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A THEORETICAL-NUMERICAL STUDY IN SINGLE CRYSTALS LARGE DEFORMATION PLASTICITY

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Abstract. *The present work addresses theoretical and numerical aspects of single crystal elasto-viscoplasticity. A rate formulation of the constitutive equations is considered, where two types of self-hardening rules are studied. Cubic crystals under plane stress with two slip planes are considered in the present study. Numerical procedures related to the integration of the resulting constitutive relation is described in detail. Some quasi-static evolution problems are solved using the finite element method.*

Keywords. *Single Crystals, Viscoplasticity, Large Deformation, Numerical Methods.*

1 Introduction

The study on the mechanical behavior of single crystals is motivated basically by the fact that it supplies the basis to understand most of the phenomena observed in polycrystalline materials. Further, single crystals have been used as structural materials, as for instance, in the case of some turbine blades.

The theory of plasticity in metals in the infinitesimal setting, based on an additive decomposition of the total strain, cannot describe satisfactorily large plastic deformations of such materials. Lee and Liu [1967] and Lee [1969] introduced a multiplicative decomposition of the gradient of deformation of the form $\mathbf{F} = \mathbf{F}^e\mathbf{F}^p$. Such decomposition was considered years later in studies performed by Hill and Havner [1982] and by Peirce et al [1982] for the description of the elastoplastic behavior of single crystals. On the other hand, it is worth mentioning that both papers were strongly influenced by the constitutive formulation proposed by Hill and Rice [1972].

Because of its highly nonlinear character, plasticity in metals imposes certain difficulties in finding analytical solutions to some practical problems. However, the development of computer technology, together with advances in numerical techniques, allowed the obtention of solutions to complicated problems in many areas of science and technology. In plasticity and viscoplasticity, contributions from authors like Simo and Hughes [1998] to the algorithmic formulation of their constitutive behaviours made feasible the search for solutions in complex geometries and boundary conditions.

This work presents a very simple study on elastoviscoplasticity in single crystals subjected to finite deformations in the framework of the ideas pesented by Hill and Rice [1972]. A computational algorithm is implemented in order to study the behavior of the rate formulated constitutive equations — where two types of self-hardening rules are studied — and also to obtain results in some quasi-static evolution problems using the finite element method. For the sake of simplicity, only cubic crystals under plane stress with two slip planes are considered in the present study.

2 Multiplicative Decomposition

Let a deformable body $\mathfrak{B}$ be described in two distinct states: a reference configuration $\mathcal{B}_0$ at time t_0 and a spatial configuration $\mathcal{B}_t$ at time t. Let $\mathbf{dX}$ be an infinitesimal line of $\mathcal{B}_0$ and $\mathbf{dx}$ the corresponding infinitesimal line in $\mathcal{B}_t$ such that

$$\mathbf{dx} = \mathbf{F}\mathbf{dX}, \tag{1}$$

where $\mathbf{F}$ is the gradient of deformation, which describes the variation in length of all material lines of the body $\mathfrak{B}$ during a deformation process.

Following LEE[1969], let us assume as valid the Multiplicative Decomposition:

$$\mathbf{F} = \mathbf{F}^{\mathrm{e}}\mathbf{F}^{\mathrm{p}}, \tag{2}$$

where $\mathbf{F}^{\mathrm{e}}$ and $\mathbf{F}^{\mathrm{p}}$ account respectively for the elastic and the plastic part of the gradient of deformation $\mathbf{F}$.

The velocity gradient $\mathbf{L}$ can be related to the gradient of deformation as:

$$\mathbf{L} := \dot{\mathbf{F}}\mathbf{F}^{-1}, \tag{3}$$

and, as a consequence, we have:

$$\mathbf{L} = \dot{\mathbf{F}}^{\mathrm{e}}\mathbf{F}^{\mathrm{e}-1} + \mathbf{F}^{\mathrm{e}}\dot{\mathbf{F}}^{\mathrm{p}}\mathbf{F}^{\mathrm{p}-1}\mathbf{F}^{\mathrm{e}-1}. \tag{4}$$

Decomposition of $\mathbf{L}$ into symmetric and antisymmetric parts $\mathbf{D}$ and Ω, respectively, leads to:

$$\mathbf{L} = \mathbf{D} + \Omega = \mathbf{L}^{\mathrm{e}} + \mathbf{L}^{\mathrm{p}}, \tag{5}$$

where

$$\mathbf{L}^{\mathrm{e}} := \dot{\mathbf{F}}^{\mathrm{e}}\mathbf{F}^{\mathrm{e}-1} \quad \text{and} \quad \mathbf{L}^{\mathrm{p}} := \mathbf{F}^{\mathrm{e}}\dot{\mathbf{F}}^{\mathrm{p}}\mathbf{F}^{\mathrm{p}-1}\mathbf{F}^{\mathrm{e}-1}. \tag{6}$$

