



ANALYTICAL MODELING OF MULTIPASS WELDING PROCESS WITH DISTRIBUTED HEAT SOURCE

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***Abstract.** In the welding process, the most interesting regions for the heat transfer study are the fusion zone (FZ) and the heat affected zone (HAZ), where high temperatures are reached. These temperature levels cause phase transformations and alterations in the mechanical properties of weldment. The analysis to estimate the temperature distribution in the multiple pass welding is more complex than in the single pass welding, due to superimposed thermal effects of one pass over the previous passes. In this work, a comparison was made between thermal cycles obtained from analytical models regarding point (concentrated) and Gaussian (distributed) heat sources. The use of distributed heat source prevents infinite temperatures near the fusion zone. The comparison showed that the thermal cycles obtained from the distributed heat source model are more reliable than those obtained from the concentrated heat source model.*

***Keywords :** Distributed source, Analytical modeling, Multipass welding*

1. INTRODUCTION

Most of the published work on temperature distribution study in welds consider that the heat source is concentrated in a very small volume of the material. In this case, solutions are obtained assuming a point, a line or a plane heat source, as the solutions proposed by Rosenthal (1941). The measured temperatures in the fusion and heat affected zones differ significantly from the temperatures obtained by those solutions, since the singularity located at the source origin results in infinite temperature. Results present higher precision in regions where temperature does not exceed twenty percent of the material melting point (Goldak, Bibby and Chakravarti, 1984).

In order to avoid the occurrence of infinite temperatures at the center and in the vicinity of the fusion zone (FZ), it is important to consider a distributed heat source in the model development. In reality, the heat source is distributed in a finite region of the material, which is relevant for the measured temperatures near the FZ. There are several models for the heat

source distribution. The Gaussian distribution, firstly suggested by Pavelic et al. (1969), is the most used model. Although solutions considering the heat source as distributed can be reached in an analytical or a numerical way, there is an increasing tendency to the use of numerical methods. This work presents a new analytical solution to estimate temperature fields in the multipass welding, as generated by Gaussian heat sources. The solution was obtained from the known forms for the multipass welding, for point heat sources.

2. MODELING

This section is divided in two sub-sections: the first one is intended for the analytical development of the model, for multipass welding. The second one shows the parameters used to obtain the thermal cycles from simulation.

2.1 Analytical development

In the one-dimensional model, it is considered that the heat flux occurs only in the y direction, as shown in the coordinate system of Fig. 1. The following assumptions have been made : the heat source moves in high speed (no heat flux in the x direction), and each weld pass fulfils the whole material thickness (no heat flux in the z direction).

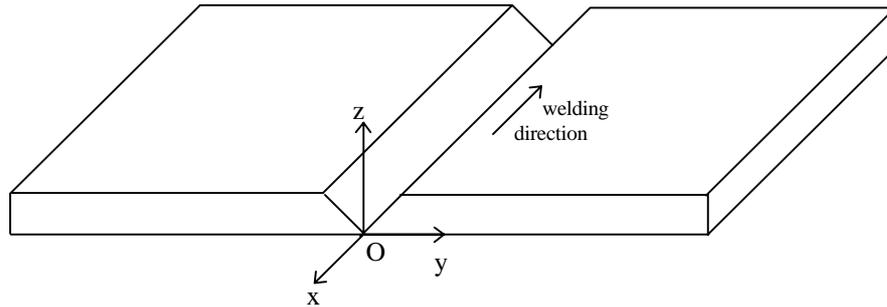


Figure 1 - Coordinate system used in the model development.

The formulation of the problem to the first weld pass is composed by the one-dimensional transient heat conduction equation, and its boundary and initial conditions. It is similar to the formulation of the point heat source problem. In the θ variable ($\theta = T - T_o$), one has:

$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial y^2} \quad (1)$$

$$\theta(t = 0) = 0 \quad (2)$$

$$\theta(y \rightarrow \infty) = 0 \quad (3)$$

$$\int_{-\infty}^{\infty} \theta dy = \frac{Q_1}{\rho c} \quad (4)$$

where:

