EQUATION ERROR METHOD FOR AIRCRAFT PARAMETER ESTIMATION

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Abstract. *In recent works the authors have been successful in applying the output and the filter error methods for aircraft parameter estimation. Basically, these 2 methods require the integration of the state space model, in order to determine the system outputs. These outputs are then compared with their predicted values, thereby defining the prediction error, which drives the estimation procedure. The actual parameter estimates are obtained by an optimization procedure, in the case of the output error method, or by the Kalman filter in the filter error method. In this paper a different, complementary and simpler approach based on the equation error method is employed: the aerodynamic coefficients are parameterized directly as a linear regression of the unknown parameters. Thence the problem boils down to a least squares one, and the powerful SVD (Singular Value Decomposition) method can be brought to bear. Experimental results, using flight data for the lateral-directional maneuver, are presented and discussed. It is concluded that preprocessing of the data, for removing instrument biases, is necessary and care must be taken while evaluating the parameter estimates quality.*

Keywords: *Equation Error Method, Aircraft Identification, Parameter Estimation.*

1. INTRODUCTION

Experimental flight test results concerning parameter and state estimation have been reported recently by the authors, see Mendonça and Hoff (2003), Mendonça *et al*. (2004), Hemerly and Mendonça (2005), Hemerly *et al*. (2006) and Mendonça *et al*. (2007). These results were obtained by 2 different techniques: output error method and filtering approach. A detailed account about these techniques can be found in the recent book Jategaonkar (2006), and a common feature they share is the use of dynamic equations for modeling the data generating system, which must be integrated during the estimation procedure. One of the main benefits of these approaches is the possibility of estimating initial state conditions as well, and then the identified model integration can match actual measured responses from the aircraft.

A complementary and simpler technique for aircraft parameter estimation is known in the literature as equation error method, see Jategaonkar (2006) and Morelli (2006): no dynamic equation is required, since the aerodynamic coefficients are parameterized directly as a linear regression of the unknown parameter. Hence any variant of the Least Squares method can be used. In the equation error method the theoretical global aerodynamic coefficients are fitted to the estimated ones by calculating a pre-determined set of aerodynamic derivatives. On the other hand, the output error method estimates the derivatives through a cost function based on the model and measured outputs. Consequently, this requires care while evaluating the equation error identification performance: the user can not simply insert the estimated parameters into the dynamic model and expect the prediction error to be small, since any mismatch will cause divergence in the integration procedure for generating the system predicted output. However, if the experimental data has been properly preprocessed, then the equation error method is bound to produce estimates which match the true aerodynamic coefficients better than output equation methods.

The previous remarks provide guidelines for adequately using the equation error method in aircraft applications: the experimental data must be preprocessed for removing biases and any other factors which may spoil the least squares estimates, and the quality of the estimated parameters should not be evaluated as if they had been obtained by the output error method.

In this work the equation error method is employed for parameter estimation with experimental flight data, for the lateral-directional maneuver. For ensuring good estimates, before applying the method a flight path reconstruction procedure is used, in order to remove biases from the instrumentation. The least squares method, based on the SVD approach, can then be applied. The following contribuitions can be stated: 1) employment of a mode more complete than that presented in Jategaonkar (2006); 2) use of SVD instead of the standard LS, and 3) performance evaluation with real flight data and performance comparison with parameter estimates obtained via the output error technique based on the Levenberg-Marquardt method.

2. AERODYNAMIC FORCE AND MOMENT COEFFICIENTS

The basic information required for calculating the aerodynamic coefficients are: measured accelerations, angular velocities, thrust and structural parameters, such as CG position.

The measured accelerations are transferred to CG by using

$$
\begin{aligned}
a_{xCG} &= a_{xm} + \left(q^2 + r^2\right).X_a - \left(pq - \dot{r}\right).Y_a - \left(pr + \dot{q}\right).Z_a \\
a_{yxCG} &= a_{ym} - \left(pq + \dot{r}\right).X_a + \left(p^2 + r^2\right).Y_a - \left(qr - \dot{p}\right).Z_a \\
a_{zCG} &= a_{zm} - \left(pr - \dot{q}\right).X_a - \left(qr + \dot{p}\right).Y_a + \left(p^2 + q^2\right).Z_a
\end{aligned} \tag{1}
$$