From (5), we can also write:

$$\mathbf{D} = \mathbf{D}^{\mathrm{e}} + \mathbf{D}^{\mathrm{p}} \quad \text{and} \quad \Omega = \Omega^{\mathrm{e}} + \Omega^{\mathrm{p}}, \tag{7}$$

where

$$\mathbf{D}^{\mathrm{e}} = \frac{1}{2}\left[\dot{\mathbf{F}}^{\mathrm{e}}\mathbf{F}^{\mathrm{e}-1} + \left(\dot{\mathbf{F}}^{\mathrm{e}}\mathbf{F}^{\mathrm{e}-1}\right)^{\mathrm{T}}\right], \quad \mathbf{D}^{\mathrm{p}} = \tfrac{1}{2}\left[\mathbf{F}^{\mathrm{e}}\dot{\mathbf{F}}^{\mathrm{p}}\mathbf{F}^{\mathrm{p}-1}\mathbf{F}^{\mathrm{e}-1} + \left(\mathbf{F}^{\mathrm{e}}\dot{\mathbf{F}}^{\mathrm{p}}\mathbf{F}^{\mathrm{p}-1}\mathbf{F}^{\mathrm{e}-1}\right)^{\mathrm{T}}\right],$$
$$\Omega^{\mathrm{e}} = \mathbf{L}^{\mathrm{e}} - \mathbf{D}^{\mathrm{e}}, \qquad \Omega^{\mathrm{p}} = \mathbf{L}^{\mathrm{p}} - \mathbf{D}^{\mathrm{p}}.$$

The tensor $\mathbf{D}$ is the total stretching tensor while Ω is the total spin tensor.

3 Large Crystalline Deformation

In this work, the description of the elastoviscoplastic deformation of single crystals is based on the following assumptions proposed by HILL AND RICE [1972, p.401]: "... (i) distortion of the lattice is effectively elastic; (ii) the crystal also deforms by simple shears relative to specific lattice planes and directions; (iii) such 'slip systems' are active only when the corresponding shear stresses attain critical values; and (iv) each value is a functional of the entire slip history of the crystal."

3.1 Kinematics

The tensorial quantities $\mathbf{F}^{\mathrm{e}}$ and $\mathbf{F}^{\mathrm{p}}$ defined by the Lee multiplicative decomposition perform the following actions:

a) The Viscoplastic Deformation Gradient $\mathbf{F}^{\mathrm{p}}$ is responsible for the viscoplastic deformation of the single crystal, characterized by the slipping of lines or planes. This deformation does not produce rotation and it occurs with no volume change, i.e. $J^{\mathrm{p}} := \det(\mathbf{F}^{\mathrm{p}}) = 1$.

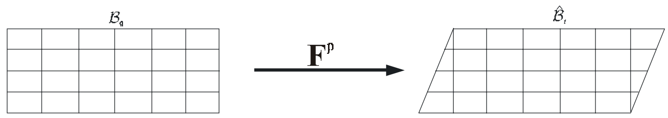

Figure 1: Action of the Viscoplastic Deformation Gradient

b) The Elastic Deformation Gradient$\mathbf{F}^{\mathrm{e}}$ is responsible for the elastic deformation of the single crystal, i.e., for the distortion of the crystalline lattice. Stretching and rotation are representative motions of this deformation which can be associated with changes in volume, i.e. $J := \det(\mathbf{F}) = \det(\mathbf{F}^{\mathrm{e}})$.

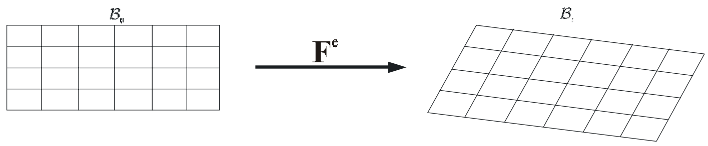

Figure 2: Action of the Elastic Deformation Gradient

The process of elastoplastic deformation is described by the combined effects of $\mathbf{F}^{\mathrm{p}}$ acting on $\mathcal{B}_0$ into the "intermediate configuration" $\hat{\mathcal{B}}_t$ and of $\mathbf{F}^{\mathrm{e}}$ acting on $\hat{\mathcal{B}}_t$ into the configuration $\mathcal{B}_t$.

