ON NUMERICAL SIMULATION OF THE NONLINEAR DYNAMICS OF A JET PLANE FLIGHT CHARACTERISTICS

Danilo Carlos Pereira, danilocp@pop.com.br
Department of Mechanical Design, FEM, UNICAMP, 13083-970, Campinas, SP, Brazil.

José Manoel Blthazar, jmbaltha@rc.unesp.br.
Department of Statistics, Applied Mathematics and Computation, IGCE, UNESP, 13506 - 700, Rio Claro, SP and visiting professor of the Department of Mechanical Design, FEM, UNICAMP, 13083-970, Campinas, SP, Brazil.

Reyolando M.L.R.F. Brasil, reyolando.brasil@poli.usp.br
Department of Structural and Geotechnical Engineering, Polytechnic School, The University of São Paulo – USP, SP, Brazil.

Fábio Roberto Chavarete, fabioch@rc.unesp.br.
Department of Statistics, Applied Mathematics and Computation, IGCE, UNESP, 13506 - 700, Rio Claro, SP, Brazil.

Abstract. *As the several existing jet planes vary widely in their dynamic characteristics, it is not a practical endeavor to develop mathematical models for each of them. In this paper, we present a general model takinginto account all the forces and moments involved in the modeling. We analyzed the effect of sudden wind changes due to atmospheric turbulence on the dynamic response of the aircraft and its structural design. Atmospheric turbulence was taken as a random process that we modeled by a series of super imposed harmonic functions. As an application, we considered the equations of horizontal leveled motion of the F-8 jet aircraft*

Keywords: *Longitudinal Flight Dynamics, Wind Spectrum,*

1. INTRODUCTION

As the several existing jet planes vary widely in their dynamic characteristics, it is not a practical endeavor to develop mathematical models for each of them. We pretend, in this research, to present a general model taking into account all the forces and moments involved in the considered modeling. It is treated of a preliminary study for the introduction of wind gusts in the model of longitudinal flight adopted in Garrard (1977).

We analyze the effect of sudden wind changes, due to the atmospheric turbulence on the dynamic response of the aircraft and its structural design.

As an application, we consider the equations of horizontal leveled motion of the F-8 jet aircraft. We consider the drag forces small in comparison with the trust forces and its 9773 kg mass and, we adopt the altitude as 9144 m. The moment of inertia about the pitch axis is taken as proportional to mass of the aircraft.

The governing differential equations of motion of the aircraft ($\dot{x} = f(x,v)$), are written in terms of four variables ($x = (u,\alpha,\theta,q)$, where u: forward flight velocity; α: attack angle; θ: pitch angle; q = pitch rate; and v = a variation of the wind velocity, composed by a constant average level (v_0) and a fluctuating part. Atmospheric turbulence is actually a random process that we are modeling by a series of superimposed harmonic functions. We simulate the interaction of the aircraft with the wind velocity field that generates forces and moments.

2. NOMENCLATURE

The used nomenclature on flight dynamics is: C_{L_w}, C_{L_t} = coefficients of wing and tail lift forces, respectively; $C^i_{L_w}, C^i_{L_t}$ = coefficients of approximated wing and tail lift forces, respectively; W = adjustment term to the wing lift force coefficient; c = damping coefficient; $c\dot{\theta}$ = damping moment; g =gravitation constant; I_x, I_y, I_z =moments of inertia about axes x, y and z, respectively; L_t, L_w = wing and tail lift forces, respectively; l =distance between wing aerodynamic center (a.c) and aircraft's center of gravity (c.g.); l_t = distance between tail a.c. and aircraft's c.g; M_w = wing pitching moment; m = mass of aircraft; p,q,r = roll, pitch and yaw rates, respectively; $\overline{q}$ = dynamic pressure; ρ = atmospheric density; S = wing area; S_t = horizontal tail area ; u,v,w = velocity components along x, y and z axes, respectively; α = wing angle of attack (rad); α_t = tail angle of attack (rad); θ = pitch angle (rad) and δ_e = deflection of elevator (rad).

3. GOVERNING EQUATIONS OF LONGITUDINAL MOTION

It is known that a mathematical model that defines the dynamics of an aircraft is extremely important in the study of its dynamics and control. An aircraft is a dynamic system whose complexity is expressed in bodies collection connected so that the rigid and elastic body relative actions can come to occur. It stresses that the flight dynamics study (or stability and control of an aircraft) worries with the dynamic global behavior of an aircraft:

- Stability,
- Controllability;
- Dynamic Answer,
- Control Qualities, etc.

However the analysis of flight dynamics requires a comprehensive aircraft model. This model must be valid for all the combinations of angle-of-attack, number of "Mach", "g" and altitude in which the aircraft operates. This operational space" is called Flight Level of Aircraft.

