NUMERICAL PROCEDURE TO OBTAIN GENERALIZED DARCY MODELS FOR NON-NEWTONIAN POWER-LAWFLOWS THROUGH POROUS MEDIA: APPLICATION TO A THROAT BETWEEN SPHERES

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Abstract. *The injection of polymeric liquids in porous media for the oil recovery in reservoirs is becoming an important technique in this process. The classical Darcy law is widely used to represent the relation between pressure drop and flow rate for Newtonian fluids in porous media. An interesting feature of this model is the fact that rheology (viscosity) and pore structure (permeability) are independent components of mobility. This model is generalized in the present paper to account for non-Newtonian effects specially concerning extensional flows which are known to be predominant in porous media. A numerical procedure is created to capture the mobility sensitiveness to rheological and pore structure parameters. The idealized pore geometry tested is a throat between two spheres of equal diameters. This contraction-expansion geometry provides the desired extensional flow. The non-Newtonian constitutive equation employed is a power-law model. Some of the hypothesis considered to model the non-Newtonian flow in this idealized porous media were tested in a Finite Element code which has supported the assumptions. Although we have not come to a closed form for the mobility we have obtained a numerical constitutive model which can be adapted to different idealized pore structures and different constitutive models for the non-Newtonian behavior of the fluid..*

Keywords*: displacement of a liquid, non-Newtonian Fluids, bypass flow, finite element method, surface tension.*

1. INTRODUCTION

1.1- Motivation and objective

Oil is energy on demand and it has increasing prices. That makes the industry turns its attention to new and not yet explored fields, as heavy oil ones, high viscosity ones, which in the past were not viable. In Brazil, most of the oil fields explored until now are heavy oil fields. The advances in technology in the industry, such as drilling fluids, new kind of bits and casings, wireline logging tools increasing the recovery factor of this fields.

Reservoir engineering studies, reservoir characterization, properties of the rocks and of the fluids that flow through them together with the way fluids interact inside these rocks, are phenomenological studies that are becoming more intense with this new scenario stated above. All this effort has the objective to maximize hydrocarbons production with lower prices.

Reservoirs formed by gravitational flow in deep waters contains most of the oil reserves of oil in the Brazilian Continental Edge, specially in Campos Basin.

The distribution and flowing of fluids inside the reservoir are mainly controlled

(1) by the shape (Geometry) of the sedimentary bodies and

(2) how these bodies are arranged

Because of that, it is necessary that, during the modeling procedures of reservoirs, every production unit (zone or sub-zone) be presented using parameters that describes accordingly its geometrical and architectural characteristics.

All reservoir models are made from subsurface data (seismic, wireline logging, cores) extracted directly from the reservoir, complemented by conceptual geologic models and dimensional parameters, obtained from analog systems (ex. depositional systems). The main reason to use analog systems is the possibility to fulfill the spaces between the subsurface data that comes from the distance between wells (which can be about many kilometers in offshore deepwater fields) and by the limits of seismic. The correct integration between subsurface data with analog systems data means the increasing in the assurance of geological models used in the characterization procedures, modeling and reservoir management.

In some applications of secondary and tertiary oil recovery is sometimes useful to inject a viscoelastic fluid to displace the oil inside the porous media. However, the porous media is, generally, made of pores that have different diameters during the trajectory of the fluid and, therefore this fluid experiences a type of flow that has a strong extensional character. Astarita (1979) and Thompson and Souza Mendes (2005) present interesting discussions on flow classification. The mechanical power loss of a liquid due to viscous action while flowing through channels is of the form $Q\Delta p$, where Q is the volume flow rate and Δp the pressure drop. Since viscoelastic fluids are generally extensional-thickening the classical Darcy law is not able to capture the interaction between the fluid and the porous media as it does when a Newtonian fluid flows inside a porous media.

In this paper our goal is to develop a numerical method to obtain a modified Darcy model that takes into consideration non-Newtonian aspects of the fluid that flows inside a porous media. More specifically we imagine an ideal porous media, with a pre-determined converging-diverging geometry and develop an algorithm that has the mobility of the porous media as an output. The present article deals with a constitutive relation between the pressure gradient and the average velocity. Besides that, it carries information about the fluid behavior in the extension.