and then the aerodynamics force coefficients in the body frame are given by

$$
\begin{aligned}
CX_{AER} &= \frac{mg}{\bar{q}S_W}\left\{a_{xCG} - \left[\left(\frac{F_{Tx}}{mg}\right)_{right`eng.} + \left(\frac{F_{Tx}}{mg}\right)_{left\ eng.}\right]\right\} \\
CY_{AER} &= \frac{mg}{\bar{q}S_W}\left\{a_{yCG} - \left[\left(\frac{F_{Ty}}{mg}\right)_{right`eng.} + \left(\frac{F_{Ty}}{mg}\right)_{left\ eng.}\right]\right\} \\
CZ_{AER} &= \frac{mg}{\bar{q}S_W}\left\{a_{zCG} - \left[\left(\frac{F_{Tz}}{mg}\right)_{right`eng.} + \left(\frac{F_{Tz}}{mg}\right)_{left\ eng.}\right]\right\}
\end{aligned} \tag{2}
$$

which are converted to components in the wind frame by using

$$
\begin{bmatrix} -CD \\ CY \\ -CL \end{bmatrix} = S.\begin{bmatrix} CX_{AER} \\ CY_{AER} \\ CZ_{AER} \end{bmatrix}, \text{ where } S = \begin{bmatrix} \cos\alpha\cos\beta & \sin\beta & \sin\alpha\cos\beta \\ -\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \tag{3}
$$

As far as the moment coefficients are concerned, they are given directly in the body frame as

$$
\begin{aligned}
Cl &= \frac{I_{XX}}{\bar{q}S_W b_w}\left[\dot{p} - \left(\frac{I_{YY} - I_{ZZ}}{I_{XX}}\right).qr - \frac{I_{XZ}}{I_{XX}}\left(pq + \dot{r}\right)\right] - \left[\left(\frac{M_{Tx}}{\bar{q}S_W b_w}\right)_{right\ eng.} + \left(\frac{M_{Tx}}{\bar{q}S_W b_w}\right)_{left\ eng.}\right] \\
Cm &= \frac{I_{YY}}{\bar{q}S_W b_w}\left[\dot{q} - \left(\frac{I_{ZZ} - I_{XX}}{I_{YY}}\right).pr - \frac{I_{XZ}}{I_{YY}}\left(r^2 - p^2\right)\right] - \left[\left(\frac{M_{Ty}}{\bar{q}S_W \bar{c}}\right)_{right\ eng.} + \left(\frac{M_{Ty}}{\bar{q}S_W \bar{c}}\right)_{left\ eng.}\right] \\
Cn &= \frac{I_{ZZ}}{\bar{q}S_W b_w}\left[\dot{r} - \left(\frac{I_{XX} - I_{YY}}{I_{ZZ}}\right).pq - \frac{I_{XZ}}{I_{ZZ}}\left(\dot{p} - qr\right)\right] - \left[\left(\frac{M_{Tz}}{\bar{q}S_W b_w}\right)_{right\ eng.} + \left(\frac{M_{Tz}}{\bar{q}S_W b_w}\right)_{left\ eng.}\right]
\end{aligned} \tag{4}
$$

Now that the aerodynamic force and moment coefficients are calculated from measured data, the parameter estimation problem can be written as a linear regression problem, by using the expansion

$$
\begin{aligned}
CD &= CD_0 + K_1.CL + K_2.CL^2 + C_{Dds}.\delta_s \\
CY &= CY_0 + C_{Y\beta}.\beta + C_{Yp}.p_n + C_{Yr}.r_n + C_{Yda}.\delta_a + C_{Ydr}.\delta_r + C_{Yds}.\delta_s \\
CL &= CL_0 + C_{L\alpha}.\alpha + C_{Lq}.q_n + C_{L\alpha dot}.\dot{\alpha}_n + C_{Lde}.\delta_e + C_{LdiH}.\delta_{iH} + C_{Lds}.\delta_s \\
Cl &= Cl_0 + C_{l\beta}.\beta + C_{lp}.p_n + C_{lr}.r_n + C_{lda}.\delta_a + C_{ldr}.\delta_r + C_{lds}.\delta_s \\
Cm &= Cm_0 + C_{m\alpha}.\alpha + C_{mq}.q_n + C_{m\alpha dot}.\dot{\alpha}_n + C_{mde}.\delta_e + C_{mdiH}.\delta_{iH} + C_{mds}.\delta_s \\
Cn &= Cn_0 + C_{n\beta}.\beta + C_{np}.p_n + C_{nr}.r_n + C_{nda}.\delta_a + C_{ndr}.\delta_r + C_{nds}.\delta_s + C_{n\beta dot}.\dot{\beta}_n
\end{aligned} \tag{5}
$$