T = temperature ($^{\circ}\text{C}$)
 T_o = ambient temperature ($^{\circ}\text{C}$)
 θ = temperature difference ($^{\circ}\text{C}$)
 t = time (s)
 y = coordinate (m)
 Q_1 = thermal energy per unit area (J/m^2)
 α = thermal diffusivity of the material (m^2/s)
 ρ = density of the material (kg/m^3)
 c = specific heat of the material ($\text{J}/\text{kg}^{\circ}\text{C}$)

The solution to this problem is known (Rosenthal,1941), and it is given by:

$$\theta(y, t) = \frac{Q_1}{\rho c \sqrt{4\pi\alpha t}} \exp\left(-\frac{y^2}{4\alpha t}\right) \quad (5)$$

To take into account the distributed heat source consider now Fig. 2, where a source with normal or Gaussian distribution is instantaneously applied at $t = 0$ to the surface of a plate. The center C of the source coincides with the origin O of the coordinate system xyz . The total power of the source is given by:

$$Q = \int_{-\infty}^{\infty} q_s(y) dy \quad (6)$$

where:

Q = total power of the source (W)
 q_s = power of the source per unit length (W/m)

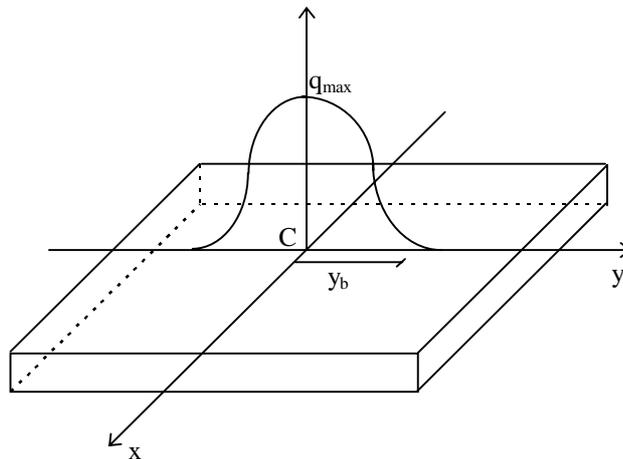


Figure 2 - Gaussian heat source.

In the one-dimensional case, the Gaussian distribution of the heat source along the y direction occurs simultaneously in all points of the x direction of welding. The power $q_s(y)$ may be expressed by:

$$q_s(y) = q_{\max} \exp(-Ay^2) \quad (7)$$

where:

$q_{\max} = q_s$ maximum value (W/m)

A = coefficient of arc concentration (1/m)

Coefficient A is determined considering a distance y_b in Eq. (7), which corresponds to the distance where the power is reduced to five percent of its maximum value (Fig. 2). Thus,

$$A = \frac{3}{y_b^2} \quad (8)$$

When y_b is large, $q_s(y)$ decreases slowly with y . Substituting Eq. (8) in Eq. (7) and then in Eq. (6), and integrating this equation between $-y_b$ and y_b limits, one obtains:

$$q_{\max} = \frac{\sqrt{3}Q}{\sqrt{\pi}y_b \text{Erf}(\sqrt{3})} \quad (9)$$

Equation (7) may then be written as :

$$q_s(y) = \frac{\sqrt{3}Q}{\sqrt{\pi}y_b \text{Erf}(\sqrt{3})} \exp\left(-\frac{3y^2}{y_b^2}\right) \quad (10)$$

The diffusion process of an instantaneous Gaussian heat source applied to the surface of the material may be obtained by the source' method. Let the y coordinate where the heat source acts be divided in small elements dy' . The heat $dQ = q_s(y')dy'$ is supplied to the element dy' at $t = 0$, and may be regarded as an instantaneous point heat source. According to Eq. (5), the diffusion process to an instantaneous heat source is:

$$d\theta(y, t) = \frac{dQ}{\rho c \sqrt{4\pi\alpha t}} \exp\left(-\frac{y^2}{4\alpha t}\right)$$

or

$$d\theta(y, t) = \frac{q_s(y')dy'}{\rho c \sqrt{4\pi\alpha t}} \exp\left(-\frac{d^2}{4\alpha t}\right) \quad (11)$$

where d is the distance between the instantaneous source and a point located on the y axis, that is,

$$d^2 = (y-y')^2 \quad (12)$$

Substituting Eqs. (10) and (12) in Eq. (11):

$$d\theta(y, t) = \frac{\sqrt{3}Q}{\pi y_b \rho c \sqrt{4\alpha t} \text{Erf}(\sqrt{3})} \exp\left(-\frac{3y^2}{y_b^2}\right) \exp\left[-\frac{(y-y')^2}{4\alpha t}\right] \quad (13)$$

