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A BAND-LIMITED H∞ CONTROLLER FOR ACTIVE STRUCTURAL NOISE CONTROL

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Abstract. *This work presents an Active Structural Noise Control method, ASNC, for the mid-frequency range. This control approach aims to attenuate the modes of a structural system that are source of acoustical noise. Called Band-Limited H_∞ Controller, this approach allows to design a reduced order H_∞ controller to attenuate only noise radiating modes. This controller has the characteristic of keeping the dynamics of the residual modes unchanged, including a robustness characteristics with respect to the residual dynamics in the design. Therefore, by adding a robustness requirement with respect to parametric uncertainties, the performance of the closed-loop system is preserved under variations of the natural frequency and damping of the controlled modes. This approach is tested in a real experiment with a plate.*

Key-Words: *Active Structural Noise Control, Robust Control, Feedback Control, Active Control*

1. INTRODUCTION

The techniques to actively suppress noise are usually related with the feedforward approach and to the adaptive filter theory. This approach, defined as Active Noise Control, ANC, uses acoustic transducers to cancel the noise source in the acoustic field. However, in a large number of applications, the noise is originated by a mechanical vibration. Recently, Fuller and Flotow (1995) introduced a new concept about this problem, controlling the vibration in the structure in order to reduce the overall sound radiation. This technique, named Active Structural Acoustic Control, ASAC, uses structural actuators to control the mechanical vibrations while the noise radiation is minimized. Fuller's research, coming from the ANC theory, is a feedforward approach based on the model of wave propagation in the structure, Fuller *et al.* (1996).

On the other hand, there is the correlated area of Active Vibration Control, AVC. The AVC problem comes from the problem of the control of flexible structures, involving low frequency dynamics. It is mainly based in the feedback theory, Meirovitch (1990) and Inman (1989). However, with the fast progress of the modern control theory, the frequency range of the AVC problem has been increased, and now it is overlapping the ANC problem. In this scenario, the Active Noise and Vibration Control problem (ANVC) was introduced, Ross and Purver (1997). The ANVC uses structural transducers to control the vibration in the noise source as the Fuller ASAC problem. However, in ANVC, the model of the system is based in the dynamics of the structural modes, instead of structural waves.

These two approaches, ASAC and ANVC, have demonstrated the potential of mixing the AVC and the ANC problems, which can now be treated as a dual theory of waves and modes. This paper is a feedback approach contribution to this unification process. This work covers the problem defined as Active Structural Noise Control, ASNC. It means the active attenuation of structural modes of a mechanical system which are source of acoustical noise. It covers a serious lack in the feedback control for structural noise attenuation, the problem of controlling the modes in a frequency band in the mid-frequency range.

The concept of active vibration control is usually based upon a base-band controller. The control system has a low-pass filter characteristics. In this case, all modes from DC up to a specified cross-over is controlled. A truncated model of the structure is generally used, in which case it is necessary to guarantee the robustness to the residual dynamics. Using an H_∞ approach, it is possible to introduce in the design a weighting function, under the control signal, to roll off the high frequency dynamics, Leo *et al.* (1993) and Kajiwara & Nagamatsu (1995). This requires iterative adjustment of the weights to satisfy the robustness requirement. Applied to the control in mid-range frequencies, this approach requires a model of the system with all modes up to the chosen frequency to be controlled. The result is a model with an order that is greater than the order that would be necessary if the uncontrolled low frequency modes had not been considered.

In this paper, a band-limited H_∞ controller which has a system model with less modes than the traditional approach (base-band model) is proposed. This model includes only the modes to be controlled in a reduced design model, a band-limited model restricted to the region $f_{min} < f < f_{max}$. The residual low and high frequency dynamics is modeled as unstructured additive uncertainty. In addition, the robustness to the parameter variations is included in the design. The result is a controller to attenuate generic distinct modes which is robust to imperfections in the system modeling, or variations in the dynamics of the controlled system.

In summary, this work presents the band-limited H_∞ approach to ANVC. First, a model partition of the nominal reduced model and the residual dynamics is introduced. Then, concepts of robustness with respect to residual modes are presented. Finally, the robustness with respect to the parameter variations is added to the design. An experimental example is used to illustrate the application of the proposed approach.

2. THE BAND-LIMITED H_∞ APPROACH

The design model of the structure includes only the modes to be controlled. The unmodeled modes are included in a residual model to make sure that the dynamics of this residual model does not interfere with the controlled system. Thus, a robustness requirement with respect to the residual dynamics is included in the design model. Otherwise, the closed-loop system could become unstable.