where the subscript "n" stands for normalized value. The parameter vector to be identified is then composed by 40 parameters, i.e.,

$$\Theta = [CD_0\ K_1\ K_2\ C_{Dds}\ CY_0\ C_{Y\beta}\ C_{Yp}\ C_{Yr}\ C_{Yda}\ C_{Ydr}\ C_{Yds}\ CL_0\ C_{L\alpha}\ C_{Lq}\ C_{L\alpha dot}\ C_{Lde}\ C_{LdiH}\ C_{Lds}\ Cl_0\ C_{l\beta}\ C_{lp}\ C_{lr}\ C_{lda}\ C_{ldr}\ C_{lds}\ Cm_0\ C_{m\alpha}\ C_{mq}\ C_{m\alpha dot}\ C_{mde}\ C_{mdiH}\ C_{mds}\ Cn_0\ C_{n\beta}\ C_{np}\ C_{nr}\ C_{nda}\ C_{ndr}\ C_{nds}\ C_{n\beta dot}\ ]^T \tag{6}$$

Hence the problem can then be summarized as follows: suppose N readings are available. Then from equations (5) and (6) we obtain a linear regression model

$$Y = M.\Theta \text{ , where } Y \in R^{6Nx1} \text{ , } M \in R^{6Nx40} \text{ and } \Theta \in R^{40x1} \tag{7}$$

which can be solved for the unknow parameter vector Θ by using the SVD approach for the underlying least squares problem. See Golub and Van Loan, (1984), for details about the SVD computation.

3. EXPERIMENTAL RESULTS FOR THE LATERAL-DIRECTIONAL MANEUVER

The parameter estimation problem is now taylored for the lateral-directional maneuver, in which case there exist only the aerodynamic coefficients CY, Cl and Cn. Therefore, there would be 22 parameters to estimate. However, since in this experiment the spoiler is not activated, the parameters C_{Yds}, C_{lds} and C_{nds} are set to zero, hence only 19 parameters has to be identified in (7). The equation error method was coded in C and for visualization Matlab graphics were used.

The signals regarding the control surfaces and the angular rates for roll and yaw are shown in Fig. 1, were the vertical scales are normalized since the data comes from an Embraer regional jet, and is then proprietary.

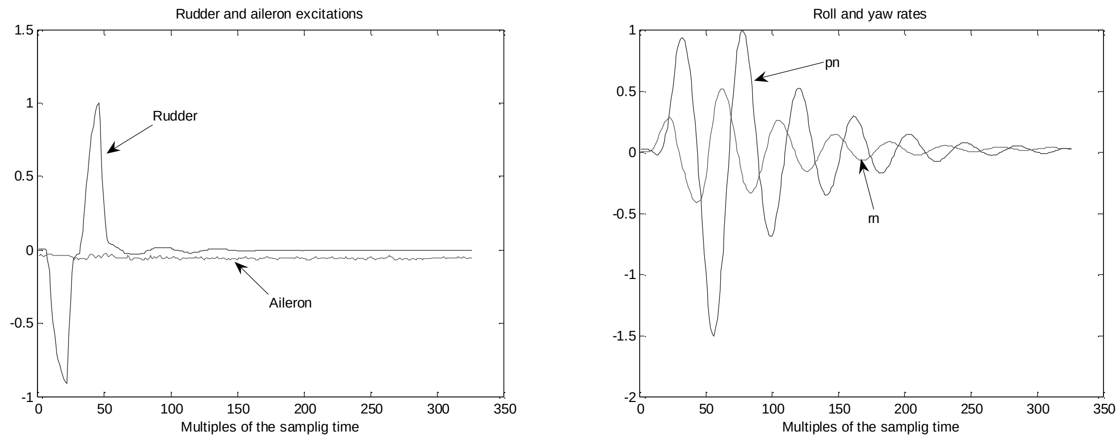

Figure 1. Rudder and aileron excitations and roll and yaw angular rates for the lateral-directional maneuver.

By using the SVD method to solve (70, the following parameter vetor estimate was obtained,

$$\hat{\Theta}_{SVD} = (0.0037, -1.2644, -0.4599, -0.8395, 0.0836, 0.2757, 0.0008, -0.1808, -0.2989, 0.4511, 0.0300, 0.0518, -0.0012, 0.3166, 0.1068, -0.1828, 0.0019, -0.1974, -0.1723) \tag{8}$$

By using the estimate (8), the prediction errors can be evaluated by using (7). The results for aerodynamic coefficients CY, Cl and Cn are shown in Fig. 2.