This relation is developed in two steps. Firstly the pressure drop / flow rate relationship for an ideal pore channel is obtained. Then a capillary model theory is applied to obtain the sought-for constitutive relation. The proposed relation needs to be validated comparing pressure drop / flow rate results with experimental data obtained in a geometry similar to the one used in theoretical model.

2.THE POROUS MEDIA MODEL

2.1. Porous media characterization

Figure (1a) represents a portion of frontal area *A* and length *2L* of an idealized homogeneous porous media which is disposed in such a way that it allows the passage of the fluid only in one (*z*) direction. In this area *A* there is a number N of pores. The elementary pore, in the present work, is idealized as a convergent-divergent spherical throat as despicted in Fig. (1b).

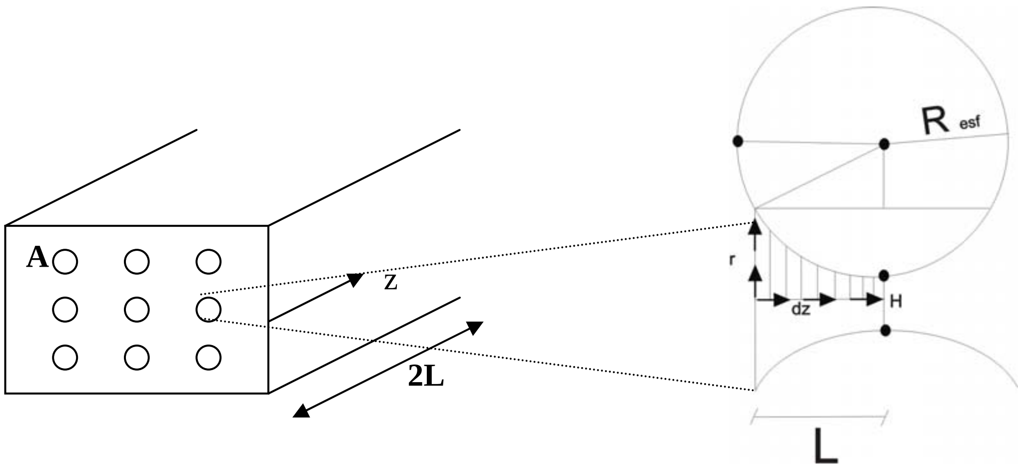


Figure 1.1 – Schematic of the idealized porous media.

The equation of the pore curve is given by

$$R_{esf}{}^2 = (z-L)^2 + (R(z) - H - R_{esf})^2 \quad (1)$$

Where *L* is half of the length of the porous element in *z* direction and $R(z)$ is the position of the solid surface. The geometrical restrictions are

$$z \le L \le R_{esf} \quad (2)$$

$$0 \le L - z \le R_{esf} - z \le R_{esf} \quad (3)$$

Solving for $R(z)$ and chosing the lower side, we have

$$R(z) = H + R_{esf} - \sqrt{R_{esf}^2 - (L-z)^2} \tag{4}$$

and the volume of one half-pore ($\forall_1$) is given by

$$\forall_1 = \pi L \left\{ \left(H + R\_esf\right)^2 + (R\_esf)^2 - \frac{L^2}{3} - (H + R\_esf)[\sqrt{(R\_esf)^2 - L^2} + (R\_esf)^2 / L * (\frac{\pi}{2} - \cos^{-1}\frac{L}{R\_esf})] \right\} \tag{5}$$

Therefore we can have a relation between microscale and macroscale through the porosity ϕ of this porous media

$$\phi = \frac{N\forall_1}{AL} \tag{6}$$

Besides that, the relation between global flow rate, Q ,and local flow rate, Q_1, as a function of the porosity is given by

$$Q = NQ_1 = \frac{AL\phi}{\forall_1} Q_1 \tag{7}$$

2.1. Hypothesis

2.2.1 Incompressible material

$$u(z)R^2(z) = C \Rightarrow \frac{du}{dz}(z) = -2u(z)\frac{R'(z)}{R(z)} \tag{8}$$

where $u(z)$ is the mean velocity at position *z*, *C* is a constant, and $R'(z)$ is the derivative of *R* with respec to *z*.