In the mathematical model of an aircraft are the motion's equations of the rigid body. Considering an aircraft like rigid, the mathematical model has six freedom degrees, giving origin to a dynamic problem of 12 first-order equations.

Four of these states (the aircraft space bearing and your "its heading angle") don't have effect in the interest dynamic behavior.

Flight dynamicists differentiate between several axis systems could be driven the several possible different combinations of the state variables.

In this work, they use just the longitudinal motion equations, which ones will be objects of our studies. The model of longitudinal flight dynamic used here follows Garrard's mathematical model (Garrard, 1977).

Considering the system of coordinated for longitudinal flight dynamic mention in Garrard (1977), suppose that drag is neglected because it is small if compared with the lift force and the weight of aircraft. For that, it will be disregarded in this analysis. The used coordinates system and the considered forces shown in the Fig. 1, to follow. Like already told previously, the drag will be disregarded and the lift force will be separated in two components, one of wing and to other of tail (Liaw and Song, 2001).

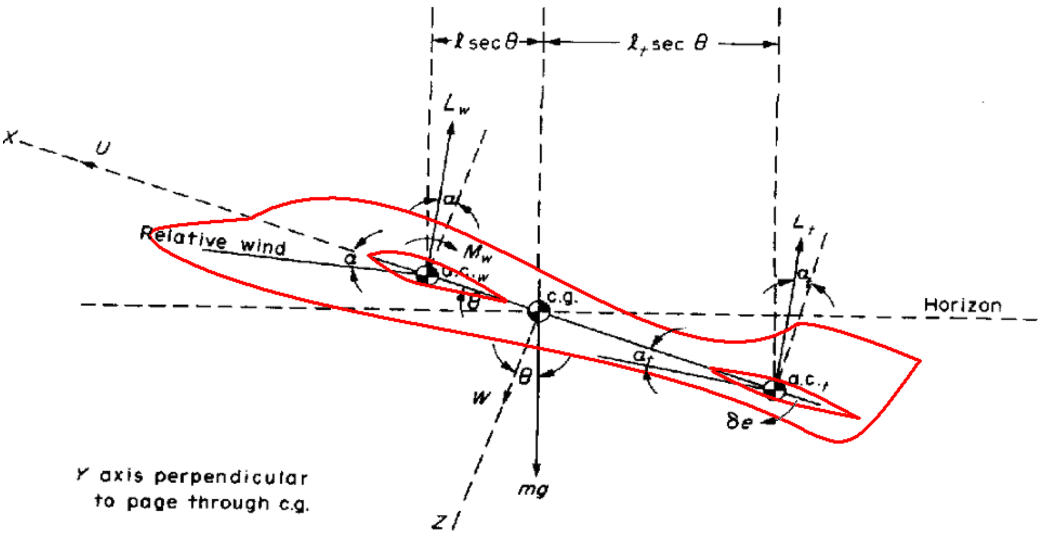


Figure 1. Aircraft Dynamical Model.

In general terms:

$$\dot{x} = F(x, V(t))$$
$$x = (x_1, x_2, x_3) = (\alpha, \theta, q)$$

where α is the angle-of-attack, θ is the pitch angle and q is the pitch rate.

The basic governing differential equations of longitudinal motion of the aircraft, with drag considerable very small if compared with lift force and weight of the airplane are given as:

$$m\left(\dot{u} + w\dot{\theta}\right) = -mg\sin\theta + L_w \sin\alpha + L_t \sin\alpha_t \quad (1)$$

$$m\left(\dot{w} - u\dot{\theta}\right) = mg\cos\theta - L_w \cos\alpha - L_t \cos\alpha_t \quad (2)$$

$$I_y \ddot{\theta} = M_w + lL_w \cos\alpha - l_t L_t \cos\alpha_t - c\dot{\theta} \quad (3)$$

Where:

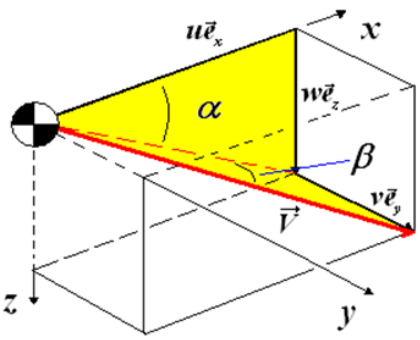
$$L_w = \overline{q}S(C_{L_w}^0 + C_{L_w}^1\alpha - C_{L_w}^2\alpha^3).W$$
$$L_t = \overline{q}S_t C_{L_t} = \overline{q}S_t(C_{L_t}^0 + C_{L_t}^1\alpha_t - C_{L_t}^2\alpha_t^3 + a_e\delta_e)$$

$$W = \left\{ \frac{1}{1+\left(\frac{\alpha}{0,41}\right)^{60}} \right\}, \quad \overline{q} = \frac{1}{2}\rho V_t^2 = \frac{1}{2}\rho_0 V_e^2 \text{ and, } \overline{q} = \frac{1}{2}\rho V_t^2 = \frac{1}{2}\rho_0 V_e^2 \Rightarrow V_t = \sqrt{\frac{\rho_0}{\rho}}V_e$$

Where V is velocity, ρ is atmospheric density.