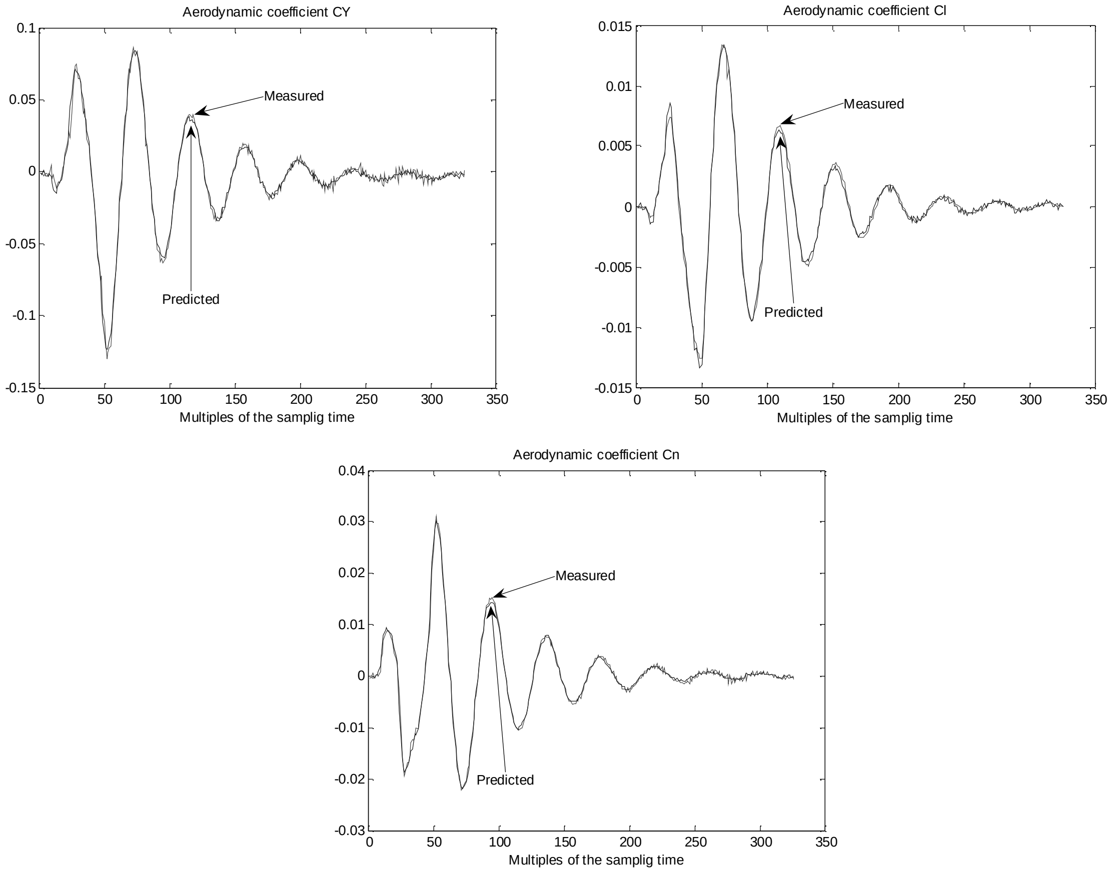

Figure 2. Measured and predicted values for the aerodynamic coefficients CY , Cl and Cn .

From Fig. 2 it is concluded that the prediction errors are small, as it should indeed be if the true data is accordingly to (5) and that the SVD is correctly implemented. A more significant test is to investigate if the parameter estimates given in (8) also provide good fitting for the lateral-directional dynamic model, i.e., if they can be also use to match actual aircraft measured responses. As already mentioned before, the problem here has to do with initial conditions: any mismatch can produce large errors during the integration procedure. Hence care must be taken: here we first run the output error method based on Levenberg-Marquardt algorithm for fitting the experimental data, which provides estimates for the parameters and also initial conditions for the states. Then, the initial state conditions are frozen and the output error parameter estimates are replaced by the equation error estimates (8). The behaviours for yaw rate and roll angle are shown in Fig. 3.