2.2.2 Lubrication approximation

$$Q_1 = -\frac{\pi R^4(z)}{8\eta(z)}\frac{dp}{dz}(z) \Rightarrow u(z) = -\frac{R^2(z)}{8\eta(z)}\frac{dp}{dz}(z) \tag{9}$$

where Q_1 is the flow rate through one pore, $\eta(z)$ is the viscosity of the material at position *z*, and $\frac{dp}{dz}(z)$ is the pressure gradient. This approximation also carries a symmetry of the problem in *z* = *L*, indicating that the mean velocity and pressure gradient have the same values for *z* and *2L-z*.

2.2.2 Constitutive equation for the liquid

$$\eta(z) = K\left|\frac{du}{dz}\right|^{n-1} \tag{10}$$

where *K* is a consistency index and *n* is a behavior index.

2.3. Mathematical formulation

Equation (9) can be integrated through half of the domain

$$\frac{p(0)-p(L)}{L}=\frac{\Delta P}{L}=\frac{1}{L}\int_0^L \frac{8\eta(z)u(z)}{R^2(z)}dz \qquad (11)$$

Given a flow rate, Q, the geometrical parameters, and the porosity, ϕ, we can integrate, numerically, Eq.(11). Therefore, we can find the mobility *M* of the porous media calculating the following ratio

$$M=\frac{\frac{Q}{A}}{\frac{\Delta P}{L}}=\frac{Q}{A}\frac{L}{\int_0^L \frac{8\eta(z)u(z)}{R^2(z)}dz}$$

3. HYPOTHESIS VALIDATION

The validation of the hypothesis made was done by a finite element galerkin numerical analysis applied at half of the physical domain for the case where $L=R_{esf}$. Figure (3.1) shows a representative mesh of the problem. Figure (3.2) shows a comparison between the mobility obtained by for the domain between $0\le z\le 2L$ and half of this, $0\le z\le L$. As expected, the mobility for the Newtonian fluid is constant while for non-Newtonian fluids, the mobility varies whith the pressure loss. It can also be seen that there is a certain point above which the mobility changes its general tendency.

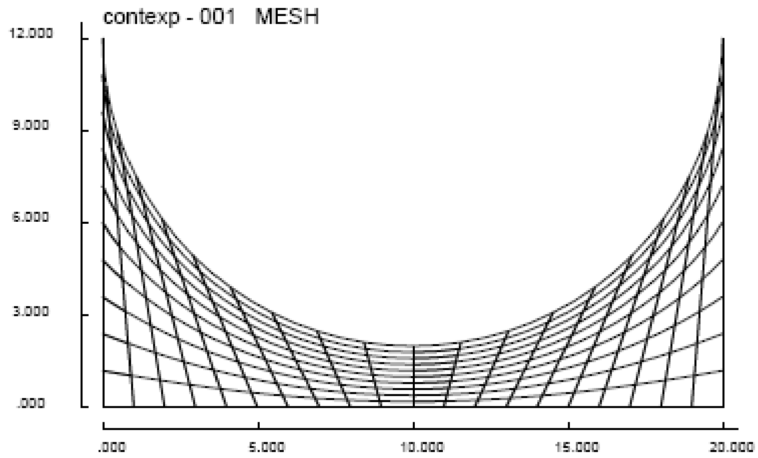


Figure 3.1 – Representative mesh of the half of the domain indicated in Figure 1.1