Based upon the uncertainty theory, the system model is partitioned in a reduced model and a residual model, such that

$$\begin{bmatrix} \dot{x}_r \\ \dot{x}_m \end{bmatrix} = \begin{bmatrix} \mathbf{A}_r & 0 \\ 0 & \mathbf{A}_m \end{bmatrix} \begin{bmatrix} x_r \\ x_m \end{bmatrix} + \begin{bmatrix} \mathbf{B}_r \\ \mathbf{B}_m \end{bmatrix} u$$
$$y = \begin{bmatrix} \mathbf{C}_r & \mathbf{C}_m \end{bmatrix} \begin{bmatrix} x_r \\ x_m \end{bmatrix} \tag{1}$$

where the subscript m is related to the nominal model, with the modes in the frequency region $f_{min} < f < f_{max}$ to be controlled, and the subscript r is related to the residual model of low and high frequency.

The residual dynamics is defined as

$$\begin{bmatrix} \dot{x}_{lf} \\ \dot{x}_{hf} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{lf} & 0 \\ 0 & \mathbf{A}_{hf} \end{bmatrix} \begin{bmatrix} x_{lf} \\ x_{hf} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{lf} \\ \mathbf{B}_{hf} \end{bmatrix} u$$
$$y_r = \begin{bmatrix} \mathbf{C}_{lf} & \mathbf{C}_{hf} \end{bmatrix} \begin{bmatrix} x_{lf} \\ x_{hf} \end{bmatrix} \tag{2}$$

where the subscript lf denotes the low frequency dynamics, $f<f_{min}$, and the subscript hf denotes the high frequency dynamics, $f>f_{max}$., and $x_r=[x_{lf}^T \; x_{hf}^T]^T$ and $\mathbf{C_r=[C_{lf} \;\; C_{hf}]}$.

Based on the uncertainty theory, the model given by Eq. (2) represents an unstructured uncertainty in the additive form. The additive unstructured uncertainty form is a straightforward way to introduce the robustness relative to the effects of the truncation in the control problem. In a transfer function form, it yields

$$\mathbf{G} = \mathbf{G}_m + \mathbf{G}_r \tag{3}$$

where $\mathbf{G}$ is the real system (infinite-dimensional), $\mathbf{G}_r=\mathbf{C}_r(\mathbf{I}s-\mathbf{A}_r)^{-1}\mathbf{B}_r$ is the residual model and $\mathbf{G}_m=\mathbf{C}_m(\mathbf{I}s-\mathbf{A}_m)^{-1}\mathbf{B}_m$ is the reduced model.

This particular uncertainty can be defined as a set of parameter-dependent models, such that, it can be possible to define a function $\mathbf{G}_{up}$ that represents a low order upper bound for the residual dynamics,

$$\|\mathbf{G}_{up}\|_\infty > \|\mathbf{G}_r\|_\infty \tag{4}$$

In this case $\mathbf{G}r_i$ represents a high order function (with all unmodeled modes of the low frequency and the high frequency region). Replacing $\mathbf{G}r$ by $\mathbf{G}_{up}$ in the design model, if there exist a controller that satisfies the robustness requirements with the uncertainty model $\mathbf{G}_{up}$, it assures the robustness of the controlled system to the residual dynamics $\mathbf{G}_r$, since $\|\mathbf{G}_r\|_\infty<\|\mathbf{G}_{up}\|_\infty$, *i. e.*, $\mathbf{G}_{up}$ is an upper bound function to $\mathbf{G}_r$ (Boyd and Barrat, 1990). Once that $\mathbf{G}_r$ is know at the moment the system is truncated, $\mathbf{G}_{up}$ can be defined before the design.

Partitioning the system, the robustness to the residual dynamics can be included in the controller design. The perturbed system scheme is founded as shown in Figure 1, an external feedback block Δ is added to the model without lost of generality, with $\|\Delta\|_\infty=1$.

