimization variables, the mutation factor (f_m), the crossover rate (CR), and the stopping criterion of maximum number of iterations (generations), G_{max}.

<u>Step 2:</u> *Initialization of an individual population*

Set generation k = 0. Initialize a population of i = 1, .., M individuals (real-valued n-dimensional solution vectors) with random values generated according to a uniform probability distribution in the n dimensional problem space. These initial individual values are chosen at random from within user-defined bounds (boundary constraints).

<u>Step 3:</u> *Evaluation of the individual population*

Evaluate the fitness value of each individual.

<u>Step 4:</u> *Mutation operation (or differential operation)*

Mutation is an operation that adds a vector differential to a population vector of individuals according to the following Eq. (13),

$$z_i(k+1) = x_{i,r_1}(k) + f_m\,[x_{i,r_2}(k) - x_{i,r_3}(k)]\,, \tag{13}$$

where i =1, 2, ..., M is the individual's index of population; k is the generation; $x_i(k) = \left[x_{i_1}(k), x_{i_2}(k), \ldots, x_{i_n}(k)\right]^{\mathrm{T}}$ stands for the position of the i-th individual of population of N real-valued n-dimensional vectors; $z_i(k) = \left[z_{i_1}(k), z_{i_2}(k), \ldots, z_{i_n}(k)\right]^{\mathrm{T}}$ stands for the position of the i-th individual of a *mutant vector*; r_1, r_2 and r_3 are mutually different integers and also different from the running index, i, randomly selected with uniform distribution from the set $\{1, 2, \cdots, i-1, i+1, \cdots, N\}$; f_m > 0 is a real parameter called *mutation factor*, which controls the amplification of the difference between two individuals so as to avoid search stagnation and is usually taken from the range [0.1, 1].

Step 5: *Recombination operation*

Following the mutation operation, recombination is applied to the population. Recombination is employed to generate a trial vector by replacing certain parameters of the target vector with the corresponding parameters of a randomly generated donor vector.

For each vector, $z_i(k+1)$, an index $rnbr(i) \in \{1, 2, \cdots, n\}$ is randomly chosen using uniform distribution, and a *trial vector*, $u_i(k+1) = [u_{i_1}(k+1), u_{i_2}(k+1), ..., u_{i_n}(k+1)]^T$, is generated with

$$u_{i_j}(k+1) = \begin{cases} z_{i_j}(k+1), & \text{if } randb(j) \leq CR \text{ or } j = rnbr(i), \\ x_{i_j}(k), & \text{if } randb(j) > CR \text{ or } j \neq rnbr(i). \end{cases} \tag{14}$$

In the above equations, *randb(j)* is the *j*-th evaluation of a uniform random number generation with [0, 1] and *CR* is a *crossover* or *recombination rate* in the range [0, 1]. The performance of a DE algorithm usually depends on three variables: the population size *N*, the mutation factor f_m, and the recombination rate *CR*.

Step 6: *Selection operation*

Selection is the procedure of producing better offspring. To decide whether or not the vector $u_i(k+1)$ should be a member of the population comprising the next generation, it is compared with the corresponding vector $x_i(k)$. Thus, if *f* denotes the objective function under minimization, then

$$x_i(k+1) = \begin{cases} u_i(k+1), & if \quad f(u(k+1)) < f(x_i(k)), \\ x_i(k), & \text{otherwise.} \end{cases} \tag{15}$$

In this case, the cost of each trial vector $u_i(k+1)$ is compared with that of its parent target vector $x_i(k)$. If the cost, *f*, of the target vector $x_i(k)$ is lower than that of the trial vector, the target is allowed to advance to the next generation. Otherwise, the target vector is replaced by trial vector in the next generation.

Step 7: *Verification of stop criterion*

Set the generation number for $k = k + 1$. Proceed to Step 3 until a stopping criterion is met, usually G_{max}. The stopping criterion depends on the type of problem.

