EVALUATION OF BOUNDARY CONDITIONS FOR AEROACOUSTICS

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Abstract. *The present work describes a numerical investigation of boundary conditions for computational aeroacoustics (CAA) problems. Such approach is crucial for developing aeroacoustics codes. Initially an overview of boundary conditions suited for CAA is presented in order to introduce the numerical activities performed herein.*
The Linearized Euler Equations (LEE) have been discretized with optimized high order finite difference schemes taking into account usual conditions of consistency, stability and convergence. The spatial discretization scheme is known as DRP (Dispersion-Relation-Preserving) which seems to be suitable for aeroacoustics simulations. To advance the solution in time, a 3rd order Runge-Kutta scheme is applied. In order to validate the numerical scheme, an initial disturbance is employed, which is then propagated in a 2D Cartesian mesh. At the boundaries of the computational domain the boundary conditions equations should permit that the waves leave the domain without reflection. In this work boundary conditions of Radiation and Outflow type have been used. The numerical scheme is deeply discussed shedding light to the relevant parameters which are necessary to achieve a good aeroacoustic simulation. The numerical results generated in this work are compared with the exact analytical solutions showing a good agreement.

Keywords*: boundary conditions, linearized Euler equations, numerical schemes, aeroacoustics.*

1. Introduction

It's well known that in Computational Fluid Dynamics (CFD), as traditionally is performed, the governing equations are solved with spatial and temporal numerical schemes in order to represent the derivatives. Besides that, additional equations arise to describe boundary conditions since the solution is obtained in a limited fluid domain. This is not different in the new area of Computational Aeroacoustics (CAA), however with more severe restrictions when dealing with boundary conditions. Indeed, at the boundaries of the limited acoustic domain a properly set of boundary conditions should take place in order to allow the radiated sound waves and the convected vorticity and entropy waves to leave the computational domain without reflection. In the aeroacoustics solution, such kind of reflecting waves, inside the domain, leads to numerical instabilities which surely will jeopardize the final results.

As stated by Tam (1997), in CAA, numerical boundary conditions are often developed for idealized model problems. In practical applications, they must be modified or extended to account for the presence of a nonuniforrn and sometimes unknown mean flow. In this way, many authors devoted special attention for understanding and to modify boundary conditions for aeroacoustics purposes.

In the literature, some authors prefer to classify and to group boundaries conditions for CAA. One of these classifications is also presented by Tam (1997) as:

(1) Radiation boundary conditions.
(2) Outflow boundary conditions.
(3) Wall boundary conditions.
(4) Impedance boundary conditions.
(5) Radiation/outflow boundary conditions with incoming acoustics or vorticity waves
(6) Radiation boundary conditions for ducted environments.

It's also clear that the use of one type of boundary conditions is strictly related to the numerical simulation pursued. In this work the focus of the discussion will be on boundary conditions of type (5) since it appears to be unique to CAA computations. Details about the other boundary conditions can be found in the papers of Tam & Dong (1994), Ozyoruk & Long (1996), Givoli (1991) among others.

When dealing with boundary conditions of radiation and outflow types, it's important to keep in mind for what purpose they have been derived for. In summary, these boundary conditions were created based on asymptotic solutions of the Linearized Euler Equations (LEE) by using Fourier-Laplace transforms as proposed by Tam & Webb (1993). Such approach was sought because the LEE, when applied for aeroacoustics codes, could support three types of disturbances been propagated through the computational domain. They are the acoustic waves, the vorticity waves and the entropy waves. Obviously, for aeroacoustics codes, the most important wave pattern is the acoustic. These acoustic waves propagate at sound speed relative to the mean flow and must leave any boundary of the computational domain without reflections. However, as said before, vorticity and entropy waves are commonly present in a CFD calculation. The vorticity as well as the entropy waves are frozen patterns advected downstream by the mean flow and must also leave an outflow boundary of the computational domain without spurious reflexions. Because of the presence of the three types of wave disturbances, each having distinct propagation characteristics, the outgoing disturbances present at outflow boundaries could be very different. Depending on the application, boundary conditions for aeroacoustics should deal not only with acoustic waves leaving the domain, but also with vorticity and entropy waves or any combination among them.