If the system is not accurately modeled or if its dynamics can undergo variation, robustness to parameter uncertainty is also added to the design. This robustness characteristic is translated into a fractional representation of the uncertainty in the matrices **A**, **B**, **C** e **D**. Thus, decomposing the reduced model, $\mathbf{G}_m$, in a nominal model and a set of l uncertainty conditions in the fractional form, the generic form of the nominal model yields

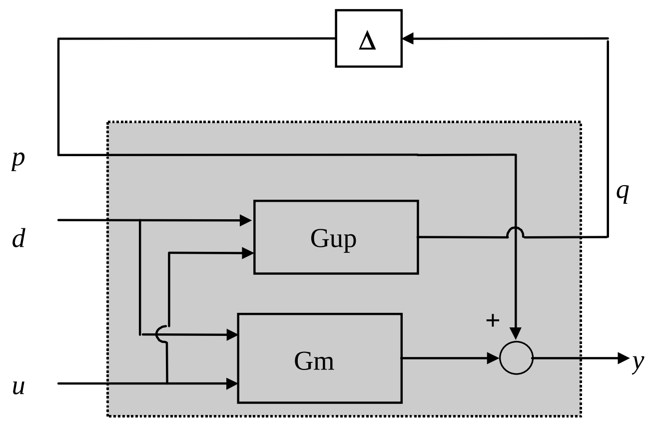

Figure 1: Perturbed system with an additive uncertainty.

$$\begin{bmatrix} \mathbf{A}_{\mathrm{j}} & \mathbf{B}_{\mathrm{j}} \\ \mathbf{C}_{\mathrm{j}} & \mathbf{D}_{\mathrm{j}} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{m} & \mathbf{B}_{m} \\ \mathbf{C}_{m} & \mathbf{D}_{m} \end{bmatrix} + \begin{bmatrix} \mathbf{H}_{\mathrm{o}} \\ \mathbf{H}_{\mathrm{i}} \end{bmatrix}_{\mathrm{j}} [\Delta]_{\mathrm{j}} [\mathbf{E}_{\mathrm{o}} \quad \mathbf{E}_{\mathrm{i}}]_{\mathrm{j}}\,, \qquad \|\Delta_{\mathrm{j}}\| = 1,\ 1 \le \mathrm{j} \le l \tag{6}$$

According to Xie (1996), this parameterization of the uncertainty can be translated into additional inputs and outputs in the system. These inputs and outputs are related with another external feedback loop, as occurred in the residual uncertainty case. Therefore, the system with parametric uncertainty is equivalent to

$$\begin{aligned} \dot{x} &= \mathbf{A}\,x + [\mathbf{B} \;\; \varepsilon\,\mathbf{H}_{\mathrm{o}}] \begin{bmatrix} w \\ \delta_{\mathrm{i}} \end{bmatrix} \\ \begin{bmatrix} z \\ \delta_{\mathrm{o}} \end{bmatrix} &= \begin{bmatrix} \mathbf{C} \\ \varepsilon^{-1}\,\mathbf{E}_{\mathrm{o}} \end{bmatrix} x + \begin{bmatrix} \mathbf{D} & \varepsilon\,\mathbf{H}_{\mathrm{i}} \\ \varepsilon^{-1}\,\mathbf{E}_{\mathrm{i}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} w \\ \delta_{\mathrm{i}} \end{bmatrix} \end{aligned} \tag{7}$$

where δ_i and δ_o are additional input and output vectors, related to the parameter uncertainties, and $\varepsilon>0$ is a scaling parameter.

To obtain the final design model, first one observes that the disturbance input d in the residual dynamics $\mathbf{G}_r$ in Figure 1 does not interfere in the feedback loop, *i. e.*, the control can not act on it. Hence, this specific input can be eliminated from the model. Additionally, to handle the problem of vibration attenuation, an output y, weighted by a low pass function $\mathbf{W}_1$, is added to the above diagram. Finally, introducing a scale factor K_2 in the residual uncertainty input and output, this problem can be converted in a scaled H_∞ problem. The final design block diagram is shown in Figure 2, where the exogenous inputs are $w=[d^T\ p^T\ \delta_i^T]^T$ and the regulated outputs are $z=[\ y'^T\ q^T\ \delta_o^T]^T$, where:

- d and y' are the disturbance input and the weighted measured output vectors, related to the damping increase requirement
- p and q are the additive uncertainty input and output vectors
- δ_i and δ_o are the input and output vectors associated to the parameter uncertainty.

Furthermore, observing Figure 2, the design parameters in this specific H_∞ problem become the gain γ, the attenuation K_2, the factor ε and the weighting function $\mathbf{W}_1$. The control targets are:

- to increase the damping of the required modes, adjusting functions $\mathbf{W}_1$ and γ
- to assure robustness to the residual dynamics, specifying $\mathbf{G}_{up}$ and setting K_2
- to restrain the performance degradation due to parameter variations, setting ε.