Each optimization approach was implemented in environment computational Matlab (MathWorks). To illustrate the effectiveness of the optimization procedure several simulations were performed. The program was run on a 3.8 GHz Pentium IV processor with 2 GB of RAM. In the tests, 30 independent runs were made for the optimization method involving 30 different initial trial solutions. In optimization tests, the setup of DE used was the following: $f_m = 0.3$, $CR = 0.8$, the population size *N* was 10 and the stopping criterion G_{max} was 200 generations for the DE.

2.4 Thermophysical Properties

The experimental results for temperature used in this study were obtained from Pérez (1998), whose work presents results for six experiments with different conditions of temperature and relative humidity to banana's drying, which are presented in Tab. 1, where X_0 is initial moisture content (kg_w/kg_{dm}) and X_e is equilibrium moisture content (kg_w/kg_{dm}).

Table 1 – Air drying conditions and parameters used in the experimental tests.

Test	T_e	R	X_0	X_e	t
1	29.9	0.01613	3.43	0.1428	121.9
2	39.9	0.01569	3.17	0.0664	72.0
3	49.9	0.01522	3.21	0.0579	40.8
4	60.2	0.01530	2.96	0.0426	35.3
5	60.5	0.01506	3.04	0.0211	27.8
6	68.4	0.01545	2.95	0.0121	27.6

The heat transfer coefficient is dependent of fluid velocity, fluid properties, surface rugosity, body shape and temperature between the body surface and the fluid. In foods with higher moisture content there are heat and mass transfer difficulting the experimental measure of heat transfer coefficient. Saravacos and Kostaropoulos (1995) studied the heat transfer coefficient observing changes in the heat transfer coefficient from 10 to 200 W/m^{2o}C when the velocity changes from 0.1 m/s to 5 m/s. Thus this coefficient is enough sensitive the small changes in velocity.

The values for heat transfer convective coefficient were obtained based on the Nusselt number,

$$h = \frac{kNu}{d}, \tag{16}$$

where k (kg/m^3) is the air thermal conductivity, d (m) is the diameter of the banana, Nu is the Nusselt number given by $Nu = 0.97 + 0.68Re^{0.52}Pr^{1/3}$, Pr is the Prandtl number, Re is the Reynolds number calculated by $Re = \rho vd/\mu$, ρ (kg/m^3) is the air density and μ (Pa.s) is the air viscosity (Kreith and Bohn, 2003). The numerical simulations were performed for values of the heat transfer convective coefficient, h, calculated from Eq. (16), in the range between 15 W/m^{2o}C and 35 W/m^{2o}C, while the values of air velocities, v, are in the range between 0.33 m/s and 0.39 m/s.

When ones study the drying of products with bigger moisture content, specifically the vegetables and fruits, in the mathematical model is need to consider, besides of heat and mass transfer, the food shrinkage too. Thus the coefficients determinate using such hypothesis has more applicability. The shrinkage phenomenon was included in this work where an empirical equation (Eq. (17))

$$R = \left[0.4721 + 0.1819X_e + 0.1819(\overline{X} - X_e)\right]R_0, \tag{17}$$

developed through an experimental test correlating the banana mean diameter to its moisture content was obtained in Queiroz and Nebra (2001). The mean radius and the moisture content were fitting by a linear regression, whose coefficients of correlation were higher than 0.97. Using the Eq. (17) the radius, R, could be continuously recalculated according to the new average moisture content, $\overline{X}$, and initial radius, R_0. Numerically, the shrinkage was treated like an elastic grid. This means that the number of nodes in the radius was maintained constant and the radial subinterval size was changed at each time step. The shrinkage is strong and fast at surface since X decrease quite fast main at the beginning of the drying.