In this field was natural, for some researches, to develop asymptotic or characteristics solutions for the LEE in order to construct a set of boundary conditions. These solutions are essentially applied for farfield sound evaluations, where the sources are sufficiently far from receivers. Radiation boundary conditions were derived to deal only with acoustic waves out coming from the noise source. On the other hand, Outflow boundary conditions could deal not only with traveling acoustic waves but also with vorticity and entropy waves.

The present paper is devoted to study the numerical propagation of an acoustic pulse throughout a 2D (two-dimensional) computational domain. The main focus was given to the numerical investigation of boundary conditions and its effects on the final solution. To accomplish the task of evacuating the acoustic pulse through the boundaries of the computational domain, Radiation and Outflow boundary conditions were implemented and tested. The propagation and evacuation of the acoustic pulse is finally validated against the analytical solution for LEE.

In the following two sections, it's presented a summary of radiation and outflow boundary conditions without going to deep details on its construction. Such details can be found in the work of Tam & Webb (1993). A general description about the numerical approach for the pulse propagation is given in section 4. The review of the numerical scheme used in the 2D simulations is provided in section 5. Finally, the numerical results are shown in section 6.

2. Radiation Boundary Conditions

Essentially, Radiation boundary conditions were based on farfield asymptotic solutions for the Linearized Euler Equations (LEE), as presented by investigators like Bayliss & Turkel (1982) and summarized in Tam & Webb (1993). In this paper, it will not be presented the mathematical construction of these boundary conditions, so the reader should consult references given above.

At the boundaries where there is only outgoing acoustic waves the solution is given by the equation (1), in polar coordinates:

$$\left(\frac{1}{V(\theta)}\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+\frac{1}{2r}\right)\begin{bmatrix}\rho\\u\\v\\p\end{bmatrix}=0 \qquad \text{Polar coordinates (1)}$$

These are the radiation boundary conditions for two-dimensional time domain computations. Where (r,θ) are polar coordinates centered near the middle of the computation domain.

Writing (1) in Cartesian coordinates:

$$\frac{1}{V(\theta)}\frac{\partial\rho}{\partial t}+\cos\theta\frac{\partial\rho}{\partial x}+\sin\theta\frac{\partial\rho}{\partial y}+\frac{\rho}{2r}=0 \qquad \text{Continuity (2)}$$

$$\frac{1}{V(\theta)}\frac{\partial u}{\partial t}+\cos\theta\frac{\partial u}{\partial x}+\sin\theta\frac{\partial u}{\partial y}+\frac{u}{2r}=0 \qquad \text{Momentum x (3)}$$

$$\frac{1}{V(\theta)}\frac{\partial v}{\partial t}+\cos\theta\frac{\partial v}{\partial x}+\sin\theta\frac{\partial v}{\partial y}+\frac{v}{2r}=0 \qquad \text{Momentum y (4)}$$

$$\frac{1}{V(\theta)}\frac{\partial p}{\partial t}+\cos\theta\frac{\partial p}{\partial x}+\sin\theta\frac{\partial p}{\partial y}+\frac{p}{2r}=0 \qquad \text{Pressure (5)}$$

The temporal derivatives of these equations should have the same discretization of the main LEE equations taken into the interior points as they are consecutively time advanced with the interior points. The discretization for the spatial derivatives is slightly different, since additional points in the boundary are needed (see section 4).

3. Outflow Boundary Conditions

As they have been created, outflow boundary conditions were also based on asymptotic solutions for the (LEE).
In Cartesian coordinates:

$$\frac{\partial \rho}{\partial t} + u_0 \frac{\partial \rho}{\partial x} = \frac{1}{{a_0}^2}\left(\frac{\partial p}{\partial t} + u_0 \frac{\partial p}{\partial x}\right) \qquad \text{Continuity (6)}$$

$$\frac{\partial u}{\partial t} + u_0 \frac{\partial u}{\partial x} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \qquad \text{Momentum x (7)}$$

$$\frac{\partial v}{\partial t} + u_0 \frac{\partial v}{\partial x} = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} \qquad \text{Momentum y (8)}$$