By the superposition principle, the temperature change in the y point may be obtained by summing the contributions of all instantaneous concentrated sources dQ, acting along the y coordinate of the material, between $-y_b$ and y_b points:

$$\theta(y, t) = \frac{\sqrt{3}Q}{\pi y_b \rho c \sqrt{4\alpha t} \text{Erf}(\sqrt{3})} \int_{-y_b}^{y_b} \exp\left(-\frac{3y'^2}{y_b^2}\right) \exp\left[-\frac{(y-y')^2}{4\alpha t}\right] dy'$$

Solving the integral and rearranging the solution, one obtains:

$$\theta(y, t) = \frac{\sqrt{3}Q}{2\rho c \sqrt{\pi(12\alpha t + y_b^2)} \text{Erf}(\sqrt{3})} \left\{ \begin{array}{l} \exp\left[-\frac{y^2}{4\alpha t} + \frac{y^2 y_b^2}{4\alpha t(12\alpha t + y_b^2)}\right] \text{Erf}\left(\frac{12\alpha t - yy_b + y_b^2}{2\sqrt{\alpha t(12\alpha t + y_b^2)}}\right) + \\ \exp\left[-\frac{y^2}{4\alpha t} + \frac{y^2 y_b^2}{4\alpha t(12\alpha t + y_b^2)}\right] \text{Erf}\left(\frac{12\alpha t + yy_b + y_b^2}{2\sqrt{\alpha t(12\alpha t + y_b^2)}}\right) \end{array} \right\} \quad (14)$$

Equation (14) is the solution to the first weld pass, regarding a heat source with Gaussian distribution. The solution to the second pass is obtained from the point heat source solution (Suzuki,1996):

$$d\theta(y, t) = \frac{q_{s1}(y')dy'}{\rho c \sqrt{4\pi\alpha t}} \exp\left(-\frac{d^2}{4\alpha t}\right) + \frac{q_{s2}(y')dy'}{\rho c \sqrt{4\pi\alpha(t-t_p)}} \exp\left[-\frac{d^2}{4\alpha(t-t_p)}\right] \quad (15)$$

In Eq. (15), it may be observed that the t variable was displaced by a value t_p , which corresponds to the sum of the welding and waiting times to the beginning of the second pass. The use of indices 1 and 2 in the q_s variable is due to the existence or not of variation in the heat input for each pass. The same steps applied to obtain Eq. (14) are used to reach the solution to the second pass, and so on. Analogously, the general solution to n passes, in the T variable, is given by:

$$T(y, t) = T_o + \frac{\sqrt{3}}{2\rho c \sqrt{\pi}} \sum_{i=1}^n \frac{Q_i}{\sqrt{12\alpha[t-(i-1)t_p] + y_b^2}} * \left\{ \begin{array}{l} \exp\left[-\frac{y^2}{4\alpha[t-(i-1)t_p]} + \frac{y^2 y_b^2}{4\alpha[t-(i-1)t_p]\{12\alpha[t-(i-1)t_p]\}}\right] \text{Erf}\left\{\frac{12\alpha[t-(i-1)t_p] - yy_b + y_b^2}{2\sqrt{\alpha[t-(i-1)t_p]\{12\alpha[t-(i-1)t_p] + y_b^2}\}}\right\} + \\ \exp\left[-\frac{y^2}{4\alpha[t-(i-1)t_p]} + \frac{y^2 y_b^2}{4\alpha[t-(i-1)t_p]\{12\alpha[t-(i-1)t_p]\}}\right] \text{Erf}\left\{\frac{12\alpha[t-(i-1)t_p] + yy_b + y_b^2}{2\sqrt{\alpha[t-(i-1)t_p]\{12\alpha[t-(i-1)t_p] + y_b^2}\}}\right\} \end{array} \right\} \quad (16)$$

Equation (16) is the solution to the temperature distribution in the one-dimensional multipass welding process, with the Gaussian heat source. Far from the heat source, i.e., for

distances where y has the same magnitude as y_b , Eq. (16) is similar to the solution obtained for the point heat source. However, near the FZ and HAZ ($y \ll y_b$), the correction introduced by the distributed heat source mode in Eq. (16) allows to better predicting the temperatures in these regions.