In order to study a specific slipping system, it is convenient to consider a reference that keeps its orthogonality during the deformation process. In this sense, let $(\mathbf{s_0}, \mathbf{m_0})$ be an orthonormal system defined at the configuration $\mathcal{B}_0$. The vector $\mathbf{s}_0$ is parallel to the slip direction and $\mathbf{m}_0$ is normal to the slip plane. The tensor $\mathbf{F}^{\mathrm{p}}$ does not modify the orthonormality of $(\mathbf{s_0}, \mathbf{m_0})$. Thus, if $(\mathbf{s}, \mathbf{m})$ is related to the intermediate system $\hat{\mathcal{B}}_t$, then it follows that $(\mathbf{s_0}, \mathbf{m_0}) \equiv (\mathbf{s}, \mathbf{m})$. Making use of this equivalence and considering $(\mathbf{s}_*, \mathbf{m}_*)$ the system $(\mathbf{s_0}, \mathbf{m_0})$ at the configuration $\mathcal{B}_t$, the following can be stated:

$$(\mathbf{s}_*, \mathbf{m}_*) = \left(\mathbf{F}^{\mathrm{e}}\mathbf{s}, \mathbf{F}^{\mathrm{e}-\mathrm{T}}\mathbf{m}\right) . \tag{8}$$

where $(\mathbf{s}_*, \mathbf{m}_*)$ preserve orthogonality. The schematic representation of the elastoviscoplastic deformation process can be observed in the figure 3.

The amount of slipping in a determined plane (or line) α can be measured by a variable related to the shearing angle $\beta^{(\alpha)}$. For this purpose, the measure $\gamma^{(\alpha)}$ of slip can be defined as

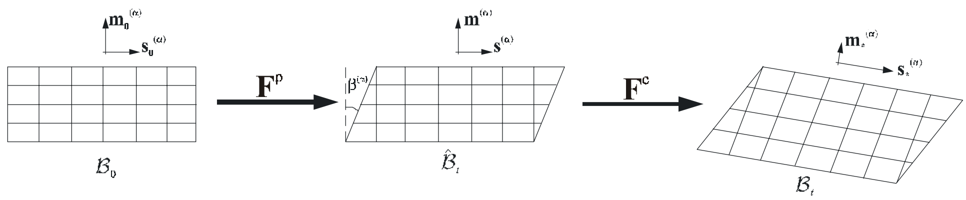

Figure 3: Large Single Crystal Deformation

the tangent of $\beta^{(\alpha)}$ when a unitary vertical distance is assumed. This micromechanic variable can be related to a macromechanical one through the following equality:

$$\mathbf{F}^{\mathrm{p}}_{\alpha} = \mathbf{I} + \gamma^{(\alpha)} \left(\mathbf{s_0}^{(\alpha)} \otimes \mathbf{m_0}^{(\alpha)} \right) . \tag{9}$$

Using (9), we obtain:

$$\dot{\mathbf{F}}^{\mathrm{p}}_{\alpha} \mathbf{F}^{\mathrm{p}-1}_{\alpha} = \dot{\gamma}^{(\alpha)} \left(\mathbf{s}^{(\alpha)} \otimes \mathbf{m}^{(\alpha)} \right) .$$

Assuming "n" slipping planes (or lines) we have:

$$\dot{\mathbf{F}}^{\mathrm{p}} \mathbf{F}^{\mathrm{p}-1} = \sum_{\alpha=1}^{\mathrm{n}} \dot{\gamma}^{(\alpha)} \left(\mathbf{s}^{(\alpha)} \otimes \mathbf{m}^{(\alpha)} \right) . \tag{10}$$

From (6) and (10), we obtain:

$$\begin{aligned} \mathbf{L}^{\mathrm{p}} &= \sum_{\alpha=1}^{\mathrm{n}} \dot{\gamma}^{(\alpha)} \left(\mathbf{s}_*^{(\alpha)} \otimes \mathbf{m}_*^{(\alpha)} \right) , \\ \therefore \quad \mathbf{D}^{\mathrm{p}} &= \frac{1}{2} \sum_{\alpha=1}^{\mathrm{n}} \dot{\gamma}^{(\alpha)} \left[\left(\mathbf{s}_*^{(\alpha)} \otimes \mathbf{m}_*^{(\alpha)} \right) + \left(\mathbf{m}_*^{(\alpha)} \otimes \mathbf{s}_*^{(\alpha)} \right) \right] . \end{aligned}$$

3.2 Constitutive Formulation

3.2.1 Stress-Strain Relation

The authors CUITIÑO AND ORTIZ [1992] proposed the following stress-strain hypoelastic relation for the elastic deformation of crystalline solids:

$$L_v^{\mathrm{e}} \tau = \mathbb{C} \mathbf{D}^{\mathrm{e}}, \tag{11}$$

where $L_v^{\mathrm{e}} \tau$ is called the Lie Derivative of the Kirchhoff stress τ, $\mathbb{C}$ is the stiffness fourth order tensor and $\mathbf{D}^{\mathrm{e}} = \mathbf{D} - \mathbf{D}^{\mathrm{p}}$. The objective stress rate $L_v^{\mathrm{e}} \tau$ is defined as:

$$L_v^{\mathrm{e}} \tau := \mathbf{F}^{\mathrm{e}} \left\{ \frac{\mathrm{d}}{\mathrm{d}t} \left[\mathbf{F}^{\mathrm{e}-1} \tau \mathbf{F}^{\mathrm{e}-\mathrm{T}} \right] \right\} \mathbf{F}^{\mathrm{eT}}. \tag{12}$$