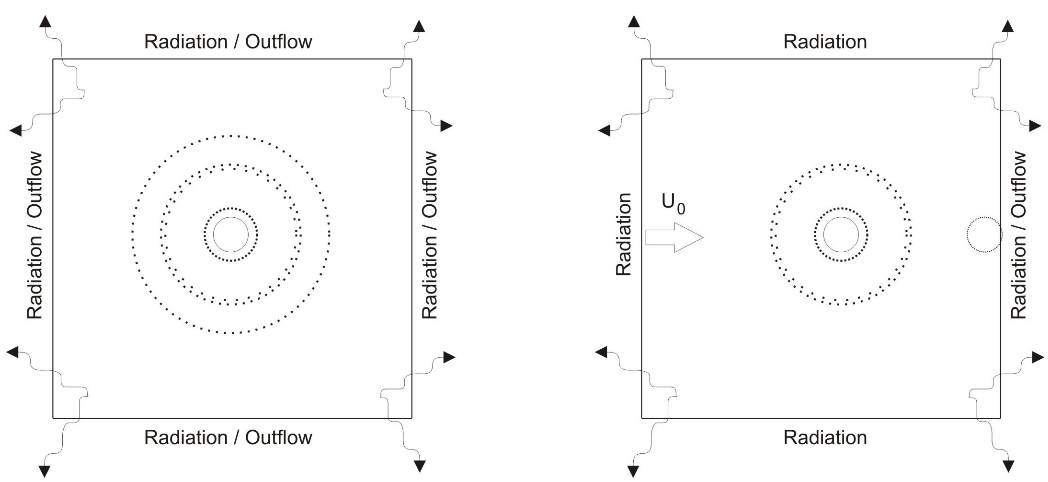
$$\frac{1}{V(\theta)}\frac{\partial p}{\partial t} + \cos\theta \frac{\partial p}{\partial x} + \sin\theta \frac{\partial p}{\partial y} + \frac{p}{2r} = 0 \qquad \text{Pressure (9)}$$

The equations above apply at the boundaries where outgoing disturbances consist of a combination of acoustic, vorticity and entropy waves.

4. Propagation of an Acoustic Pulse

In order to evaluate the numerical propagation of an acoustic pulse, the Linearized Euler Equations (LEE) has been solved in a square 2D computational domain. At the time $t = 0$ an acoustic pulse is generated at the center of the domain and then consecutively propagated through it until reaching the boundaries. At the boundaries points of the computational domain, Radiation and Outflow equations, as described in sections 2 and 3, have been solved in order to allow the acoustic waves to leave the domain.

Two test conditions were analyzed based on the Mach number of the mean flow. These two cases provided conditions to check the effectiveness of Radiation and Outflow boundary conditions. Figure 1(a) and (b) shows a description of the test cases.



(a) No mean Flow (only acoustic waves) (b) Mean Flow (acoustic + entropy waves)

Figure 1 - Test cases to check Radiation and Outflow BC effectiveness.

In the first case - case (A) - there is no uniform flow u_0 and a_0. An unique outgoing acoustic pulse in the center of the domain is supposed to be propagated in all directions reaching all the four boundaries at the same computational time. In this case, all the four boundaries (top, bottom, left and right) were treated with both Radiation and Outflow boundaries conditions one at the time of simulation. The numerical results for such simulations were compared against each other and against the analytical solution for the LEE.

On the other hand, in the case (B) there is an uniform flow of velocity u_0 and a_0 coming to the left that reaches the pressure and entropy pulses. The entropy pulse is released at the same time and at a distance of about 1/3 of the length of the computational domain. This is done in order to force both perturbations to be at same time leaving the domain through the right boundary as they are propagated. As the acoustic pulse expands radially, it is advected to the right until reaching the right boundary. During its expansion at the sound speed, the acoustic pulse reaches the entropy pulse which is being only advected to the right with the mean flow velocity u_0. Thus, at the right boundary, the Radiation and Outflow boundary conditions should evacuate not only an acoustic pulse but also an entropy one. To simulate case (B), at the top, bottom and left boundaries, the Radiation boundary condition was imposed. The right boundary was switched between Radiation and Outflow boundary conditions.

It's important to emphasize that the left boundary could also be treated with Outflow boundary conditions, however considerations about the incoming velocity should have been taken – Tam (1997). In this work, such approach was not considered.

The pressure and entropy pulses were given by a Gaussian distribution:

$$t = 0, \qquad f(x,y) = \varepsilon_1 \exp\left[-\ln 2\left(\frac{x+y}{b}\right)^2\right] \tag{10}$$

The following parameters were applied:
Pressure pulse amplitude $\varepsilon_1 = 0.01$
Half-width b = 3.0
Entropy pulse amplitude $\varepsilon_2 = 0.001$
Half-width b = 5.0

According to Roeck (2004) the Gaussian function is used to evaluate the general dispersion and dissipation properties and the stability ranges of the scheme.