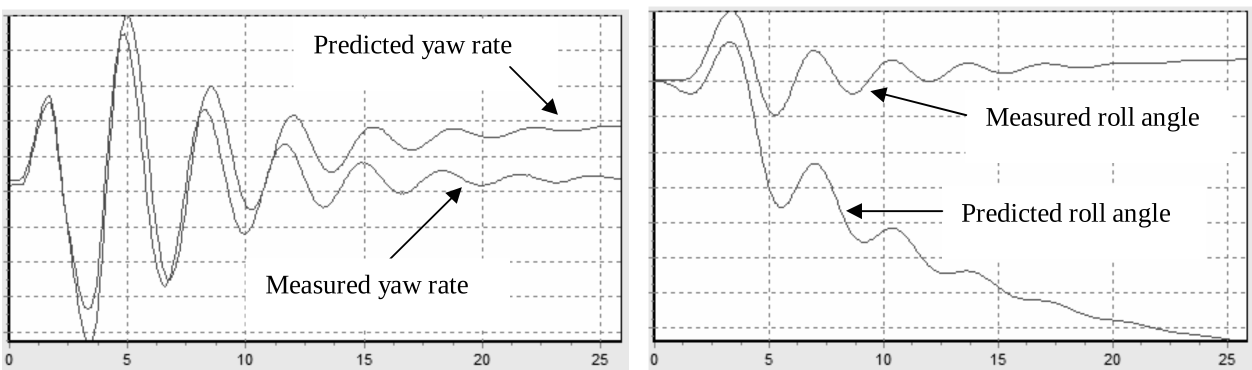

Figure 3. Measured and predicted values of yaw rate and roll angle (horizontal scale in seconds), when the equation erro estimates are used for performing output prediction.

From Fig. 3 it is concluded that the prediction error for the roll angle is worst than that for yaw rate, what is expected since the predicted roll angle is obtained through a double integration procedure. Hence any mismatch will produce a growing prediction error. This indicates what must be done to reduce the prediction error in Fig. 3: the equation error parameters must now be frozen and the output error method must be ran again, in order to recalculate the state initial conditions and some other bias. By doing so, the graphics in Fig. 4 are obtained.

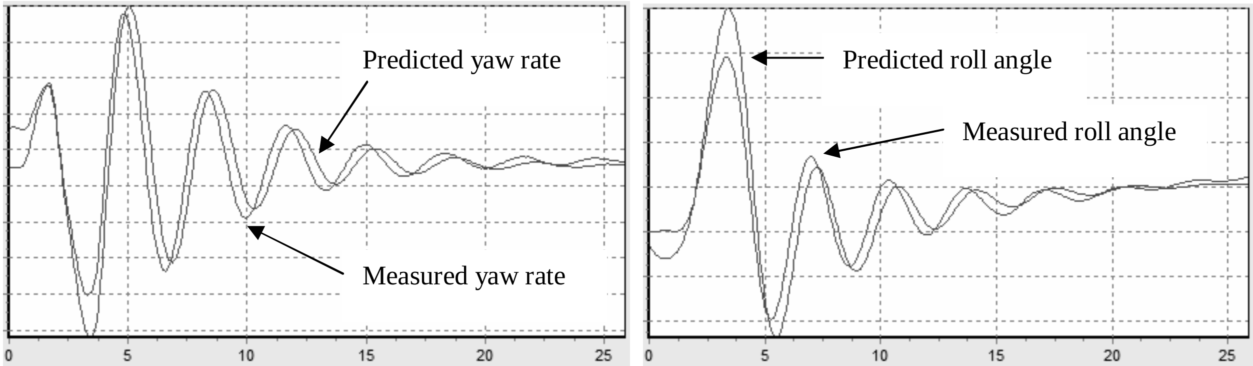

Figure 4. Same variables as in Fig. 3, but by using the output equation error method to refine the inititial state estimates (horizontal scale in seconds).

The differences between figure 3 and 4 illustrates clearly a warning put forward by Morelli (2006), concerning how to evaluate the equation error performance: it is incorrect to blame the equation error estimates for the poor model fitting shown in Fig. 3, since these estimates are being used out of context. More precisely, the context here is defined by equations (5), (6) and (7), and not by model fitting capability.

4. CONCLUSIONS

This paper used experimental flight data for evaluating the performance of the equation error method, in a lateral-directional maneuver. Data from an Embraer regional jet was employed and the least squares problem was solved via SVD. The small prediction errors indicate good performance. Then another use of the equation error estimate is considered: its model fitting capability, i.e., for actually matching measured aircraft outputs. Here some care must be exercised: large prediction errors can be produced due to inadequate state initial conditions, which are not estimated by the equation error method. The solution adopted here is to employ the output error method based on Levenberg-Marquardt method to estimate the state initial conditions, while the equation error parameters are frozen. By doing so, the mismatch between measured and predicted outputs are considerably reduced.

5. ACKNOWLEDGEMENTS

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