3.2.2 Schmid's Law

The so called Schmid Stress $\tau^{(\alpha)}$ is the component of the applied stress acting on the direction of slip in the plane α. Using energy balance considerations, it is possible to obtain the following micro-macro correspondence:

$$\tau^{(\alpha)} = \tau : \left(\mathbf{s}_*^{(\alpha)} \otimes \mathbf{m}_*^{(\alpha)} \right) . \tag{13}$$

The Critical Schmid Stress $\tau_{\mathrm{c}}^{(\alpha)}$ is the limiting value for $\tau^{(\alpha)}$ in such a way that when $\tau^{(\alpha)} = \tau_{\mathrm{c}}^{(\alpha)}$, slipping occurs. This fact is known as Schmid's Law.

3.2.3 Flow Law

The flow law represents the evolution equation for the viscoplastic deformation. In this work, we consider a law proposed by HUTCHINSON [1976]:

$$\dot{\gamma}^{(\alpha)} = \dot{\gamma}_0 \frac{\tau^{(\alpha)}}{\tau_c^{(\alpha)}} \left[\left| \frac{\tau^{(\alpha)}}{\tau_c^{(\alpha)}} \right| \right]^{\frac{1}{m}-1} , \tag{14}$$

where $\dot{\gamma}_0$ is the reference rate of slipping, $\tau^{(\alpha)}$ is the Schmid Stress in plane α, $\tau_c^{(\alpha)}$ is the Critical Schmid Stress in plane α and m is the material rate sensitivity.

3.2.4 Strain Hardening

Tests performed with single crystals reveals that slip systems need increasing loads to keep slipping. It is also observed that the stress in a determined system depends not only on its slipping history (self-hardening), but on the global slipping history of all other slip systems (latent hardening). In order to model these phenomena, HILL [1966] proposed the following hardening law:

$$\dot{\tau}_c^{(\alpha)} = \sum_{\beta=1}^{n} h_{\alpha\beta} \left| \dot{\gamma}^{(\beta)} \right| , \tag{15}$$

where $h_{\alpha\beta}$ is the hardening modulus. Among some possible hardening laws described in the literature, the present study adopts the one proposed by PEIRCE ET AL [1982] :

$$h_{\alpha\beta} = [q + (1-q)\,\delta_{\alpha\beta}]\, h^{(\beta)} , \tag{16}$$

where q is the parameter that ranges in $1 < q < 1,4$ and h is the self hardening material parameter.

Two distinct laws describing the parameter $h^{(\beta)}$ are considered. The first one was proposed by PEIRCE ET AL [1982] and is given by:

$$h^{(\beta)} = H_0 \operatorname{sech}^2 \left(\frac{H_0 \gamma}{\tau_s - \tau_0} \right) \quad \text{when} \quad \gamma = \sum_{\alpha=1}^{n} \gamma^{(\alpha)} , \tag{17}$$

where H_0 is initial hardening modulus, τ_s is the saturation value for the Schmid Stress and τ_0 is the initial critical Schmid Stress. The other hardening law, proposed by ANAND AND KOTHARI [1996], is given by:

$$h^{(\beta)} = H_0 \left(1 - \frac{\tau_c^{(\beta)}}{\tau_s} \right)^a , \tag{18}$$

where a is a material exponent.

4 Numerical Solutions

4.1 Solution of the Incremental Constitutive Equations

This work uses the Newton-Raphson iterative method to solve the Implicit Euler discretized constitutive equations. In the case of a scalar function $g(x)$, the method is described by following scheme:

$$x_{k+1} = x_k - \frac{g(x_k)}{g'(x_k)} . \tag{19}$$

If a tensorial function $G(\mathbf{A})$ is considered, then the method can be written as:

$$\text{grad}\, G(\mathbf{A}_k)\,[\mathbf{A}_{k+1} - \mathbf{A}_k] \;=\; -G(\mathbf{A}_k)\,. \tag{20}$$

By considering (10), we obtain:

$$\Phi\left((\mathbf{F}^{\mathrm{p}})_{n+1}\right) := \frac{(\mathbf{F}^{\mathrm{p}})_{n+1} - (\mathbf{F}^{\mathrm{p}})_n}{\Delta t} - \left[\sum_{\alpha=1}^{\mathrm{n}} \frac{(\gamma^{(\alpha)})_{n+1} - (\gamma^{(\alpha)})_n}{\Delta t}\left(\mathbf{s}^{(\alpha)} \otimes \mathbf{m}^{(\alpha)}\right)\right](\mathbf{F}^{\mathrm{p}})_{n+1} = 0\,. \tag{21}$$