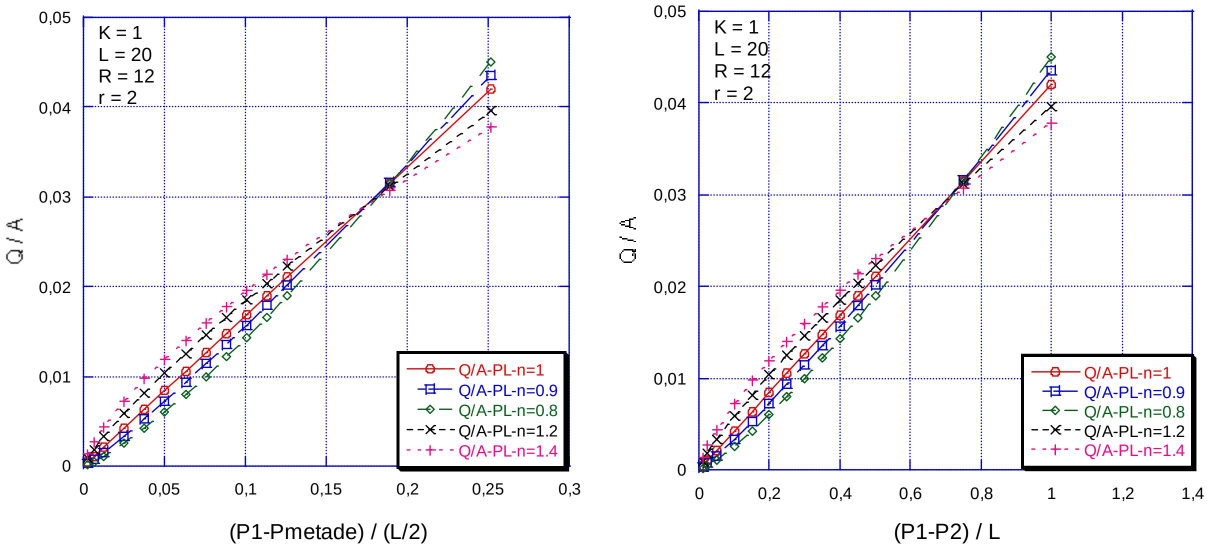
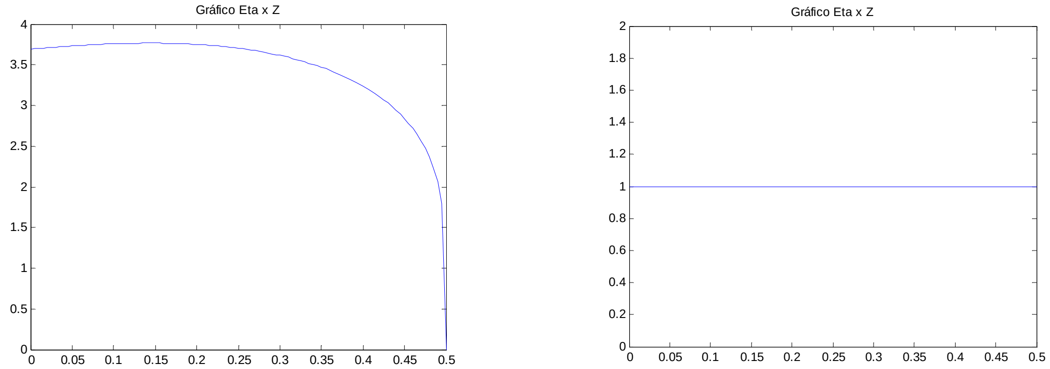


Figure 3.2 – Mobility for (a) the hole domain; (b) half of the domain.

4. PRELIMINARY RESULTS

Figure 4.1 shows the plot of the viscosity with the position inside de pore. As expected, the viscosity of the Newtonian fluid Fig. 4.1(b) (n=1) is constant over the entire range. On the other hand, for an extensional-thickening fluid as the one represented by Fig. 4.1 (a) (n=1.2), there is a variation on the resistance it opposes to the flow, since the mean rate of deformation is not constant throughout the pore.



(a) n=1.2 (b) n=1

Figure 4.1 – Variation of viscosity with the longitudinal position z (a) for an extensional power-law model with the behavior index n = 1.2; (b) for a Newtonian fluid.

This result is better explored in Fig. (4.2) where there is a representation of the variation of the viscosity for a diversity of power-law fluids. As it can be seen, for viscosity-thinning fluids, Fig.(4.2a), the viscosity increases for higher values of *z*; while the opposite tendency is shown when a viscosity-thickening fluid passes through the pore, Fig. (4.2b,c). This happens because in this range of *z*, the deformation rate decreases when we increase *z*.