The Fig. 12 below shown the velocity components (*u, v* e *w)* throughout the (*x, y* e *z*) axis and its relations:

$$u = V\cos\beta\cos\alpha$$
$$v = V\sin\beta$$
$$w = V\cos\beta\sin\alpha$$



$$V = \left(u^2 + v^2 + w^2\right)^{\frac{1}{2}}$$

Figure 2: Components of velocity.

This way, let be $w = u\tan\alpha$ and $\dot{w} = \dot{u}\tan\alpha + u\dot{\alpha}\sec^2\alpha$, and this way, we can rewrite Eq.(1), Eq. (2) and Eq. (3) as:

$$\begin{cases} \dot{u} = -u\dot{\theta}\tan\alpha - g\sin\theta + (L_w/m)\sin\alpha + (L_t/m)\sin\alpha_t \\ \dot{\alpha} = \dot{\theta}\cos^2\alpha + (g/u)\cos^2\alpha\cos\theta - L_w/(um)\cos^3\alpha - \\ \quad - L_t/(um)\cos^2\alpha\cos\alpha_t - (\dot{u}/u)\sin\alpha\cos\alpha \\ \ddot{\theta} = M_w/I_y + (lL_w/I_y)\cos\alpha - (l_tL_t/I_y)\cos\alpha_t - (c/I_y)\dot{\theta} \end{cases} \quad (4)$$

That, replacing $\dot{u} = -u\dot{\theta}\tan\alpha - g\sin\theta + (L_w/m)\sin\alpha + (L_t/m)\sin\alpha_t$ in. (4) becomes:

$$\begin{cases} \dot{u} = -u\dot{\theta}\tan\alpha - g\sin\theta + (L_w/m)\sin\alpha + (L_t/m)\sin\alpha_t \\ \dot{\alpha} = -\left(\frac{\dot{u}}{u}\right)\cos\alpha\sin\alpha + \dot{\theta}\cos^2\alpha + \left(\frac{g}{u}\right)\cos^2\alpha\cos\theta - \\ \quad -\left(\frac{L_w}{um}\right)\cos^3\alpha - \left(\frac{L_t}{um}\right)\cos^2\alpha\cos\alpha_t \\ \ddot{\theta} = \left(\frac{M_w}{I_y}\right) + \left(\frac{lL_w}{I_y}\right)\cos\alpha - \left(\frac{l_tL_t}{I_y}\right)\cos\alpha_t - \left(\frac{c}{I_y}\right)\dot{\theta} \end{cases} \quad (5)$$

To u not constant, we have $\dot{u} \neq 0$. Therefore, taking the $\dot{\alpha}$ equation we have:

$$\dot{\alpha} = -\frac{1}{u}\cos\alpha\sin\alpha\left[-u\dot{\theta}\frac{\sin\alpha}{\cos\alpha} - g\sin\theta + \left(\frac{L_w}{m}\right)\sin\alpha + \left(\frac{L_t}{m}\right)\sin\alpha_t\right] + \dot{\theta}\cos^2\alpha + \left(\frac{g}{u}\right)\cos^2\alpha\cos\theta - -\left(\frac{L_w}{um}\right)\cos^3\alpha - \left(\frac{L_t}{um}\right)\cos^2\alpha\cos\alpha_t$$

and,

$$\dot{\alpha} = \dot{\theta}\sin^2\alpha + \left(\frac{g}{u}\right)\cos\alpha\sin\alpha\sin\theta - \left(\frac{L_w}{um}\right)\cos\alpha\sin^2\alpha - \left(\frac{L_t}{um}\right)\cos\alpha\sin\alpha\sin\alpha_t + \dot{\theta}\cos^2\alpha + \left(\frac{g}{u}\right)\cos^2\alpha\cos\theta - -\left(\frac{L_w}{um}\right)\cos^3\alpha - \left(\frac{L_t}{um}\right)\cos^2\alpha\cos\alpha_t$$