$$\text{grad}\,\Phi\left([(\mathbf{F}^{\mathrm{p}})_{n+1}]_r\right)\left\{[(\mathbf{F}^{\mathrm{p}})_{n+1}]_{r+1} - [(\mathbf{F}^{\mathrm{p}})_{n+1}]_r\right\} = -\Phi\left([(\mathbf{F}^{\mathrm{p}})_{n+1}]_r\right)\,, \tag{22}$$

where

$$\text{grad}\,\Phi\left((\mathbf{F}^{\mathrm{p}})_{n+1}\right) \;=\; \frac{\partial\,\Phi_{ij}\left((\mathbf{F}^{\mathrm{p}})_{n+1}\right)}{\partial\,(F^{\mathrm{p}}_{pq})_{n+1}}\,\mathbf{e}_i \otimes \mathbf{e}_j \otimes \mathbf{e}_p \otimes \mathbf{e}_q\,. \tag{23}$$

Similarly to the scalar expressions (14) and (15), the following can be written:

$$\Upsilon\left((\tau_{\mathrm{c}}^{(\alpha)})_{n+1}\right) := \frac{(\tau_{\mathrm{c}}^{(\alpha)})_{n+1} - (\tau_{\mathrm{c}}^{(\alpha)})_n}{\Delta t} - \left\{\sum_{\beta=1}^{\mathrm{n}} (h_{\alpha\beta})_{n+1}\left|\frac{(\gamma^{(\beta)})_{n+1} - (\gamma^{(\beta)})_n}{\Delta t}\right|\right\} = 0\,, \tag{24}$$

$$[(\tau_{\mathrm{c}}^{(\alpha)})_{n+1}]_{r+1} = [(\tau_{\mathrm{c}}^{(\alpha)})_{n+1}]_r - \frac{\Upsilon\left([(\tau_{\mathrm{c}}^{(\alpha)})_{n+1}]_r\right)}{\Upsilon'\left([(\tau_{\mathrm{c}}^{(\alpha)})_{n+1}]_r\right)}\,. \tag{25}$$

$$\Lambda\left((\gamma^{(\alpha)})_{n+1}\right) := \frac{(\gamma^{(\alpha)})_{n+1} - (\gamma^{(\alpha)})_n}{\Delta t} - \dot{\gamma}_0\,\frac{(\tau^{(\alpha)})_{n+1}}{(\tau_{\mathrm{c}}^{(\alpha)})_{n+1}}\left[\left|\frac{(\tau^{(\alpha)})_{n+1}}{(\tau_{\mathrm{c}}^{(\alpha)})_{n+1}}\right|\right]^{\frac{1}{\mathrm{m}}-1} = 0\,, \tag{26}$$

$$[(\gamma^{(\alpha)})_{n+1}]_{k+1} = [(\gamma^{(\alpha)})_{n+1}]_k - \frac{\Lambda\left([(\gamma^{(\alpha)})_{n+1}]_k\right)}{\Lambda'\left([(\gamma^{(\alpha)})_{n+1}]_k\right)}\,. \tag{27}$$

4.2 Discretization of the Bidimensional Domain

In this work the finite element formulation is based on the rate form of the virtual power principle and the Galerkin Method. This principle can be written for crystalline plasticity as:

$$\int_{\mathcal{B}_t} J^{-1}\left(L_v^{\mathrm{e}}\tau - \mathbf{L}^{\mathrm{p}}\tau - \tau\mathbf{L}^{\mathrm{p}\mathrm{T}} + \mathbf{L}\tau\right) : \text{grad}\eta_t\, dV = \int_{\partial\mathcal{B}_t^t} \dot{\mathbf{t}}\cdot\eta_t\, dS + \int_{\mathcal{B}_t} \rho\dot{\mathbf{b}}\cdot\eta_t\, dV\,. \tag{28}$$

where η_t is the spatial virtual velocity, $\mathbf{t}$ is the contact force density and $\mathbf{b}$ is the body force density.

In the case of plane stresses, the stress-strain relationship is written as:

$$\begin{bmatrix} (L_v^{\mathrm{e}}\tau)_{xx} \\ (L_v^{\mathrm{e}}\tau)_{yy} \\ (L_v^{\mathrm{e}}\tau)_{xy} \end{bmatrix} = \begin{bmatrix} \zeta - \dfrac{\beta^2}{\zeta} & \beta - \dfrac{\beta^2}{\zeta} & 0 \\ \beta - \dfrac{\beta^2}{\zeta} & \zeta - \dfrac{\beta^2}{\zeta} & 0 \\ 0 & 0 & \kappa \end{bmatrix}\begin{bmatrix} \mathbf{D}^{\mathrm{e}}_{xx} \\ \mathbf{D}^{\mathrm{e}}_{yy} \\ 2\mathbf{D}^{\mathrm{e}}_{xy} \end{bmatrix} \quad \text{or} \quad \{L_v^{\mathrm{e}}\tau\} = [\mathbb{C}]\,\{\mathbf{D}^{\mathrm{e}}\}\,, \tag{29}$$