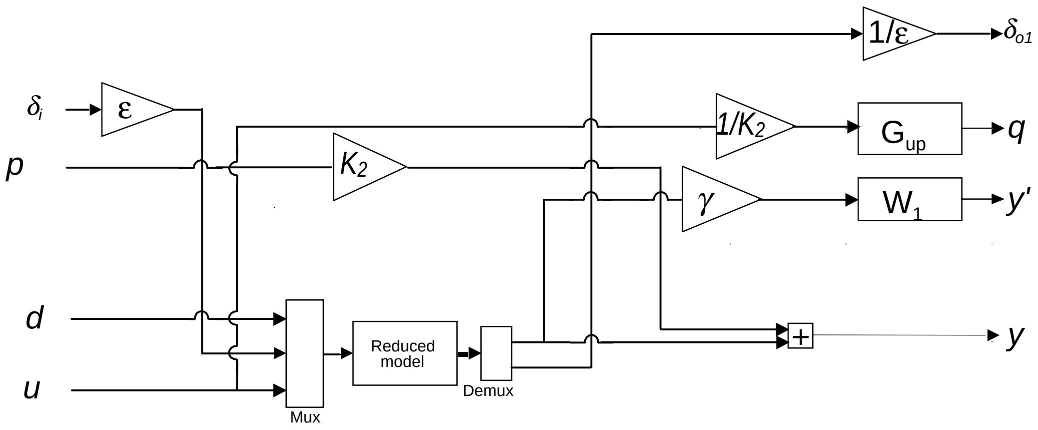

Figure 2: Design model diagram.

3. EXPERIMENTAL EXAMPLE: CONTROL OF TWO MODES OF A PLATE IN THE MID FREQUENCY RANGE

The experimental example proposed here illustrates the following problem: Consider a structure with structural borne noise due to some modes of the mid- frequency range. It is assumed that the actuators are attached in optimal positions to attenuate these specific modes. An optimal location is also determined to place the sensors. However, this position may not be available for some reason. It is the case of a microphone to measure noise in a aircraft cabin, for instance, to attenuate the noise in the passenger place. It is not possible to put the sensor in an optimal location, where the passenger head is. Rather, sensors are distributed is sub-optimal positions along the aircraft structure. Thus, in this example, a sub-optimal position is selected to measure the feedback signal. The so called optimal sensor is used only to define a cost function and to verify the attenuation obtained in the controlled system. In the implementation these optimal sensors can be suppressed once they do not appear in the feedback loop.

Plant model

To illustrate the design of a *band-limited H_∞ controller*, a plate experiment is proposed, meant to simulate the hypothetical ANVC problem described above. The structure, shown in Figure 3, is a 450*x*500*x*2 mm aluminum plate completely free along its boundaries. Two modes of this plate, located in the mid-frequency range, are considered acoustic noise sources. These two modes to be controlled have natural frequencies f_1=141 Hz and f_2=153 Hz. Two Piezo-ceramic patches, PZT, with dimensions 50*x*20*x*0.267 mm, are used as control actuators. They are positioned in optimal points, each one to attenuate one mode. Dividing the plate in square elements of dimension 50*x*50 mm, the actuators are located in the elements 32 and 68, as shown in Figure 3. An optimally localized PVDF film sensor is used to measure the vibration, located in the element 44. Suppose this optimal sensor location unrealizable to feed back the system, a sub-optimal position is determined in the element 65, where a 25*x*25*x*0.267 mm PZT patch is used as the feedback sensor. The disturbance signal is applied in the node of the elements 3, 4, 11 and 12. A hammer with a force transducer is used as the disturbance signal.

The PZT actuators use two LDS power amplifiers, model PA25E, to magnify the control signal. The controller is implemented is a dSPACE environment with a DS1003 processor board, a DS2003 DAC board and a DS2103 ADC board. The controller input signal is provided by the sub-optimal PZT sensor signal connected to a signal conditioner.

