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APPLICATION OF THE CENTER MANIFOLD THEORY TO THE SLEWING FLEXIBLE NON-IDEAL STRUCTURE UNDER LARGE DEFLECTIONS : A CASE STUDY

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Abstract. *In this paper is analised the dynamical behaviour of a slewing flexible structure under large deflections. The dynamics from the original (complete) governing equations of motion are reduced to the center manifold in the neighborhood of an equilibrium solution with the proposal of locally study the stability of the system. In this analysed equilibrium point, a Hopf bifurcation occurs. In this region, were found values for the control parameter (structural damping coeficient) where the system is unstable and values where the system stability is assured (periodic motion). This local analysis of the system reduced to the center manifold assures the stable/unstable behaviour of the original system around a known solution.*

Key words: *Center manifold, Equilibrium solution, Non-ideal dynamical systems*

1. INTRODUCTION

The study of slewing structures was formerly considered in the literature by Book *et al*. (1975). After this work, many others were developed in this field. Among these works, one may quote the study of a solar panel by (Juang, Horta, 1987).

The goal of the present work is the study of the problem of slewing maneuvers considering the existence of a mutual interaction between the energy source and the structure dynamics. This fact turns the problem a non-ideal one (Kononenko,1969).

Here, the dynamics of the nonlinear and nonideal system is analysed in the neighborhood of an equilibrium solution (fixed point). To this end, the theory of reduction to the center manifold is utilised and the governing equations of motion are simplified (Carr,1971).

2. GOVERNING EQUATIONS OF MOTION

The governing equations of motion for the slewing flexible structure (under large deflections and non-ideal) according to Fenili *et al.* (1999), are reproduced in Eq. (1) :

$$\left\{\begin{array}{l} \dot{x}_1 = x_3 \\ \dot{x}_2 = x_4 \\ \dot{x}_3 = -a_1\dot{x}_4 - a_6x_3 + a_8U - a_7x_2 + \in^2 \left(-a_2^N\dot{x}_3x_2^2 - a_3^Nx_4^2x_2 + a_4^Nx_2^2\dot{x}_4 - a_5^Nx_2x_3x_4 \right) \\ \dot{x}_4 = -b_1x_2 - b_2\dot{x}_3 + \in^2 \left(-\mu_1x_4 - b_3^Nx_3^2x_2 + b_4^Nx_2x_3x_4 - b_5^Nx_2^2\dot{x}_3 + b_6^Nx_2^3 - b_7^Nx_2^3 + b_8^Nx_4^2x_2 + \right. \\ \qquad \left. + b_8^Nx_2^2\dot{x}_4 \right) \end{array}\right. \tag{1}$$

Rewriten Eq. (1) in the form $\dot{x} = F(x)$, expanding it on Taylor series around one of the equilibrium solutions (the fixed point (x_1, x_2, x_3, x_4) = (0 , 0 , 0 , 0)) and eliminating the equation for the variable x_1 (decoupled), one can find :

$$\begin{Bmatrix} \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \dfrac{-a_7 + a_1b_1}{1-a_1b_2} & \dfrac{-a_6}{1-a_1b_2} & \dfrac{\in^2 \mu_1a_1}{1-a_1b_2} \\ \dfrac{-b_1 + a_7b_2}{1-a_1b_2} & \dfrac{a_6b_2}{1-a_1b_2} & \dfrac{-\in^2 \mu_1}{1-a_1b_2} \end{bmatrix} \begin{Bmatrix} x_2 \\ x_3 \\ x_4 \end{Bmatrix} + \left\{ \begin{Bmatrix} 0 \\ \left(\dfrac{a_8}{1-a_1b_2}\right)U \\ \left(\dfrac{-b_2a_8}{1-a_1b_2}\right)U \end{Bmatrix} + \right.$$

$$\left. + \in^2 \begin{Bmatrix} 0 \\ a_{400}^NUx_2^2 + a_{500}^Nx_2^3 + a_{600}^Nx_2^2x_3 + a_{700}^Nx_2x_3^2 + a_{800}^Nx_2x_3x_4 + a_{900}^Nx_2x_4^2 \\ a_{450}^NUx_2^2 + a_{550}^Nx_2^3 + a_{650}^Nx_2^2x_3 + a_{750}^Nx_2x_3^2 + a_{850}^Nx_2x_3x_4 + a_{950}^Nx_2x_4^2 \end{Bmatrix} \right\} \tag{2}$$