where ζ, β and κ are the coefficients of the stiffness matrix $\mathbb{C}$ when a cubic crystal is considered. The spatial discretization leads to the the description of the fields η_t^h and $\mathbf{v}^h$ as:

$$\eta_t^h = \sum_{i=1}^{m^h} \eta_{t\,i}^h \mathbf{h}_i^{\mathbf{t}} \qquad \text{and} \qquad \mathbf{v}^h = \sum_{j=1}^{m^h} v_j \mathbf{h}_j^{\mathbf{t}} + \bar{\mathbf{v}}\,, \tag{30}$$

and hence (28) can be rewritten as:

$$\begin{aligned} \int_{\mathcal{B}_t} \mathrm{B}^{\mathrm{T}}\left[\mathbb{C}^{\mathrm{plast}}\right]\mathrm{B}\mathrm{V}^h + \check{\mathrm{B}}^{\mathrm{T}}\lfloor\tau^h\rfloor\check{\mathrm{B}}\mathrm{V}^h \frac{dV}{J^h} &= \int_{\partial\mathcal{B}_t} \Theta^{\mathrm{T}}\dot{\mathbf{t}}^h\,dS + \int_{\mathcal{B}_t} \Theta^{\mathrm{T}}\rho\dot{\mathbf{b}}^h\,dV - \\ &- \int_{\mathcal{B}_t} \mathrm{B}^{\mathrm{T}}[\mathbb{C}]\,\mathrm{B}\,\{[\mathrm{grad}\,\bar{\mathbf{v}}]_S\} + \check{\mathrm{B}}^{\mathrm{T}}\lfloor\tau^h\rfloor\check{\mathrm{B}}\{\mathrm{grad}\,\bar{\mathbf{v}}\}\frac{dV}{J^h}\,, \end{aligned} \tag{31}$$

where

$$\mathrm{B} := \begin{bmatrix} \partial\mathbf{h}_1^{\mathbf{t}}/\partial x & 0 & \partial\mathbf{h}_2^{\mathbf{t}}/\partial x & 0 & \dots \\ 0 & \partial\mathbf{h}_1^{\mathbf{t}}/\partial y & 0 & \partial\mathbf{h}_2^{\mathbf{t}}/\partial y & \dots \\ \partial\mathbf{h}_1^{\mathbf{t}}/\partial y & \partial\mathbf{h}_1^{\mathbf{t}}/\partial x & \partial\mathbf{h}_2^{\mathbf{t}}/\partial y & \partial\mathbf{h}_2^{\mathbf{t}}/\partial x & \dots \end{bmatrix};$$

$$\Theta := \begin{bmatrix} \mathbf{h}_1^{\mathbf{t}} & 0 & \mathbf{h}_2^{\mathbf{t}} & 0 & \dots \\ 0 & \mathbf{h}_1^{\mathbf{t}} & 0 & \mathbf{h}_2^{\mathbf{t}} & \dots \end{bmatrix}; \qquad \mathrm{V}^h := \begin{bmatrix} v_{x1} \\ v_{y1} \\ v_{x2} \\ v_{y2} \\ \vdots \end{bmatrix}; \qquad \lfloor\tau^h\rfloor := \begin{bmatrix} \tau_{xx}^h & 0 & \tau_{xy}^h & 0 \\ 0 & \tau_{yy}^h & 0 & \tau_{xy}^h \\ \tau_{xx}^h & 0 & \tau_{yy}^h & 0 \\ 0 & \tau_{xy}^h & 0 & \tau_{xx}^h \end{bmatrix};$$

$$\check{\mathrm{B}} := \begin{bmatrix} \partial\mathbf{h}^{\mathbf{t}}{}_1/\partial x & 0 & \partial\mathbf{h}^{\mathbf{t}}{}_2/\partial x & 0 & \dots \\ 0 & \partial\mathbf{h}^{\mathbf{t}}{}_1/\partial y & 0 & \partial\mathbf{h}^{\mathbf{t}}{}_2/\partial y & \dots \\ \partial\mathbf{h}^{\mathbf{t}}{}_1/\partial y & 0 & \partial\mathbf{h}^{\mathbf{t}}{}_2/\partial y & 0 & \dots \\ 0 & \partial\mathbf{h}^{\mathbf{t}}{}_1/\partial x & 0 & \partial\mathbf{h}^{\mathbf{t}}{}_2/\partial x & \dots \end{bmatrix};$$

$$\mathbb{C}_{ijpq}^{\mathrm{plast}} = \frac{\left[L_v^e\tau^h - \mathbf{L}^{\mathrm{p}h}\tau^h - \tau^h(\mathbf{L}^{\mathrm{p}h})^{\mathrm{T}}\right]_{ij}^{\mathrm{pert}} - \left[L_v^e\tau^h - \mathbf{L}^{\mathrm{p}h}\tau^h - \tau^h(\mathbf{L}^{\mathrm{p}h})^{\mathrm{T}}\right]_{ij}}{[\mathbf{D}^h]_{pq}^{\mathrm{pert}} - \mathbf{D}_{pq}^h}\,,$$

where $[\bullet]^{\mathrm{pert}}$ represents perturbed components following CAR ET AL [1997].