2.2 Model evaluation

The comparison between concentrated and distributed heat source models was made through simulation of thermal cycles obtained from three weld passes. Equation (16) was used to estimate the temperatures near fusion zone, regarding Gaussian distributed heat sources. The multipass model with concentrated heat source is given by Eq. (15). In this case, the variable $q_s(y)$ does not have a distribution, and it is calculated by:

$$q_s = Q = \frac{\eta VI}{v\delta} \quad (17)$$

where:

η = arc efficiency (%)

V = welding voltage (V)

I = welding current (A)

v = welding speed (m/s)

δ = plate thickness (m)

The waiting time used corresponds to 60 seconds, and the welding process was simulated during a total time of 300 seconds. The ambient temperature is 25 °C. The error function in Eq. (16) was evaluated using a polynomial approximation. The parameters used in the simulation are described below.

Material. The evaluation of the proposed model was made considering the butt welding of high strength low alloy steel (HSLA) plates, with dimensions 0,013 x 0,10 x 0,25 m (thickness x length bead x width). Table 1 shows the physical properties used in the simulation. It is known that the physical properties of the metal change with temperature. However, this variation in the analytical models results in a non-linear equation, and it is not possible to obtain the solution in a closed form. Then, the physical properties are usually taken in a specific temperature, for example, in the half melting point of the material. In this work, they were calculated at 800°C. The values refer to low carbon steels, but they can be used for HSLA steel, as suggested by Hanz et al. (1989).

Table 1. Physical properties of low carbon steels (Hanz et al,1989)

k (J/ms°C)	ρc (J/m ³ °C)	α (m ² /s)
31,67	7,14x10 ⁶	4,44x10 ⁻⁶

Welding parameters. In butt welding, to fulfil the groove, it is usual to increase the heat input from one pass to the next. In this work, the increase of heat input is obtained by increasing the welding current, the other parameters in Eq. (17) remaining unaltered. However, the current increase causes efficiency to decrease. Then, a different value of efficiency was used in each pass. The choice of these values were based on the efficiency range for the Gas

Metal Arc Welding (GMAW) process, which ranges from 66 to 85% (Svensson, 1994). The welding parameters used in the simulation are in Table 2. The heat input (HI) values were determined by the Eq. (17), but not divided by the material thickness. The same HI values were used in the point and Gaussian heat source models. However, in the Gaussian heat source model, ninety-five percent of HI was applied in the weld. In order to correct this difference, the HI values in Table 2 were multiplied by the 1,05 factor for the Gaussian model.

Table 2. Welding parameters used in the three weld passes

Pass	I (A)	V (V)	v (m/s)	η (%)	HI ($\times 10^6$ J/m)
1	186	26,4		80	0,79
2	235	26,2	0,005	75	0,92
3	301	25,4		70	1,07

Parameter y_b . In order to verify the capability of the proposed model in reproducing the thermal cycles, some values were chosen for the y and y_b variables. These choices took into account the heat input used in the simulation, and also some values obtained from experimental determination of y_b . Table 3 shows the y_b values obtained by Kou and Wang (1986), Zacharia et al. (1989), and Wu (1992), as well as the heat input (HI) used in their analyses. In this work, the y_b parameter was estimated based on the heat input values showed in Table 3. In Eq. (8), the A coefficient was determined for the y_b distance where the power is reduced to five percent of its maximum value. According to this equation, the heat input increase causes y_b parameter to increase too, in order that the area below Fig. 2 be equal to ninety-nine percent of the q_{\max} value. Then, different values of y_b for each pass were considered, since the heat input increased in the second and third passes. The y_b values used in the modeling were 0,004; 0,0047 and 0,0054 m for the first, second and third pass, respectively.