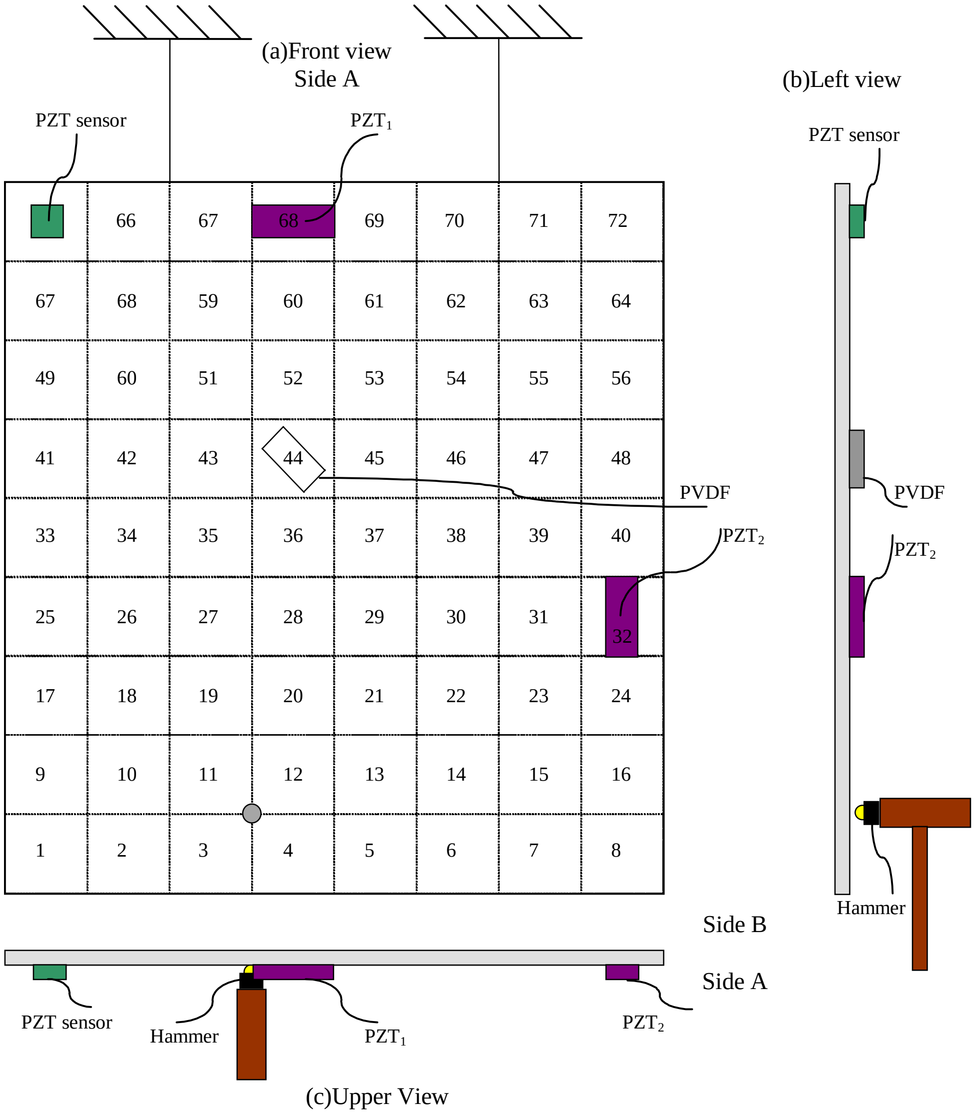

Figure 3: Free plate experiment.

An HP3566A signal analyzer is used to obtain an experimental model of the structure and to investigate the controller performance. The signals measured by the HP analyzer are the control inputs, the disturbance input, and the PVDF and PZT sensors. With these signals, a state-space model of the plate is determined by an identification program (Moreira and Arruda, 1997), using the ERA approach (Juang and Pappa, 1988). This state space realization has the disturbance signal, w, the PZT_1 actuator, u_1, and the PZT_2 actuator, u_2, as the input vector. The output vector consists of the PZT sensor signal, y_{PZT}, and the PVDF sensor signal, y_{PVDF}.

Controller design

The parameter variation is simulated by a lumped damping/mass device, with mass attached to the plate by a thick viscoelastic tape, and by small lumped mass. The nominal case is that one with the damping/mass attached to the element 8. The perturbed cases, considered the worst cases, are the plate without any lumped mass attached and the plate with two lumped mass attached in the elements 8 and 72, respectively. Thus, three models are identified experimentally.

After identifying the models, nominal and perturbed cases, the system is partitioned as described in section 2. For the controller design, a fourth-order model is assumed as the reduced nominal model. This model contains only the dynamics of the two modes to be controlled, with frequencies f_1=141 Hz and f_2=153 Hz. All the other modes, higher than the frequency f_2 and lower than frequency f_1, are considered residual dynamics. In the sequence, the uncertainty model is determined.