5 Numerical Results

The results obtained for the numerical results are based on the following plane stress scheme with double slip (two slip planes):

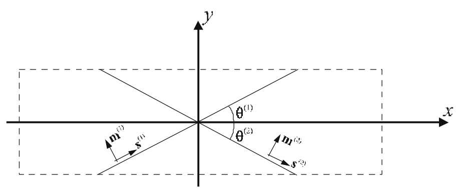

Figure 4: Model for the Numerical Solutions.

5.1 Constitutive Formulation

The values for the numerical constants are: $\dot{\gamma}_0 = 0,001$; m $= 0,05$; q$= 1,2$; $\tau_0 = 16$MPa. The self-hardening parameters H_0 and τ_s are taken to be 142,4MPa and 28,8MPa respectively when (17) is considered and 180MPa, 148MPa when (18) is considered. Such values are taken from the works of ANAND AND KOTHARI [1996] and PEIRCE ET AL [1982].

It is also adopted the following prescribed conditions:

$$\text{Stretching} \Rightarrow \mathbf{F} = \begin{bmatrix} 1+\alpha(t) & 0 \\ 0 & 1 \end{bmatrix}, \qquad \text{Shear} \Rightarrow \mathbf{F} = \begin{bmatrix} 1 & \delta(t) \\ 0 & 1 \end{bmatrix}.$$

Considering a 60% stretching, the following graphic is obtained:

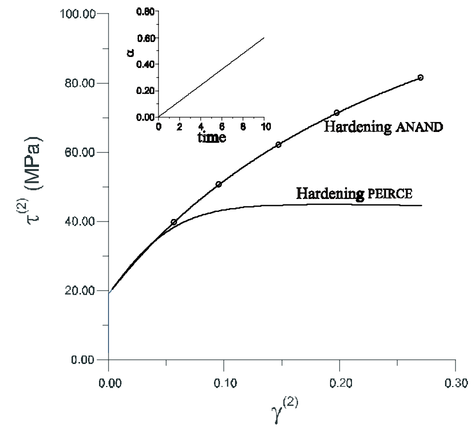

Figure 5: Schmid Stress versus Slip in stretching with $\alpha_{t_f} = 0{,}6$ and $\theta^{(1)} = \theta^{(2)} = 30°$.

where the "ANAND Hardening" (18) shows a stiffer character than the "PEIRCE Hardening" (17).

The behaviour of the "PEIRCE Hardening" curve in figure 5 can be compared with a curve presented in PEIRCE ET AL [1982], adapted to the figure 6.

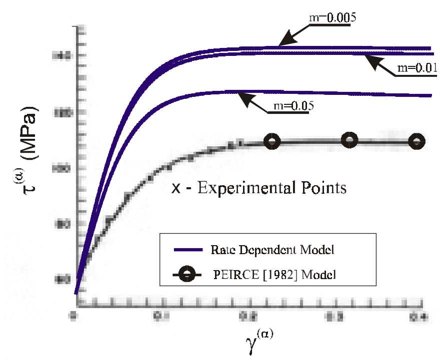

Figure 6: Schmid Stress versus Slip with $\tau_0 = 60{,}84$MPa and $\theta^{(1)} = \theta^{(2)} = 30°$ (adapted from PEIRCE ET AL [1982], Figure 10).

The PEIRCE ET AL [1982] curve shows the behaviour of a rate independent constitutive formulation, which is present in their work. The present work, however, adopts a rate dependent formulation. It can be concluded that the increasing values for the rate sensivity tends to approximate the rate dependent curve to that of the rate independent formulation.

Considering the shear prescribed condition, the curves in the figure 7 are obtained. It can be observed that no pathological effects (oscillations, for example) are present.

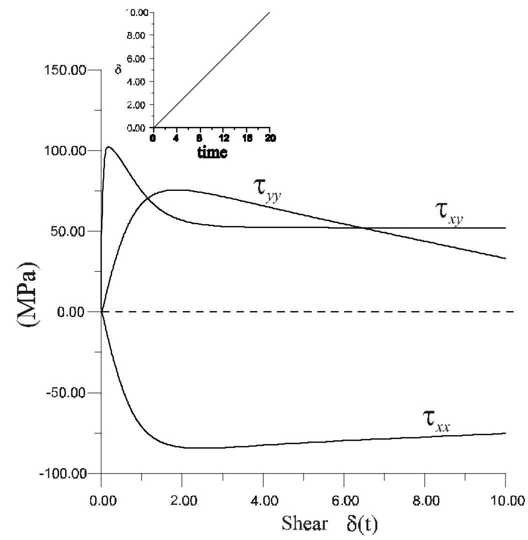

Figure 7: Kirchhoff Stress versus $\delta(t)$ in shear with $\delta_{t_f} = 10$ and $\theta^{(1)} = \theta^{(2)} = 30°$.