Table 3. Heat input and y_b values used by some authors

Author	I (A)	V (V)	v (m/s)	HI ($\times 10^6$ J/m)	y_b (m)
Kou and Wang (1986)	100	11	0,0055	0,20	0,003
Zacharia et al. (1989)	175	14	0,0034	0,72	0,003
Wu (1992)	200	20	0,010	0,40	0,002

3. RESULTS AND DISCUSSION

To compare the point and Gaussian heat source models, thermal cycles were simulated in two different y distances, namely: y equal to 0,001 m, appropriate for a point near the fusion zone ($y < y_b$), and y equal to 0,003 m, appropriate for a point of the same magnitude as the y_b parameter.

Figure 3 shows the thermal cycles obtained from the point and Gaussian heat source models, at $y = 0,001$ m. It can be observed that the peak temperatures in each pass are more elevated for the point heat source model than those obtained from Gaussian heat source model. This occurs due to the assumption that the heat input is instantaneously applied in the area composed of thickness \times width of the workpiece. In the Gaussian heat source model, it is assumed that the heat input is applied not only in the same area, but also in the y coordinate, along the y_b dimension. Then, the peak temperatures produced by this model have a tendency to be more realistic. The previous determination of the peak temperature to be reached in a

defined point is interesting, since it indicates fortuitous phase changes. The differences between the temperatures simulated by the two models are described in the Table 4. T_1 refers to maximum temperature reached in the first pass; T_2 in the second pass, and so on. It is worth noticing that the correction in the proposed model affect only the peak temperature determination, and no difference is seen in the cooling rate.

Table 4. Peak temperatures reached in each weld pass

T_{peak}	Point heat source	Gaussian heat source
T_1	2057,52	1632,28
T_2	2535,16	1881,01
T_3	3044,18	2115,27

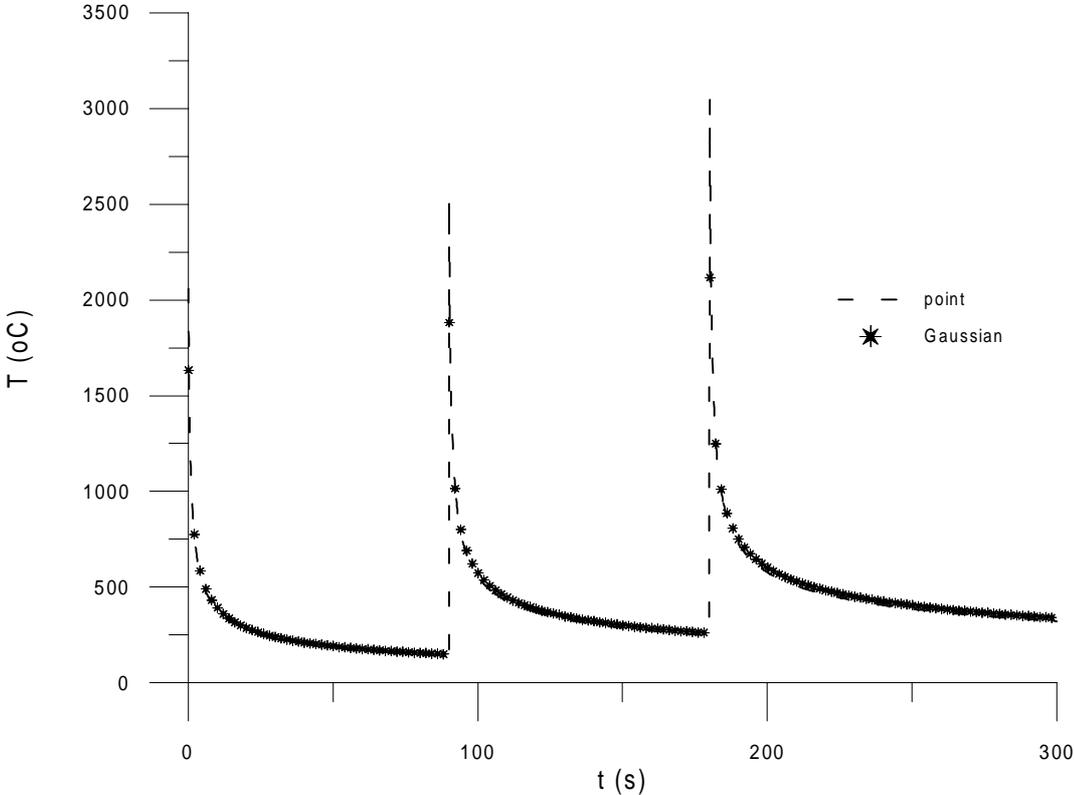


Figure 3 - Thermal cycles for the point and Gaussian heat source models, at $y = 0,001$ m.