The design model is shown in Figure 2. In this design model diagram, the input d refers to the disturbance input, the control inputs, u=$[u_1\ u_2]^T$, are the PZT_1 and PZT_2 actuators, the input p is related to the uncertainty model and $\delta_i =[\ \delta_{1i} \ldots \delta_{6i}]^T$ is related with the parametric perturbation. The output y_{PVDF} is the regulated output related to the performance requirement, *i. e.*, the mode attenuation. The other regulated outputs are $q =[q_1\ q_2]^T$ associated to the uncertainty model and $\delta_o =[\ \delta_{1o} \ldots \delta_{6o}]^T$ related with the parametric perturbation. The PZT sensor signal, y_{PZT}, is the measured output, y, used to feed back the controller.

The uncertainty is a function from the two control inputs to the measured output. Thus, the uncertainty model yields

$$\mathbf{G}_{up} = \begin{bmatrix} \mathbf{G}_{up1} \\ \mathbf{G}_{up2} \end{bmatrix} \tag{8}$$

where

$$G_{upi} = k\,\frac{(s/\omega_{zi})^2 + 2\,\xi_{zi}\ s/\omega_{zi}\ +1}{(s/\omega_{pi})^2 + 2\,\xi_{pi}\ s/\omega_{pi}\ +1}\ , \qquad i=1,2 \tag{9}$$

The uncertainty model is shown in Figure 4. The residual dynamics in the figure refers to the nominal case and to the perturbed cases. The left plot is related with the transfer function from the control input 1, u_1, to the measured output, y_{PZT}. The right plot is related with the transfer function from the control input 2, u_2, to the measured output, y_{PZT}.

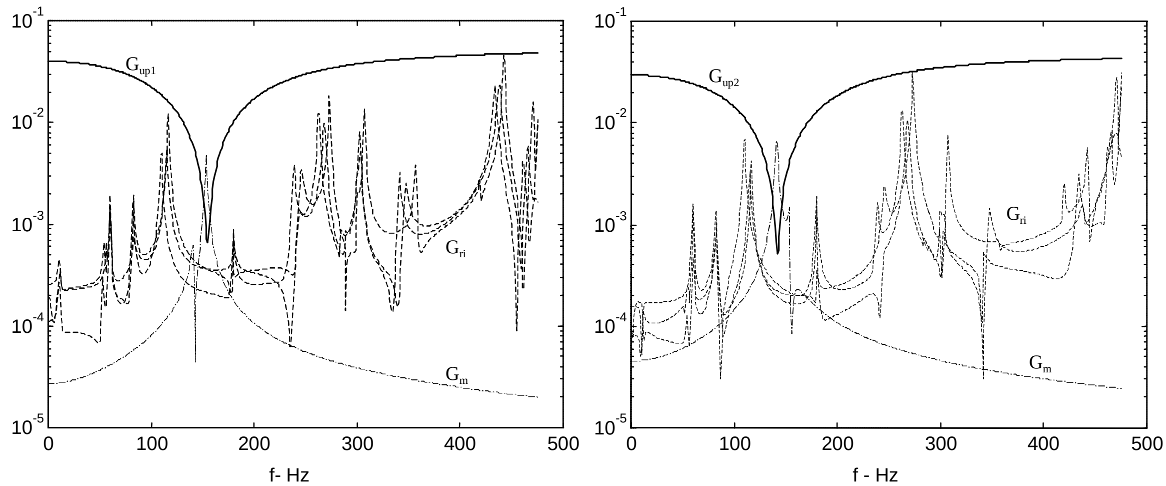

Figure 4: Frequency response to the residual uncertainty, $\mathbf{G}_{up}$, residual dynamics, $\mathbf{G}_r$ and the nominal model, $\mathbf{G}_m$.

The variation in the parameters of the model is shown in Figure 5, for the nominal case and the two perturbed cases. The left plot refers to the transfer function from the disturbance input, *d*, to the regulated output, y_{PVDF}. The right plot refers to the transfer function from the disturbance input, *d*, to the measured output, y_{PZT}.