The computation domain is filled out with 200 x 200 uniform meshes with the following global parameters:
$L/\Delta x = 200$
$\Delta t = 0.05$s
$\Delta x = \Delta y = 1$

Figure 2 depicts details about the computational domain and the numerical parameters used in this simulation. The grey zone shown is the region of extra points, normally called "ghost points", for evaluating the radiation and outflow boundary equations. More details about this procedure are given in the next section.

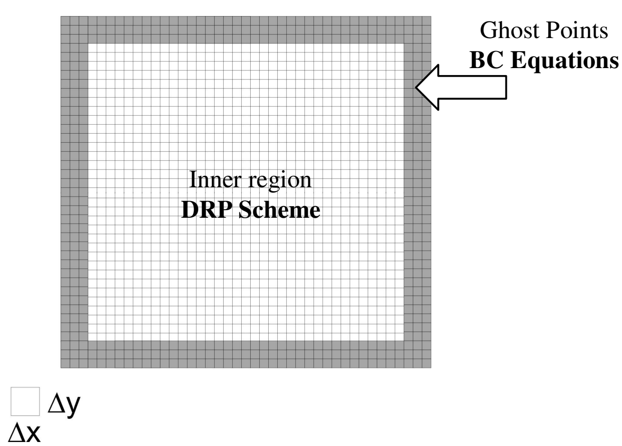


Figure 2 - Computational domain and numerical parameters.

In order to validate the propagation of an acoustic pulse in a two dimensional domain, the analytical solution for LEE was numerically implemented. The complete description of the analytical solution can also be found in the work of Tam & Webb (1993).

4.1. Review of Numerical Scheme

The complete validation of the numerical scheme employed in this work was carried out with 1D solution of a propagating pulse, according to Almeida & Souza (2006). In this section, it will be provided just a summary about the spatial and temporal discretization used on the numerical implementation.

For the temporal discretization, a 3rd order Runge Kutta scheme was applied, according to Williamson (1980). This scheme consists of 3 sub-steps for each time step Δt, with different coefficients γ and ζ. Despite questions about instabilities and numerical dispersion, this scheme performed very well when compared with other schemes Dispersion-optimized – see Almeida & Souza (2006).

Thus, the following equation applies to advance a general ϕ variable:

$$\phi^{n+1} = \phi^n + \Delta t\left[\gamma_i f(t_n, \phi^n) + \zeta_i f(t_{n-1}, \phi_{n-1})\right] \tag{11}$$

where the coefficients γ and ζ are defined according to Table 1:

Table 1 - 3rd Order Runge Kutta Coefficients – Williamson (1980).

i	γ_i	ζ_i
1	8/15	0
2	5/12	-17/60
3	3/4	-5/12

For the spatial discretization, the low-dispersion and low-dissipation numerical schemes also known as DRP (Dispersion-relation-preserving) were employed. This scheme seems to be very suitable for the propagation of an acoustic pulse. The complete description of optimized spatial discretization for the Dispersion-Relation-Preserving (DRP) schemes is given in the work of Tam & Webb (1993b).

The coefficients of the 7-point central difference scheme (DRP) for approximating spatial derivatives used in this work are:

7-point central difference scheme (DRP):

$a_0 = 0$

$a_1 = -a_1 =$ 0.77088238051822552

$a_2 = -a_2 =$ -0.166705904414580469

$a_3 = -a_3 =$ 0.02084314277031176

This scheme was applied to all internal points in the computational domain (see Figure 2). However, the use of such a scheme gives rise to ghost points at the boundary of the computational domain. Figure 3 shows the interface region between the inner domain, where DRP scheme is applied, and the region where ghost points must be applied in order to evaluate the BC equations.

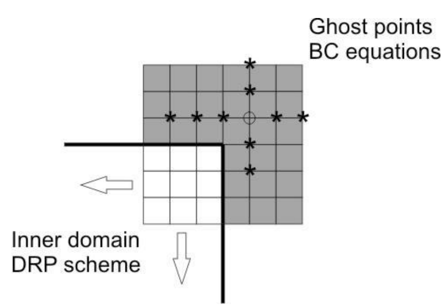


Figure 3 - Interface between inner and ghost points

For the ghost points, a 7-point backward difference scheme (DRP) was employed in order to approximate the spatial derivatives. The coefficients used for this scheme are given below, according to the stencil applied in the 3 ghost points.