5.2 Finite Elements

The initial mesh configurations and boundary conditions used in this work are represented in the figures 8 and 9.

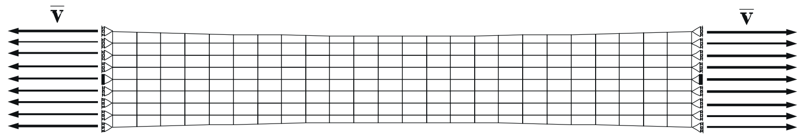

Figure 8: Boundary Conditions using prescribed velocity.

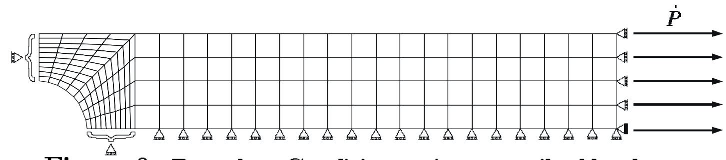

Figure 9: Boundary Conditions using prescribed load rate.

There are two degrees of freedom in each node of the quadrilateral element. It is used bi-linear shape functions. As PEIRCE ET AL [1982] suggest, it is necessary to impose a thickness inhomogeneity in meshes like that of the figure 8, in order to obtain some localization behaviour.

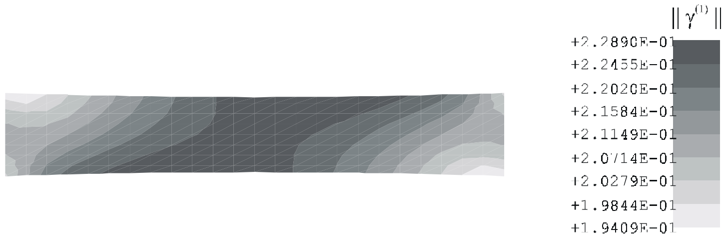

Figure 10: Distribution of the Slip Modulus $\gamma^{(1)}$ in a residual plastic deformation of $\Delta l/l_0 = 0{,}25$, $\bar{\mathbf{v}} = 0{,}01$ m/s with $\theta^{(1)} = \theta^{(2)} = 20°$.

The values for the numerical constants are: $\dot{\gamma}_0 = 0,001$; m $= 0,05$; q$= 1,2$; $\tau_0 = 16$MPa. The self-hardening parameters are defined only for (17): $H_0 = 142,4$MPa and $\tau_s = 28,8$MPa.

The results obtained, imposing $\Delta l/l_0 = 25\%$ and $\bar{\mathbf{v}} = 0,01$m/s, are shown in the figures 10 and 11.

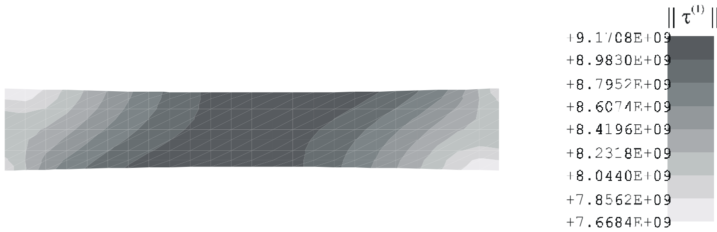

Figure 11: Distribution of the Schmid Stress Modulus $\tau^{(1)}$ in a residual plastic deformation of $\Delta l/l_0 = 0{,}25$, $\mathbf{v} = 0{,}01$ m/s with $\theta^{(1)} = \theta^{(2)} = 20°$. Values in Pa.

These graphics reveal a low level of localization, both in terms of stress and slip, near the slip line number 1 defined in the model of the figure 4. Results for the slip line 2 are similar.

Imposing a loading-unloading cycle through the load rate $\dot{P}$, the following result is obtained:

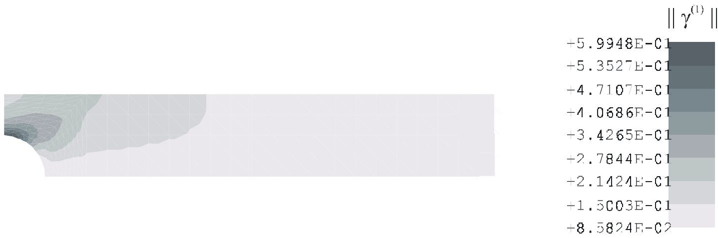

Figure 12: Distribution of the Slip Modulus $\gamma^{(1)}$ in a residual plastic deformation of $\Delta l/l_0 = 0{,}11$, for a loading-unloading cycle and $\theta^{(1)} = \theta^{(2)} = 20°$.

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