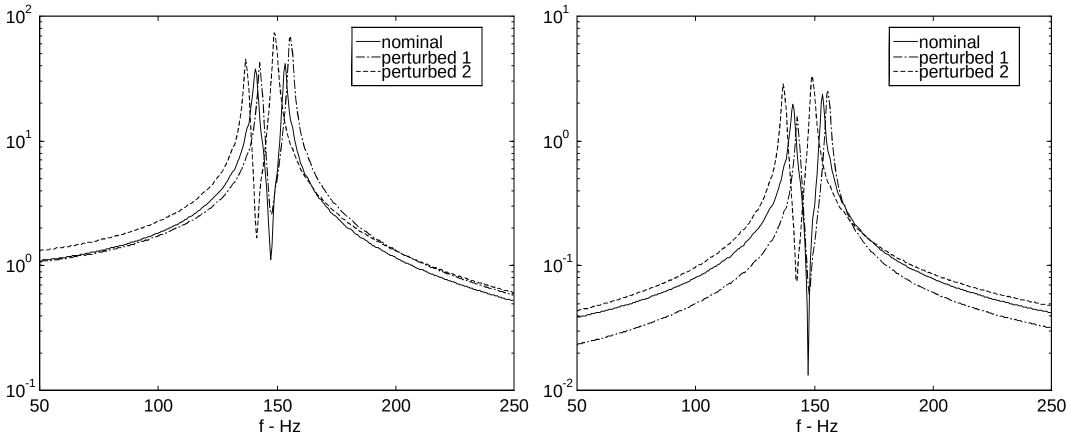

Figure 5: Dynamics of the reduced nominal model and perturbed cases

The matrices [**H**, **E**] of the fractional representation of the parametric uncertainty, Eq. (6), are determined by the singular value decomposition. The uncertainty model is assumed as the worst case of parameter variation. This case corresponds to the perturbed case, which presents the supreme singular value. After determining **H** and **E**, the augmented system is determined using Eq. (7). Thus, the design plant model is determined (see Figure. 2).

After some iteration over the free parameters, the final values chosen were γ=0.3, K2=0.01 and ε=1. The weight $\mathbf{W}_1$ is a second order function $W_1=\omega^2/(s^2+2\xi\omega s+\omega^2)$, with $\omega=2*\pi*147$ and ξ=0.1. It means a weighting function with a high gain in the region of the mode to be controlled. The parameters of the residual uncertainty, $\mathbf{G}_{up}$, are,

- to G_{up1} : k= 0.0541 , ω_{z1} = 2*π*154 rad/s, ξ_{z1} = 0.01 , ω_{p1} = 2*π*180 rad/s, ξ_{p1} = 0.7
- to G_{up2} : k= 0.0477 , ω_{z2} = 2*π*142 rad/s, ξ_{z2} = 0.01 , ω_{p2} = 2*π*180 rad/s, ξ_{p2} = 0.7

The robustness is quantified by mean of the FRF from the disturbance input to the control signals. Figure 6 shows the FRF of the non-robust closed-loop system, without the inclusion of the residual and the parametric uncertainty, in continuous line, and the robust one, in dashed line. The robustness is associated with the low magnitude of the peaks of the residual modes in the FRF for the robust control case. It means that the control energy do not excite the residual dynamics like the non-robust control.

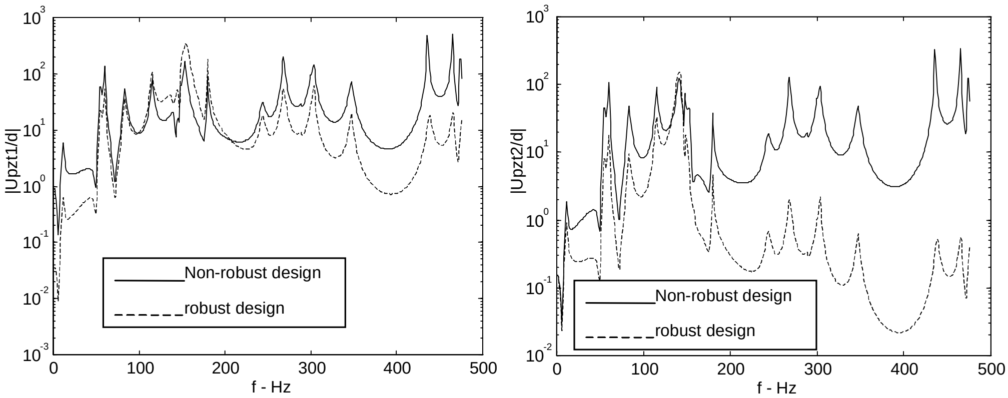

Figure 6: FRF from the disturbance input to the control signal.