Note that several authors have derived equations to predict thermophysical properties. Semi-theoretical equations (krokida *et al*, 2001; Maroulis *et al*., 2002) are simple to use however these equations are not always in agreement with experimental data. The functional forms of thermal properties are generally unknown, especially in the case of foods with multiple compositions. A preliminary choice of these functions could be an obstacle to a correct approximation of these thermal properties dependent of the temperature or moisture, even if the parameters of these functions are adjustable. A usual solution consists of representing these functions by empirical polynomials, with orders sufficiently high to correctly represent the properties variations (Chourot *et al.*, 1997; Pérez, 1998). Some authors have proposed replacing polynomials by piecewise linear functions of temperature, and to adjust the parameters by optimization approaches, generally the number of parameters to adjust is relatively large, more than 20, and the convergence of optimization approach is delicate (Jarny *et al.*, 1986; Saad and Scott, 1995).

Analyzing the temperature variation rate with the time in the center of the banana, from the experimental data, it can be observed that the thermal diffusivity must have a behavior of decreasing very accented in the first hours of drying, practically being constant in the next hours. Due to this fact it was proposed the use of a non-linear function dependent of dimensionless average moisture content in section. The parameters were adjusted by inverse method using a Differential Evolution approach, thus the number of parameters for adjust is 3, accordant with

$$\alpha(\overline{X}^*) = \frac{A_1}{A_2^{\overline{X}^*} + A_3}, \tag{18}$$

where apparent thermal diffusivity is dependent of dimensionless average moisture content in the section, $\overline{X}^*$.

3. RESULTS AND DISCUSSION

The proposed approach was analyzed for the case in which 3 parameters (see Eq. 18). The mathematical model described in Eqs. (1) to (8) considers some strict assumptions (homogeneous material and infinite cylinder). To characterize the apparent thermal diffusivity variable as a function of the moisture content, several preliminary analyses were made. The best fitness in the least square sense between the experimental temperatures and the temperatures obtained by mathematical model (Eqs. (1)-(8)) is shown in Tab. 2. Deviations between experimental and simulated temperatures were calculated using the multiple correlation coefficient (Pearson coefficient), in successive trials as,

$$R^2 = 1 - \frac{\sum \left(\tau_0^j - T_0^j(\alpha)\right)^2}{\sum \left(\tau_0^j - \overline{\tau_0^j}\right)^2} \tag{19}$$

where $\overline{\tau_0^j}$ (°C) is the mean experimental temperature of the banana at thermocouple central, $r = 0$, and time indicator j. The R^2 value of 0.9 to 1.0 is considered sufficient for that the apparent thermal diffusivity obtained in this work to be well adjusted with the experimental data, such values are shown in Tab. 2 and the values for goal function are shown in the last column in same table.

In Tab. 2 ones observe, through of R^2 values that the results predicted have a good agreement with experimental values, showing that the apparent thermal diffusivity obtained from the inverse method was fitted by function presented in Eq. (18).

Table 2 – Parameters of the Eq. (18).

Cases	$A_1.10^{11}$	A_2	A_3	R^2	f
1	6.3197	0.3027	- 0.3000	0.9971	0.0064
2	9.7950	0.3674	- 0.3658	0.9969	0.0146
3	11.9875	0.4806	- 0.4799	0.9815	0.0762
4	11.5770	0.5000	- 0.4994	0.9741	0.0985
5	17.7960	0.4622	- 0.4607	0.9937	0.0758
6	19.6654	0.4155	- 0.4141	0.9923	0.0954

Fig. 2 shows predicted and experimental temperatures at the thermal centre ($r = 0$) predicted by Eq. (18) using the parameters of the Tab. 2 for all cases. The predicted temperatures are in excellent agreement with experimental data obtained in Pérez (1998). Predicted temperatures were calculated with parameters estimated under the inverse method using the Differential Evolution. Statistical analysis through of the values of multiple correlation coefficients shows no significant differences between reported and estimated curves (see Tab. 2). In this figure the shrinkage and convective effect at banana surface are included in the mathematical model, so, this is a complete model due the incorporation of physical phenomena in the banana's drying. It is important to observe that in practice, bananas shrink by about 43 ± 47% their original diameter during drying in accordance with Queiroz and Nebra (2001). This fact reveals the importance of including this phenomenon in the theoretical model. The inclusion of shrinkage and convective effect lends more credibility to the apparent thermal diffusivity obtained and presented in Tab. 2.