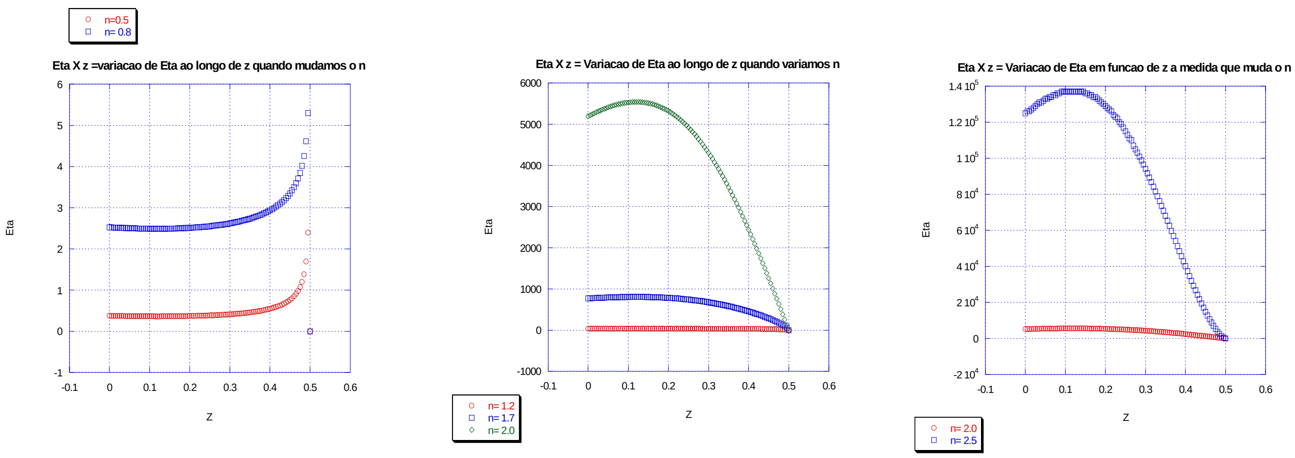


Figure 4.2 – Variation of viscosity with the longitudinal position *z* (a) comparison between two viscosity-thinning fluids, n=0.8 and n=0.5, (b) comparison between three viscosity-thickening fluids, n=1.2 , n=1.7, and n=2.0; (c) comparison between two viscosity-thickening fluids, n=2.0 and n=2.5.

The other preliminary analysis done in the present work is the study of the variation of the mobility with the geometry of the elementary pore. For this purpose, we decided to compute the sensitiviness of a dimensionless mobility with respect to dimensionless representative aspect ratios of the problem. Besides the fact that the porous space consttitutes a void between two spheres, the geometry of the pore is characterized by two dimensionless numbers, namely $L^* = \dfrac{L}{R_{esf}}$, and $H^* = \dfrac{H}{R_{esf}}$. The dimensionless mobility is formed, in the present work, by the mobility obtained for the non-Newtonian fluid divided by the correspondent Newtonian one. Figures (3.4) and (3.5) Mobility changes when we change these geometric numbers.