Putting the terms in evidence:

$$\dot{\alpha} = \dot{\theta}\left(\sin^2\alpha + \cos^2\alpha\right) + \left(\frac{g}{u}\right)\left[\cos\alpha\sin\alpha\sin\theta + \cos^2\alpha\cos\theta\right] - \left(\frac{L_w}{um}\right)\left[\cos\alpha\sin^2\alpha + \cos^3\alpha\right] - -\left(\frac{L_t}{um}\right)\left[\cos\alpha\sin\alpha\sin\alpha_t + \cos^2\alpha\cos^2\alpha_t\right]$$

In this way:

$$\dot{\alpha} = \dot{\theta} + \left(\frac{g}{u}\right)\left[\cos\alpha\sin\alpha\sin\theta + \cos^2\alpha\cos\theta\right] - \left(\frac{L_w}{um}\right)\left[\cos\alpha\sin^2\alpha + \cos^3\alpha\right] - -\left(\frac{L_t}{um}\right)\left[\cos\alpha\sin\alpha\sin\alpha_t + \cos^2\alpha\cos^2\alpha_t\right]$$

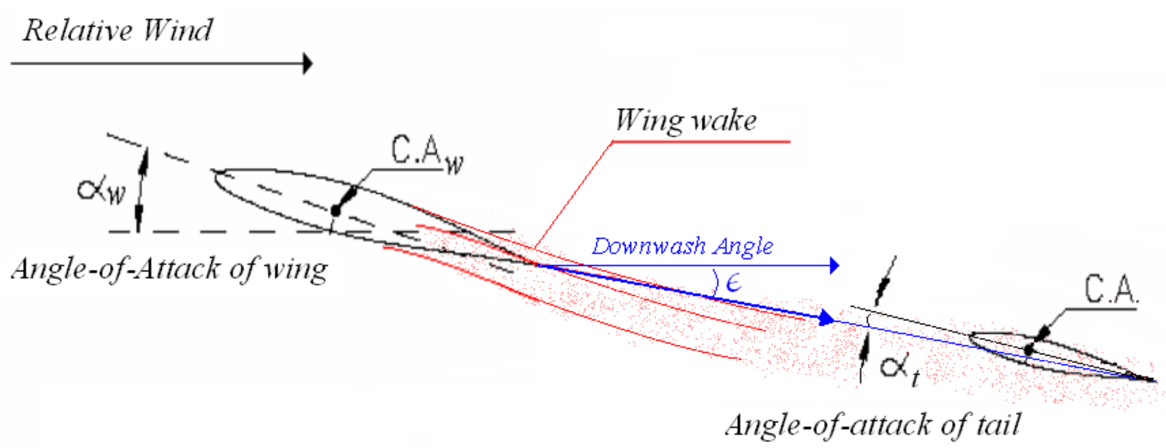


Figure 3: Angle of Downwash

Observation: The angle of attack at the horizontal tail will not be the same as the wing angle of attack because of differences in wing and tail incidence of the relative wind.

Because the horizontal tail of the F-8 aircraft is within the wing wake, as depicted in the figure 3, the downwash angle $\in$ is including for determining the angle-of-attack of the elevator.

Using the linear approximation $\in = a_{\in}\alpha$ (Liaw and Song, 2001), the α_t angle is given as:

$$\alpha_t = \alpha - \in + \delta_e \Rightarrow \alpha_t = \alpha - a_{\in}\alpha + \delta_e \Rightarrow \alpha_t = \alpha\left(1 - a_{\in}\right) + \delta_e$$

As $a_{\in} = 0,75$ we have that $\alpha_t = 0,25\alpha + \delta_e$.

This way, we have the system as:

$$\begin{cases} \dot{u} = -uq\tan\alpha - g\sin\theta + \left(L_w/m\right)\sin\alpha + \left(L_t/m\right)\sin\left(0,25\alpha+\delta_e\right) \\ \dot{\alpha} = q + \left(\dfrac{g}{u}\right)\left[\cos\alpha\sin\alpha\sin\theta + \cos^2\alpha\cos\theta\right] - \left(\dfrac{L_w}{um}\right)\left[\cos\alpha\sin^2\alpha + \cos^3\alpha\right] - \\ \quad -\left(\dfrac{L_t}{um}\right)\left[\cos\alpha\sin\alpha\sin\left(0,25\alpha+\delta_e\right) + \cos^2\alpha\cos\left(0,25\alpha+\delta_e\right)\right] \\ \dot{\theta} = q \\ \dot{q} = \left(\dfrac{M_w}{I_y}\right) + \left(\dfrac{lL_w}{I_y}\right)\cos\alpha - \left(\dfrac{l_t L_t}{I_y}\right)\cos\left(0,25\alpha+\delta_e\right)_t - \left(\dfrac{c}{I_y}\right)\dot{\theta} \end{cases} \tag{6}$$

To proceed, it is considered the data of F-8 aircraft.