Experimental analysis

The effectiveness of the controlled system is verified in a real time implementation. First the continuous controller must be converted into a discrete one. In this case, the dSPACE sample time is assumed as 0.1 ms. Again, the HP3566A signal analyzer is used to acquire the simulation data to investigate the controller performance. The signals measured by the HP analyzer are used to compute the FRF of the open-loop and closed-loop systems.

The result for the nominal case is shown in Figure 7. Figure 8 shows the experimental result for the perturbed cases. These figures present the FRFs from the input disturbance to the PZT sensor for the uncontrolled and controlled case. The increment in the damping of the controlled modes, located in f_1=141 Hz and f_2=153 Hz, is clear in Figure 8.

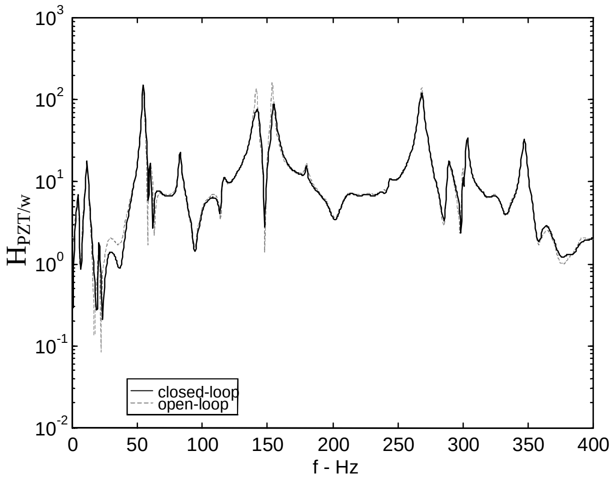

Figure 7: FRF for the nominal case.

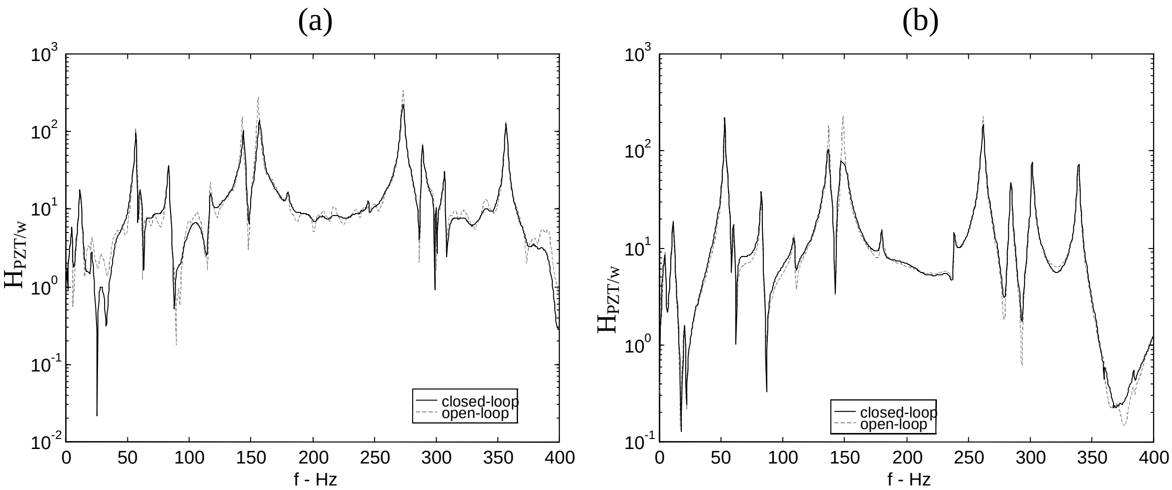

Figure 8: FRF for the (a)1st perturbed case (b) 2nd perturbed case.

As expected, the selected modes are attenuated, while the dynamics of the residual modes remains unchanged. The performance deterioration is satisfactory in the perturbed cases. Thus, all theoretical results are corroborated by the experiments.

4. CONCLUSION

A feedback approach is applied here in the attenuation of structural modes which are considered as noise sources. A band-limited H_∞ control is presented to attenuate modes in the mid-frequency range. The experimental example showed that the modes to be controlled can be attenuated without changing the dynamics of the remaining modes. Thus, the control effort is minimized since it is concentrated in the frequency band of the controlled modes. The inclusion of the robustness to residual and parametric uncertainties assures the robust performance of the closed-loop system. Moreover, the resultant controller presents a low order dynamics since a minimal order model of the structure is taken into account.

Acknowledgements

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