0-6 stencil applied to n-2 points:

$a_{-2} =$ 0.049041958
$a_{-1} =$ -0.468840357
$a_0 =$ -0.474760914
$a_1 =$ 1.273274737
$a_2 =$ -0.518484526
$a_5 =$ 0.166138533
$a_6 =$ -0.026369431

where n is the total number of points in a coordinate direction.

1-5 stencil applied to n-1 points:

$a_{-1} =$ -0.209337622
$a_0 =$ -1.084875676
$a_1 =$ 2.147776050
$a_2 =$ -1.388928322
$a_3 =$ 0.768949766
$a_4 =$ -0.28181465
$a_5 =$ 0.048230454

0-6 stencil applied to n points:

$a_0 =$ -2.192280339
$a_1 =$ 4.748611401
$a_2 =$ -5.108851915
$a_3 =$ 4.461567104
$a_4 =$ -2.833498741
$a_5 =$ 1.128328861
$a_6 =$ -0.203876371

5. Numerical Results

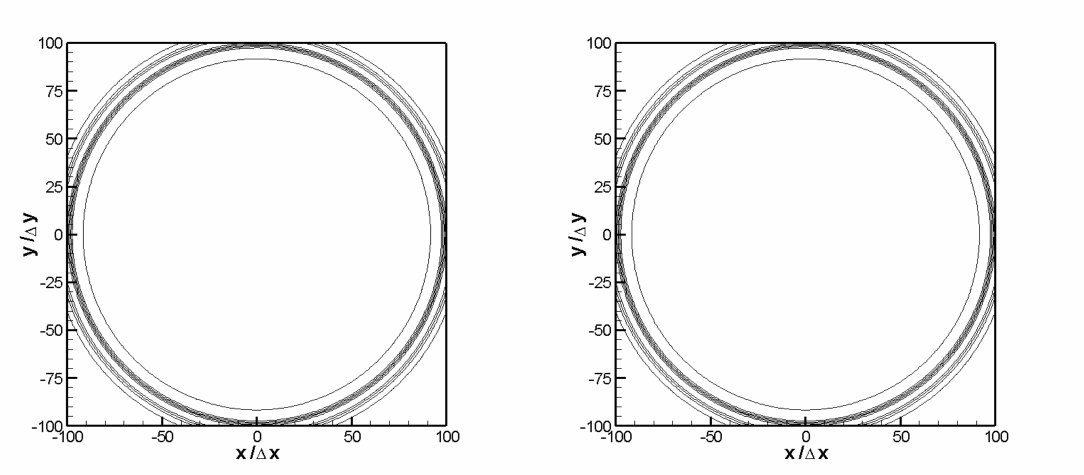
This section provides the numerical results of the present work. The results are shown separately for the cases (A) and (B), in order to check the effectiveness of Radiation and Outflow boundary conditions in a pulse propagation problem under the effects of a mean flow.

5.1. Case (A) – No mean flow

As shown in Figure 1(a), the Case (A) consists only on propagating and evacuating an acoustic wave through the 2D domain. In this case, for all boundaries, both Radiation and Outflow boundary conditions were implemented and tested separately. The analytical solution for the LEE is used to validate the numerical solutions.