Table 1: Data of Aircraft F-8 Cruzader

$C^0_{L_w} = C^0_{L_t}$	*= 0*
$C^1_{L_w} = C^1_{L_t}$	*= 4.0*
$C^2_{L_w} = C^2_{L_t}$	*= 12*
a_e	*= 0.1*
S	*= 375 ft^2 (33.75 m^2)*
S_t	*= 93.4 ft^2 (8.41 m^2)*
m	*= 667.7 slugs (9773 kg)*
$a_\in$	*= 0.75*
$\in$	*= 0*
$C_{m_{a.c.}}$	*= 0*
$\overline{c}$	*= 11.78 ft (3.53 m)*
I_y	*= 96800 slug ft^2 (127512 kg-m^2)*
l	*= 0.189 ft (0.06 m)*
l_t	*= 16.7 ft (5.01 m)*
c	*= 38332.8 lb ft s (50494.752 kg m s)*

Taking the data proposed in table 1 and substituting in Eq. (6) we will obtain:

$$\begin{cases} \dot{u} = -uq\tan\alpha - 10\sin\theta + \dfrac{\overline{q}}{m}\left\{\begin{matrix} 33.75W\sin\alpha\left(4\alpha-12\alpha^3\right) + \\ +8.41\sin\left(0.25\alpha+\delta_e\right)\left[4\left(0.25\alpha+\delta_e\right) - 12\left(0.25\alpha+\delta_e\right)^3 + 0.1\delta_e\right]\end{matrix}\right\} \\ \dot{\alpha} = q + \dfrac{10}{u}\cos\alpha\cos\left(\alpha-\theta\right) - \dfrac{\overline{q}}{mu}\cos\alpha\left\{\begin{matrix} 33.75W\left(4\alpha-12\alpha^3\right) - \\ -8.41\cos\left(0.75\alpha-\delta_e\right)\begin{pmatrix} 4\left(0.25\alpha+\delta_e\right) - \\ -12\left(0.25\alpha+\delta_e\right)^3 + 0.1\delta_e\end{pmatrix}\end{matrix}\right\} \\ \dot{\theta} = q \\ \dot{q} = \dfrac{50.1}{127512}m\cos\theta - \dfrac{171.1125\left(4\alpha-12\alpha^3\right)}{127512}\overline{q}W\cos\alpha - \dfrac{50494.752}{127512}q + \\ \quad + \dfrac{\overline{q}}{127512}\left\{\begin{matrix} 2.025\left(4\alpha-12\alpha^3\right)W\cos\alpha - \\ -42.1341\cos\left(0.25\alpha+\delta_e\right)\left[4\left(0.25\alpha+\delta_e\right) - 12\left(0.25\alpha+\delta_e\right)^3 + 0.1\delta_e\right]\end{matrix}\right\} \end{cases} \tag{7}$$

4. WIND GUSTS:

In this procedure, it is divide the action of wind in a both flotation portion and a medium portion. According with the proposed method, the medium portion is applied statically to the body and the flotation part is divided at components harmonicas series with aleatory phase angles. To the flotation part, they are used 11 harmonicas component with one of which being resonant. The frequencies of the other components are defined as multiples or submultiples of that resonant frequency of factor 2.

In an improved version, it is made that factor same to the reason among the natural frequencies of the first and of the second mode, establishing thus resonant functions with these fundamental modes of the structure. The amplitude of the each other harmonic components is obtained starting from a Spectral Density Function of Power of wind speed.

The Spectral Density Function of Power (SDFP), according with (Blessmann, 1998), indicates the energy distribution contained in a phenomenon in several frequencies. Admitting that the signal supplied for wind gusts constitute of a not-periodic complex function, by the theorem of Fourier this function can be faced as a simple functions harmonicas overlapping, as described in Newland (1975).

The simplest idealization of a SDFP to a wind gust is an ideal white noise

$$S_{\ddot{S}g}(\omega) = S_0 \ , \ \ 0 < \omega < \infty \tag{8}$$

This description is, however, little reasonable, for corresponding the infinite power. A more realistic form, which will be used in this work, is the white noise filtrate model, (Kanai, 1957) and (Tajimi, 1960), well-known like Kanai-Tajimi's model. In this model, the wind is seen as the answer in absolute acceleration of a system of a single degree of freedom under a base acceleration of an ideal white noise spectrum. It supposes that this oscillator models the wind blasts. The formulation for the model Kanai-Tajimi is found in Buchholdt (1999), with some parameters variations.

In Buchholdt, (1999), it has the representation of the SDFP of the wind acceleration by expression:

$$S_{\ddot{S}g}(\omega) = S_0 \frac{[1 + 4H^2 r^2}{\left(1 - r^2\right)^2 + 4H^2 r^2} \tag{9}$$

where S_0 is used as ideal White noise $\left[\frac{m}{s^2}\right]^2 s$, H is a factor non-dimensional related with the dumping and r the non-dimensional relationship of frequencies.