The jacobian matrix for the system (2) is :

$$J = \begin{bmatrix} 0 & 0 & 1 \\ \frac{-a_7 + a_1 b_1}{1 - a_1 b_2} & \frac{-a_6}{1 - a_1 b_2} & \frac{\in^2 \mu_1 a_1}{1 - a_1 b_2} \\ \frac{-b_1 + a_7 b_2}{1 - a_1 b_2} & \frac{a_6 b_2}{1 - a_1 b_2} & \frac{-\in^2 \mu_1}{1 - a_1 b_2} \end{bmatrix} \qquad (3)$$

In the cases where J have eigenvalues equal to zero or eigenvalues with zero real parts, the linear analysis (analysis of the matrix (3)) is not conclusive about the stability of the equilibrium solution. In these cases, some terms of superior order must be included in the analysis and the theory of center manifold reduction is utilised (Carr,1981).

According to (Fenili,1999), the conditions for the ocurrence of a Hopf bifurcation in this kind of system were studied.The following analysis is realized around these conditions.

3. A CASE STUDY

In complex systems like this one, it is very dificult to work analitically because of the length of the general expressions. Here, the small parameter, $\in$, depends on the geometrical characteristics of the structure and is not used as control parameter (will be specified for this case). In this analysis, the control parameter will be μ_1.

Some parameters for the studied case are given in Table 1.

Table 1. Some parameters of the dynamical system

Parameter	Nomenclature	Value
height (of the cross section)	h	0.010 (m)
width (of the cross section)	b	0.009 (m)
gear ratio	Ng	20
length of the structure	L	3 (m)
structural damping	μ_1	control parameter
small parameter	$\in$	7.499999999e-007

Utilizing the Hurwitz criterium, one can find the value of μ_1 that renders a pair of pure imaginary eigenvalues in the jacobian matrix. For the case in point, this value was found to be μ_1 = 5.009291768e+011. The eigenvalues of the associated linear system are: λ_1 = 0.893603065 i , λ_2 = - 0.893603065 i and λ_3 = -13.440613874.

The value of the squared perturbation (or small) parameter ($\in^2 = 5.624999999 \; 10^{-13}$) won't be involved in the calculations so the aspect of perturbation of the nonlinear terms may be clear. For a new value of $\in$, new values of *L* and *b* must be chosen (other case) .

The order of magnitude of $\in^2$ (10^{-13}) is justified in the fact that the dimensional variables of the system are rescaled by orders of $1/\in$. The equations of motion presented here are in adimensional form.

The parameter μ_1 must be substituted by (5.009291768e+011 + μ_2), because is of interest the study of the system behaviour around the critical value.

In the cases where μ_2 is equal to zero, the system operates in the critical situation of change of stability (the case where a bifurcation of the kind of Hopf occurs).