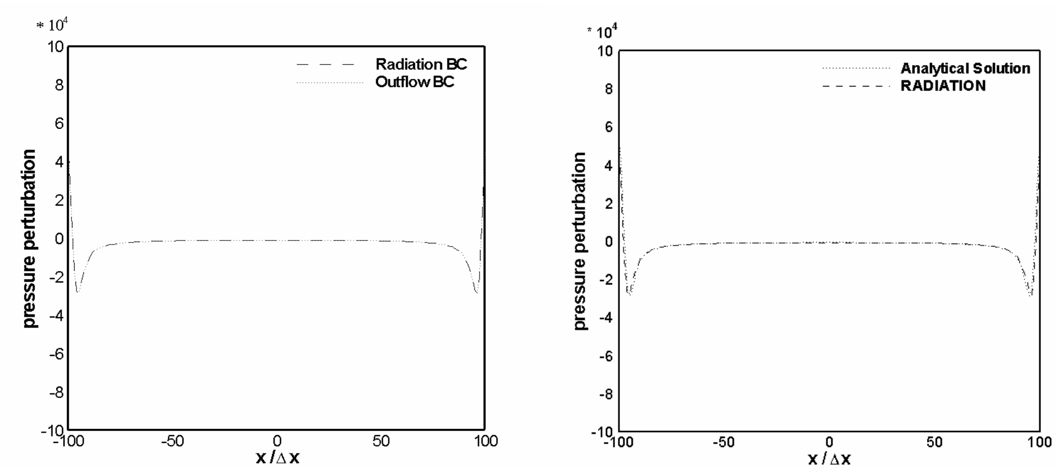
Figure 4(a) and (b) shows the acoustic pulse leaving the computational domain after 2000 time steps (t = 100) under the effects of Radiation and Outflow boundary conditions, respectively. In both contour plots there are no noticeable reflections of the pulse in the boundaries. Both solutions are identical and the boundary conditions imposed in these regions appear to be very effective. The solution was run until 3200 time steps where the acoustic pulse is completely evacuated from the domain. Again, there were not noticeable reflections during the simulation process from 2000 up to 3200 time steps.

More detailed information about the wave's propagation could be considered when looking to the waveform. Figure 5 shows the pressure waveform along the *x* axis after 2000 time steps. In figure 5(a) are compared the results obtained with Radiation and Outflow boundary conditions. On the other hand, figure 5(b) presents the comparison between the results obtained with Radiation boundary condition and the analytical solution for LEE. As said before, in figure 5(a) both solutions are quite identical and no wiggles are observed in the waveform while it's leaving the computational domain. However, in Figure 5(b) it's possible to observe little differences in the waveform, but still no reflections are noticeable. These small differences are attributed to the numerical scheme (spatial and temporal discretization) employed in this work and not really to the boundary conditions. In summary, the numerical solution has a good agreement with the exact solution of the linearized Euler equations.



(a) Radiation BC (b) Outflow BC

Figure 4 – Pressure contours at 2000 time steps – Comparison between Radiation and Outflow BC's.



(a) Radiation vs Outflow BC (b) Radiation vs Analytical

Figure 5 – Pressure waveform along the centerline at 2000 time steps – Computed vs Analytical Solution for LEE.

5.2. Case (B) – Effect of mean flow

According to Figure 1(b), the Case (B) consists on propagating and evacuating an acoustic wave through the 2D domain under the effects of a mean flow coming from the left side of the domain. The Mach number a_0 for the incoming flow was adjusted to 0.5.

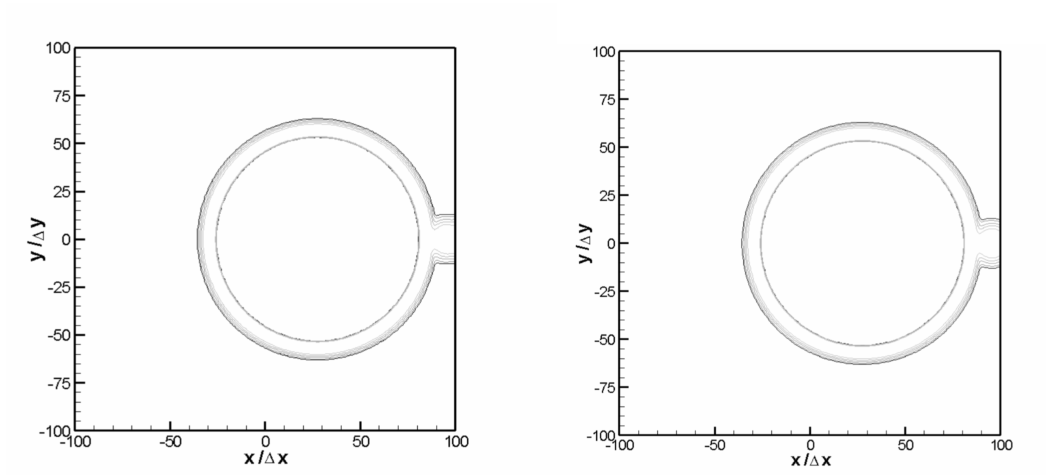
For this specific case, at the top, bottom and left boundaries, the Radiation boundary conditions were used. For the right boundary, the effectiveness of the boundary condition was tested, changing between Radiation and Outflow. Again, the analytical solution for the LEE is used to validate the numerical solutions.