The Kanai-Tajimi’s model is used in a lot works that try to esteem artificial accelerograms.

The proposal in this work is to use the following reduced spectrum:

$$S_{r\ddot{S}_g}(\omega) = \frac{\omega S_{\ddot{S}g}(\omega)}{R^2} \tag{10}$$

Where R is a constant with dimensions of acceleration.

Thus, it proposes, model mathematically the support excitation as an overlapping of n components harmonicas. The mathematical model, (Kanai, 1957) and (Tajimi, 1960), as:

$$\ddot{S}_g = R \sum_{k=1}^{n} \overline{C}_k \cos(\overline{\omega}_k t - \theta_k) \tag{11}$$

where the adimensional amplitudes are obtained from SDFP (Corbani, 2006) as:

$$C_k = \sqrt{2 S_{r\ddot{S}_g}(\omega_k) \Delta\omega} \tag{12}$$

Being R=1/11 and n=11 we obtained $\overline{C}_k$ and $\overline{\omega}_k$ are described in table 2 below (Corbani, 2006).

The phase angles θ_k are random values such that $\theta_k \in [0, 2\pi]$.

Like this, after established SDFP for the wind, can take a study of the number reasonable of harmonic functions to simulate this phenomenon. In that way, esteem any number of harmonic functions with a resonant term, in this way, the amplitude of the load for each harmonic k is shown in the Fig. 4.

Table 2. Values for $\overline{C}_k$ and $\overline{\omega}_k$.

$\overline{C}_1$ =0,18	$\overline{\omega}_1$=122,55
$\overline{C}_2$ =0,23	$\overline{\omega}_2$=77,52
$\overline{C}_3$ =0,30	$\overline{\omega}_3$=49,04
$\overline{C}_4$ =0,40	$\overline{\omega}_4$=31,02
$\overline{C}_5$ =0,53	$\overline{\omega}_5$=19,62
$\overline{C}_6$ =0,62	$\overline{\omega}_6$=12,41
$\overline{C}_7$ =0,49	$\overline{\omega}_7$=7,85
$\overline{C}_8$ =0,34	$\overline{\omega}_8$=4,97
$\overline{C}_9$ =0,25	$\overline{\omega}_9$=3,14
$\overline{C}_{10}$ =0,19	$\overline{\omega}_{10}$=1,99
$\overline{C}_{11}$ =0,15	$\overline{\omega}_{11}$=1,26

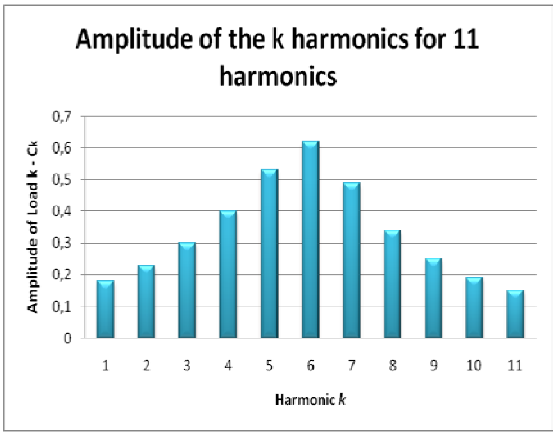


Figure 4: Amplitude of the *k* harmonics for 11 harmonics

The power spectral density have been simulated and presented in Fig. 5.

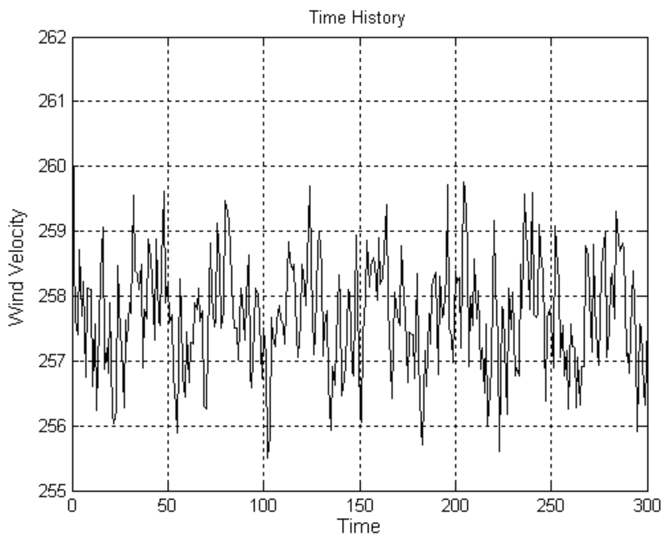


Figure 5: Load Time History Generated by Power Spectral Density Fuction.