In Figure 4, the thermal cycles were simulated by using $y = 0,003$ m, with regard to shows that, if y is close to y_b , the Gaussian heat source model is equivalent to the point heat source solution. The two models give results that are practically the same. This means that the point source model at distances far from the fusion zone can correctly predict the temperature fields.

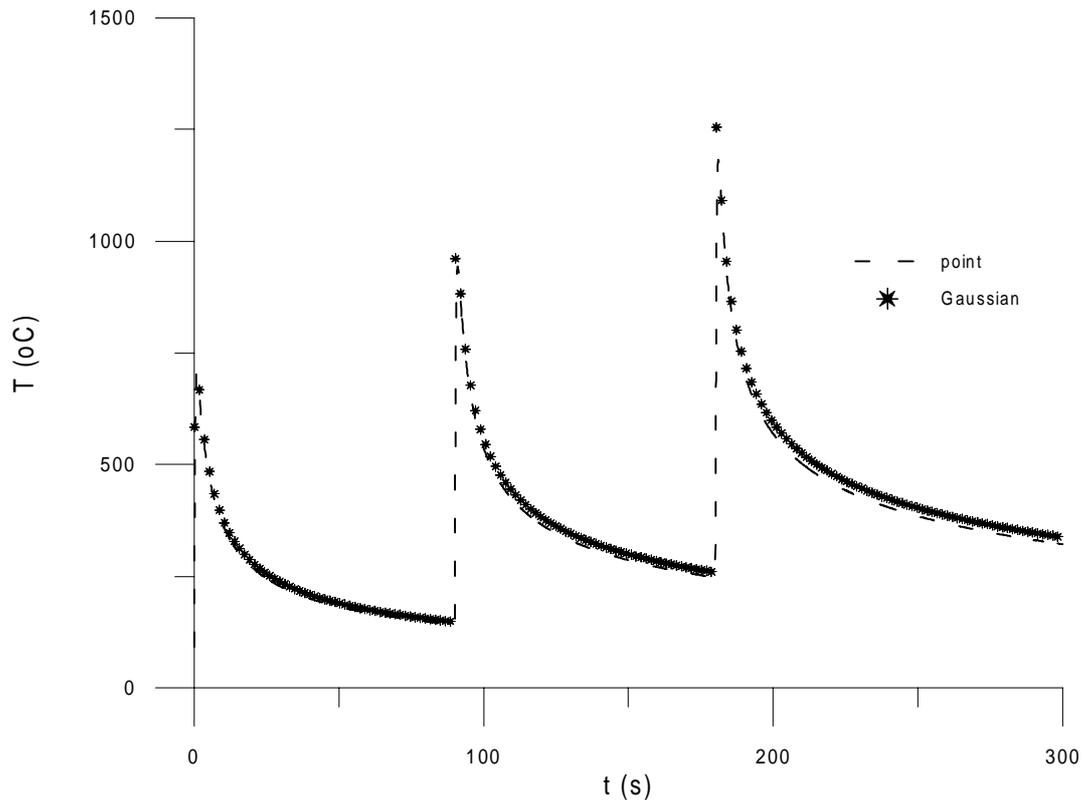


Figure 4 - Thermal cycles for the point and Gaussian heat source models, at $y = 0,003$ m.

4. CONCLUSIONS

The main conclusions of this work are:

- the closed form solution obtained allows to estimate the thermal cycles produced by multipass welding process, near the fusion and heat affected zones;
- the distributed heat source in the proposed solution is an important correction for the known model with point source, since this factor allows to obtain temperature values more realistic near to fusion zone;
- the analytical solution obtained is useful as a reference for numerical and experimental methods.

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