The Eq. (2) , applied to the case in point, is writen as :

$$\begin{Bmatrix} \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -5.657478429 & -10.732685509 & 4.255241456 \\ 2.225650928 & 6.119315772 & -2.707928365 \end{bmatrix} \begin{Bmatrix} x_2 \\ x_3 \\ x_4 \end{Bmatrix} + \left\{ \begin{Bmatrix} 0 \\ 49.508021001\,U \\ -28.227344730\,U \end{Bmatrix} + \right.$$

$$+ \quad 5.624999999\ 10^{-13} \left\{ \begin{matrix} 0 \\ 15.101683134\,\mu_2 x_4 + 457.541691079\,U x_2^2 - 116.490601066\,x_2^3 + \\ -9.610330351\,\mu_2 x_4 - 352.902024994\,U x_2^2 + 65.575628045 x_2^3 + \end{matrix} \right.$$

$$\left. \left. \begin{matrix} +24.885341256\,x_2^2 x_3 + 4.710237622\,x_2 x_3^2 - 10.735148339\,x_2 x_3 x_4 - 63.414375454\,x_2 x_4^2 \\ + \quad 5.762647167\,x_2^2 x_3 - 2.997476452\,x_2 x_3^2 + 6.120720462\,x_2 x_3 x_4 + 38.474800066\,x_2 x_4^2 \end{matrix} \right\} \right\} \qquad (4)$$

Making the coordinate transformation x = Pu, where the matrix P represents the matrix of the linear system eigenvectors (associated to λ_1 = 0.893603065 i , λ_2 = - 0.893603065 i and λ_3 = -13.440613874), one have :

$$\begin{Bmatrix} \dot{u}_2 \\ \dot{u}_3 \\ \dot{u}_4 \end{Bmatrix} = \begin{bmatrix} 0 & 0.893603064 & 0 \\ -0.893603064 & 0 & 0 \\ 0 & 0 & -13.440613874 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} + \left\{ \begin{Bmatrix} -1.366744899\ \ U \\ 2.790862524\ \ U \\ -56.118493263\ \ U \end{Bmatrix} + \right.$$

$$+\ 5.624999999\ 10^{-13}\left\{\begin{array}{l} -0.921554892\ \mu_2 u_2 - 0.377736424\ \mu_2 u_3 - 0.827702237\ \mu_2 u_4 + 0.595276586\ u_2^3 + \\ 0.205540628\ \mu_2 u_2 + 0.084249113\ \mu_2 u_3 + 0.184608066\ \mu_2 u_4 - 0.235012346\ u_2^3 + \\ -0.767790885\ \mu_2 u_2 - 4.003722875\ \mu_2 u_3 - 8.773024572\ \mu_2 u_4 + 8.021700697\ u_2^3 - \end{array}\right.$$

$$\begin{array}{l} +\ 3.179373761\ u_3^3 - 0.052877788\ u_4^3 - 1.140805981\ u_2^2 u_3 - 0.825930985\ u_2^2 u_4 - 2.063096959\ u_2 u_3^2 + \\ +\ 3.556113919\ u_3^3 - 0.002906999\ u_4^3 + 1.029431880\ u_2^2 u_3 - 0.860017084\ u_2^2 u_4 - 2.570298829\ u_2 u_3^2 + \\ -37.728289798\ u_3^3 - 0.314281273\ u_4^3 - 25.069983932\ u_2^2 u_3 + 8.732845274\ u_2^2 u_4 + 28.881768813\ u_2 u_3^2 + \end{array}$$

$$\begin{array}{l} +0.470653703\ u_2 u_4^2 - 1.410621001\ u_3 u_4^2 - 8.603821336\ u_3^2 u_4 + 5.362496367\ u_2 u_3 u_4 - 8.266001494\ U u_2^2 - \\ +0.150232469\ u_2 u_4^2 - 0.227986168\ u_3 u_4^2 - 2.790352390\ u_3^2 u_4 + 3.346028255\ u_2 u_3 u_4 - 1.218465083\ U u_2^2 - \\ +0.714802095\ u_2 u_4^2 - 5.864819030\ u_3 u_4^2 - 12.329882726\ u_3^2 u_4 - 12.224748978\ u_2 u_3 u_4 - 36.334477827\ U u_2^2 - \end{array}$$