For the simulations we will use:

$$\overline{q} = \frac{1}{2}\rho V^2$$
$$V(t) = V_0 + \ddot{S}_g \tag{13}$$

5. SIMULATIONS

To simulation the following values are used:

With initial values of: $V_0=u=845,\ 6\ ft/s$ (257, 7 m/s) and an altitude of *30000 ft* (9144m). The initial mass m of aircraft like to m_0 = 667,7 slugs (9773 Kg) and atmospheric density ρ to *9144 meters* of altitude equal to 0.4938, and initial conditions like $u=257.7m/s$, $\alpha=0.24rad,\ \theta=0.23\ rad,\ q=0\ rad/s$ and $\delta_e= -0.1\ rad.$

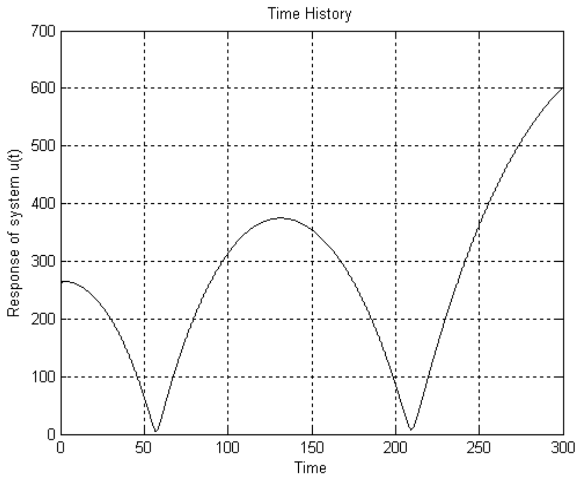


Figure 6. Time Historic of u

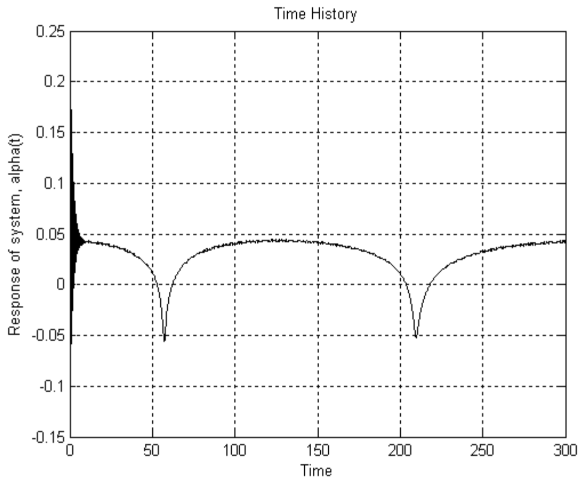


Figure 7. Time Historic of α

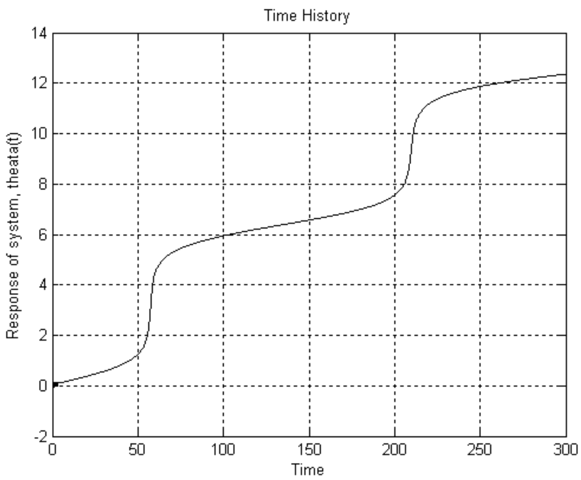


Figure 8: Time Historic of θ

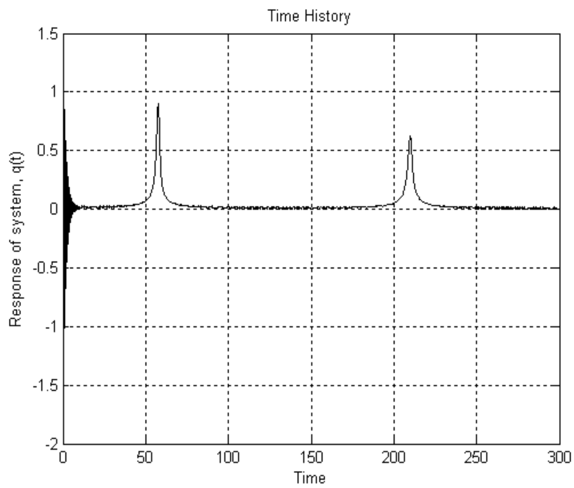


Figure 9. Time Historic of q

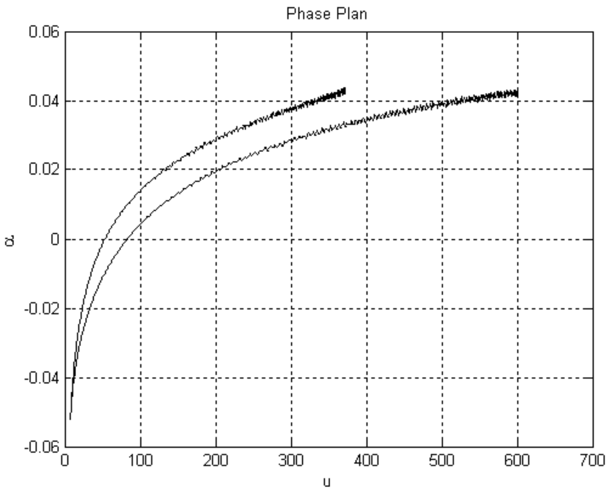


Figure 10. Phase Plan u vs α

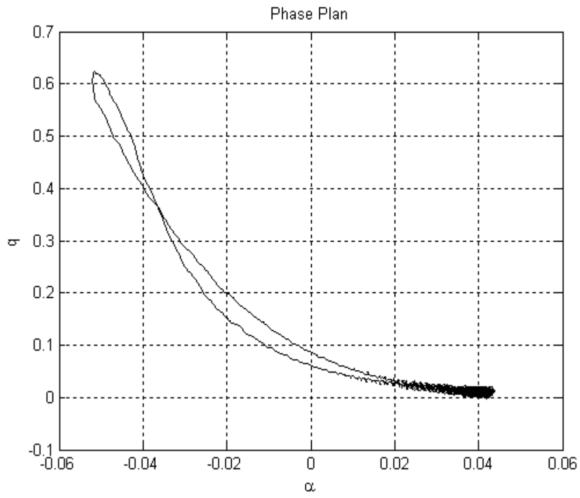


Figure 11. Phase Plan α vs q

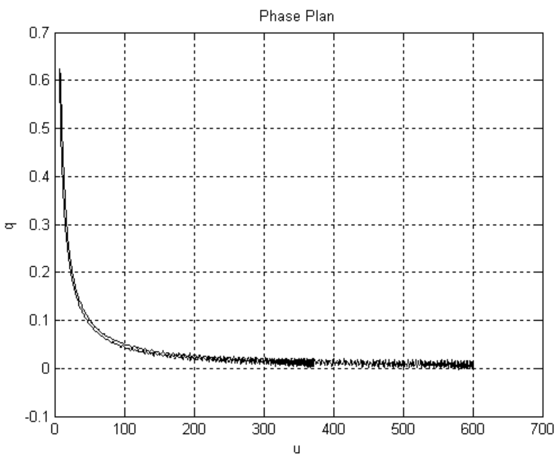


Figure 12. Phase Plan u vs. q

6. CONCLUSIONS:

As the several existing jet planes vary widely in their dynamic characteristics, it is not a practical endeavor to develop mathematical models for each of them. In this paper, we present a general model taking into account all the forces and moments involved in the modeling.

We analyzed the effect of sudden wind changes due to atmospheric turbulence on the dynamic response of the aircraft and its structural design.