In order to show the entropy pulse leaving the domain, Figure 6 displays the density contours considering a computational time of 1100 time steps. At this time, both acoustic and entropy pulse are caught together in the right boundary. Again, this is a very good test to check if the Radiation and Outflow boundary conditions could evacuate both disturbances without any considerable reflections.

As can be seen in Figure 6(a) and (b) there are not noticeable reflections in the computational domain. The order of magnitude for reflected waves in the computational domain is much less than 1% intensity relative to peak acoustic disturbance and lies well within the range of accepted patterns for aeroacoustics.

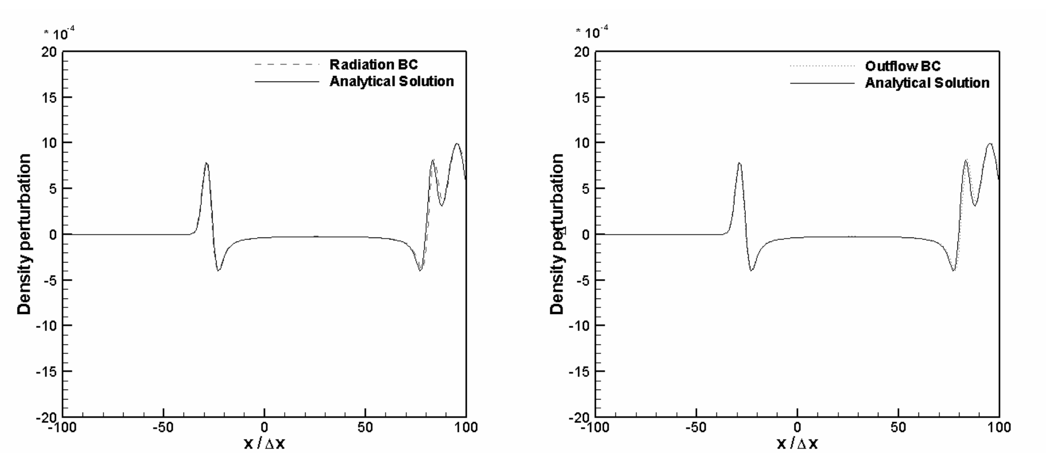
Figure 7(a) and (b) shows the comparison between computed and analytical solution for the density waveform distributed over the centerline (x-axis) of the computational domain considering a computational time of 1100 time steps. As said in the previous paragraph, the numerical results agree very well with the exact solution for the LEE, showing not only a good performance of the outflow and radiation boundary conditions in evacuating pressure and entropy disturbances, but also a good combination of the numerical scheme employed in this work.

In addition to the results presented herein, some other tests were done decreasing the size of the computational domain, in order to check the effectiveness of the boundary conditions when the pulse is close to the border of the domain. Even for a domain with only 50 points, small and negligible reflections were observed. Also, other simulations were carried out using a vorticity pulse perturbation released at the same position of the entropy one. These results are not shown here for brevity, but the results were quite good and prove once more the good applicability of the numerical scheme for aeroacoustics simulations.



(a) Radiation BC (b) Outflow BC

Figure 6 – Density contours at 1100 time steps - Comparison between Radiation and Outflow BC's.



(a) Radiation BC (b) Outflow BC

Figure 7 – Density waveform along the centerline at 1100 time steps - Computed vs Analytical Solution for LEE.

6. Conclusions

The present work described a numerical evaluation of boundary conditions for aeroacoustics. To accomplish the task, spatial DRP and temporal Runge-Kutta schemes have been implemented in order to propagate pressure and entropy disturbances throughout a 2D (two-dimensional) domain. At the boundaries, Radiation and Outflow equations were used to evacuate the pulses under the effect or not of a mean flow.

With the numerical results presented herein, was possible to identify the good effectiveness of the Radiation and Outflow boundary conditions. For all simulations performed with and without the presence of a mean flow, both types of boundary conditions were efficient in evacuating the pressure and entropy pulse perturbation with no noticeable reflections and instabilities in the computed waveforms.

In order to guarantee the right judgment for the effectiveness of the boundary conditions, the exact solutions of the Linearized Euler Equations have been used. The computed pressure and density waveforms have been compared with the exact solution showing a very good agreement.

Finally, the next step of this work will be the extension of the current numerical scheme for three dimensions.

7. Acknowledgments

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