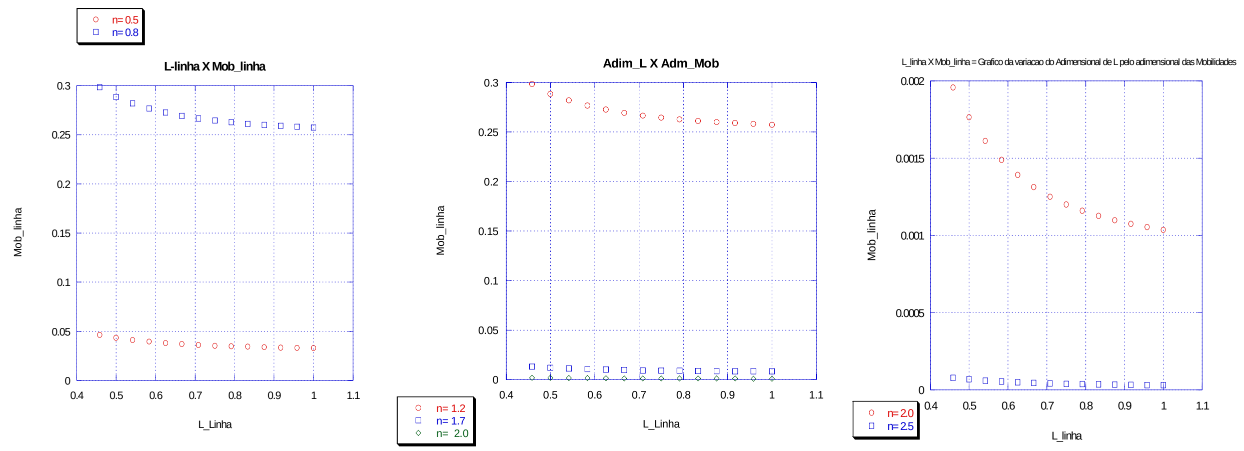


Figure 4.3 – Variation of the dimensionless mobility as a function of L^* (a) comparison between two viscosity-thinning fluids, n=0.8 and n=0.5, (b) comparison between three viscosity-thickening fluids, n=1.2 , n=1.7, and n=2.0; (c) comparison between two viscosity-thickening fluids, n=2.0 and n=2.5.

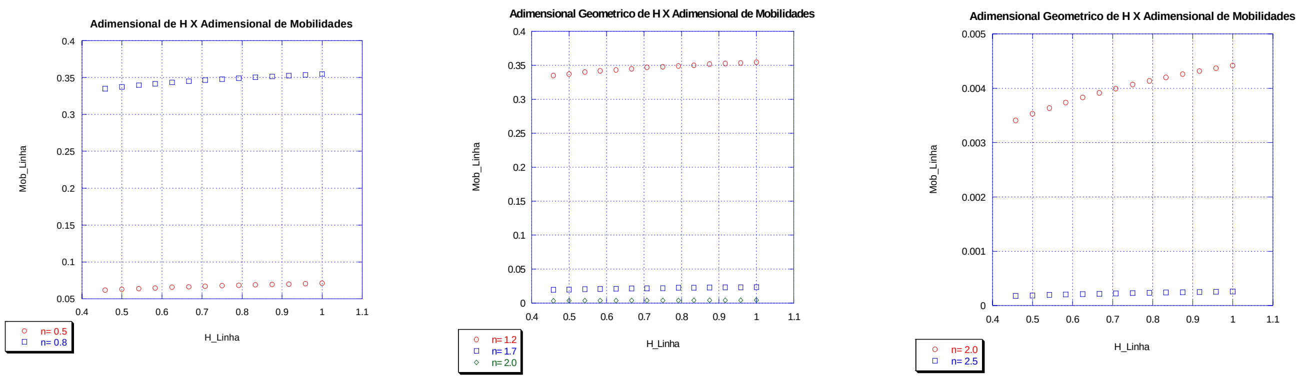


Figure 4.4 – Variation of the dimensionless mobility as a function of H^* (a) comparison between two viscosity-thinning fluids, n=0.8 and n=0.5, (b) comparison between three viscosity-thickening fluids, n=1.2 , n=1.7, and n=2.0; (c) comparison between two viscosity-thickening fluids, n=2.0 and n=2.5.

5. CONCLUSIONS

Probably the biggest problem to make a numerical evaluation of this problem is the extensional viscosity data due to its extreme difficulty to obtain a purely extensional flowing.

Typical flowing in the porous media is in a divergent-convergent way. However, flowing through real porous media cannot be classified as shear flowing. Convergent-divergent passages claims a mainly extensional kinematics. On the other hand, it is known that non newtonian fluids has an extensional viscosity that drastically increases as the extensional index. So we can imagine that flowing of these fluids through the porous media, most of the energy loss will be due to mechanical behavior of the extension.

A theoretical simple model was used to obtain a constitutive relation for flowing in extensional thickening fluids through the porous media. The Non Newtonian behavior of the fluid is considered as a generalized Newtonian fluid with Power Law viscosity and extensional index.

6. ACKNOWLEDGEMENTS

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