As an application, we considered the equations of horizontal leveled motion of the F-8 jet aircraft. We considered the drag forces as small in comparison with the trust forces and its mass and we adopt the altitude as 9144 m. The moment of inertia about the pitch axis is taken as proportional to the 9773 kg mass of the aircraft.

The governing differential equations of motion of the aircraft ($\dot{x} = f(x,v)$), were written in terms of four variables ($x = (u,\alpha,\theta,q)$, where u : forward flight velocity; α :attack angle; θ :pitch angle; q =pitch rate; and v = a variation of the wind velocity, composed by a constant average level (v_0) and a fluctuating part.

Atmospheric turbulence was taken as a random process that we modeled by a series of superimposed harmonic functions. These were generated from an adopted Power Spectrum Density Function based on suggestions by Tajimi (1960) and Kanai (1957) in the context of seismic motions, as reported in Buchholdt (1999).

It was observed that the results that were exposed in the figures of 5 - 12 were satisfactory considering the longitudinal motion of the aircraft with the speed of aircraft variable. For the simulations it was verified that this method can be made efficiently what is an advance in the aeronautical research. It observed itself with that, that this method of to simulate the wind was satisfactory for the authors and could be used for questions of this nature, being overcome physical situations more realistic.

7. ACKNOWLEDGMENTS

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