The minimum and maximum values for apparent thermal diffusivity obtained in this work using Eq. (18), for example, for fourth case were 2.49×10^{-10} (m^2/s) and 1.88×10^{-7} (m^2/s), respectively, with values lower than the obtained by Lima (1999) with minimum equals 1.44×10^{-9} (m^2/s) and maximum equals 2.7×10^{-7} (m^2/s). Considering all cases such differences occur due the different mathematical model adopted in each work.

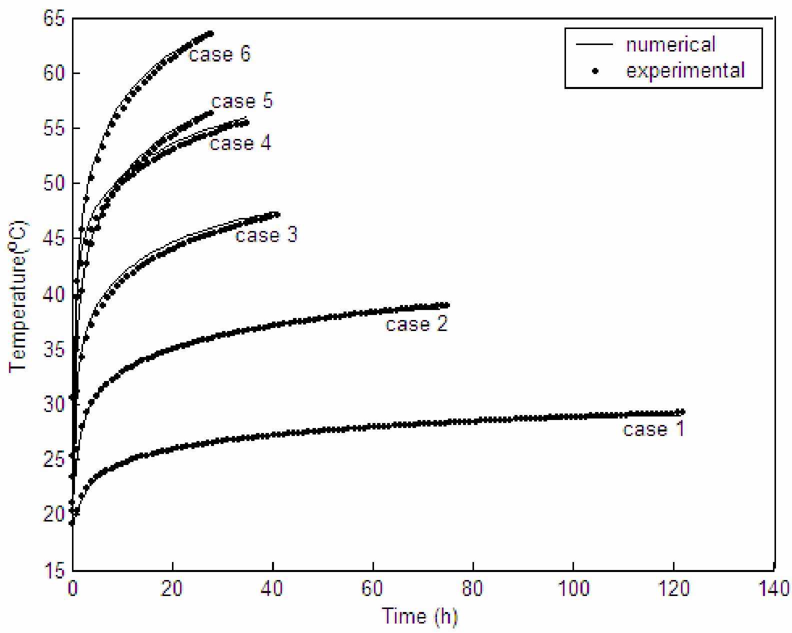

Fig. 2. Experimental validation for the inverse method using the parameters of the Tab. 2 (central temperature, in $r = 0$).

4. CONCLUSIONS

The proposed optimization procedure using Differential Evolution method was successfully applied to the determination of apparent thermal diffusivity dependent of moisture content and/or temperature average in the section radial of the banana during the drying process. A statistical analysis shows no significant differences between the predicted and experimental profiles of temperature at the thermal centre using Eq. (18). The results obtained in this work validate the proposed method as a tool to the determination of apparent thermal diffusivity in the drying temperature range. The proposed procedure can be extended to the determination of other thermophysical properties in different processes like thermal conductivity, and specific heat in drying, wetting, cooling, heating and/or freezing.

The determination of thermophysical properties from an inverse method is an attractive technique both from the experimental and methodological point of view, because of its accuracy and short time for parameters estimation. The higher value obtained in this work to apparent thermal diffusivity was approximately 1.88×10^{-7} (m^2/s) while the lower value was 9.47×10^{-11} (m^2/s) using the Eq. (18). However it is clear that diffusivity depends on several parameters, among other the composition of the product, direction of the fibres and in particular of its temperature. The values provided in this study were obtained without a examination of all these characteristics, our main objective was to validate the inverse method using Differential Evolution optimization approach coupled with a simple 1D heat transfer model besides to obtain a value acceptable for thermal diffusivity. The shrinkage and convective effects assumptions in the mathematical model provides greater reliability on the calculated thermal diffusivity. The determination of thermophysical properties from an inverse method is an attractive technique obtaining the apparent thermal diffusivity with acceptable accuracy for two functions investigated.

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