$$\left.\left.\begin{array}{l} -\ 40.332748581\ U u_2 u_3 - 2.408442910\ U u_2 u_4 - 49.199440903\ U u_3^2 + 5.875822939\ U u_3 u_4 - 0.175435404\ U u_4^2 \\ -\ 5.945322647\ U u_2 u_3 - 0.355020937\ U u_2 u_4 - 7.252333661\ U u_3^2 + 0.866136438\ U u_3 u_4 - 0.025860377\ U u_4^2 \\ -177.288784670\ U u_2 u_3 - 10.586680342\ U u_2 u_4 - 216.263691192\ U u_3^2 + 25.828081258\ U u_3 u_4 - 0.771153236\ U u_4^2 \end{array}\right\}\right\} \quad (5)$$

4. CENTER MANIFOLD REDUCTION

Utilizing the notation and the theory of reduction of the system dynamics to the center manifold in the way it is proposed in (Wiggins,1990), one has :

$$x = \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} \quad (6)$$

$$y = u_4 \equiv h_1 \quad (7)$$

$$\varepsilon = \mu_2 \quad (8)$$

$$h = h_1 \quad (9)$$

$$A = \begin{bmatrix} 0 & 0.8936064 \\ -0.8936064 & 0 \end{bmatrix} \quad (10)$$

$$B = [\ -13.440613874\ ] \quad (11)$$

$$f(x,y,\varepsilon) = \left\{\begin{array}{l} -1.366744899\ U + 5.624999999\ 10^{-13}\ \{\ -0.921554892\ \mu_2 u_2 - 0.377736424\ \mu_2 u_3 - \\ 2.790862524\ U + 5.624999999\ 10^{-13}\ \{\ 0.205540628\ \mu_2 u_2 + 0.084249113\ \mu_2 u_3 + \end{array}\right.$$

$$-0.827702237\ \mu_2 u_4 + 0.595276586\ u_2^3 + 3.179373761\ u_3^3 - 0.052877788\ u_4^3 - 1.140805981\ u_2^2 u_3 -$$
$$+0.184608066\ \mu_2 u_4 - 0.235012346\ u_2^3 + 3.556113919\ u_3^3 - 0.002906999\ u_4^3 + 1.029431880\ u_2^2 u_3 -$$

$$-0.825930985\ u_2^2 u_4 - 2.063096959\ u_2 u_3^2 + 0.470653703\ u_2 u_4^2 - 1.410621001\ u_3 u_4^2 - 8.603821336\ u_3^2 u_4 +$$
$$-0.860017084\ u_2^2 u_4 - 2.570298829\ u_2 u_3^2 + 0.150232469\ u_2 u_4^2 - 0.227986168\ u_3 u_4^2 - 2.790352390\ u_3^2 u_4 +$$

$$+\ 5.362496367\ u_2 u_3 u_4 - 8.266001494\ U u_2^2 - 40.332748581\ U u_2 u_3 - 2.408442910\ U u_2 u_4 -$$
$$+\ 3.346028255\ u_2 u_3 u_4 - 1.218465083\ U u_2^2 - 5.945322647\ U u_2 u_3 - 0.355020937\ U u_2 u_4 -$$

$$-\ 49.199440903\ U u_3^2 + 5.875822939\ U u_3 u_4 - 0.175435404\ U u_4^2 \Big\}\Big\}$$
$$-\ 7.252333661\ U u_3^2 + 0.866136438\ U u_3 u_4 - 0.025860377\ U u_4^2 \Big\}\Big\} \qquad (12)$$

$$g(x,y,\varepsilon) = \Big\{ -56.118493263\ \ U + 5.624999999\ 10^{-13}\ \Big\{ -9.767790885\ \mu_2 u_2 - 4.003722875\ \mu_2 u_3 -$$
$$-8.773024572\ \mu_2 u_4 + 8.021700697\ u_2^3 - 37.728289798\ u_3^3 - 0.314281273\ u_4^3 - 25.069983932\ u_2^2 u_3 +$$
$$+8.732845274\ u_2^2 u_4 + 28.881768813\ u_2 u_3^2 + 0.714802095\ u_2 u_4^2 - 5.864819030\ u_3 u_4^2 - 12.329882726\ u_3^2 u_4 -$$
$$-12.224748978\ u_2 u_3 u_4 - 36.334477827\ U u_2^2 - 177.288784670\ U u_2 u_3 - 10.586680342\ U u_2 u_4 -$$
$$-216.263691192\ U u_3^2 + 25.828081258\ U u_3 u_4 - 0.771153236\ U u_4^2 \Big\}\Big\} \qquad (13)$$

The expression for h_1 is given by :

$$h_1 = a u_2^2 u_3 + b u_2 u_3^2 \qquad (14)$$

The unknown parameters *a* e b in Eq (14) may be found by the expression :

$$\frac{\partial h_i(x,\varepsilon)}{\partial x} = \left[\ Ax + f(x, h(x,\varepsilon), \varepsilon)\ \right] - Bh(x,\varepsilon) - g(x, h(x,\varepsilon), \varepsilon) = 0 \qquad (15)$$

Substituting Eq. (6)-Eq. (13) in Eq. (15) and making the terms in $u_2^2 u_3$ equal to zero yields :

$$13.440613874\ a - (\ 5.624999999\ \ 10^{-13})(\ -\ 25.069983932\ ) = 0$$

And one finds :

$$a = -10.491980568\ \ 10^{-13} \qquad (16)$$

Making the terms in $u_2 u_3^2$ equal to zero yields :

$$13.440613874\ b - (\ 5.624999999\ \ 10^{-13})(\ 28.881768813\ ) = 0$$

And one find :

$$b = 12.087660286 \quad 10^{-13} \tag{17}$$

Substituting Eq. (16) and Eq. (17) in Eq. (14), one obtain :

$$h_1 = -10.491980568 10^{-13} \; u_2^2 u_3 + 12.087660286 10^{-13} \; u_2 u_3^2 \tag{18}$$

The graphic of the center manifold expressed by Eq. (18) is shown in Fig. 4.

Substituting Eq. (18) in the first and second of the Eq. (5) and expanding the resulting system of equations around $\mu_2 = 0$, one obtain the reduction of the system expressed by Eq. (1) to the center manifold (Eq. (19)).

The suspension trick (aditional equation for the control parameter (Eq. (19b))) assures that the influence of other values of this parameter on the dynamics of the system may be verified (bearing in mind that the center manifold is only valid for a specified value of μ_2).

$$\begin{Bmatrix} \dot{u}_2 \\ \dot{u}_3 \end{Bmatrix} = 5.624999999 \;\; 10^{-13} \begin{Bmatrix} (-0.921554892 \;\; u_2 - 0.377736424 \;\; u_3 + 8.684235786 \;\; 10^{-13} u_2^2 u_3 - 10.004983458 \; 10^{-13} u_2 u_3^2 \;) \mu_2 \\ (\, 0.205540628 \;\; u_2 + 0.084249113 \;\; u_3 \; - 1.936904241 \;\; 10^{-13} u_2^2 u_3 + 2.231479587 \; 10^{-13} u_2 u_3^2 \,) \mu_2 \end{Bmatrix} \tag{19a}$$

$$\dot{\mu}_2 = 0 \tag{19b}$$

There are stability theorems that proof that if a equilibrium solution of the reduced equations (Eq. (19)) is stable (unstable), the same equilibrium solution for the complete equations (Eq. (1)) is also stable (unstable) (Nayfeh,1994).

5. RESULTS

The integration of Eq. (19) in time was made beneath a fourth order Runge-Kutta integrator. Three situations were considered (one for each value of μ_2 around the critical value of μ_1 (named μ_{1C})), according to the specifications in the following figures (Fig. 1 – Fig. 3).

The variables u_2 and u_3 and the time in the Fig. 1 - Fig. 3 are adimensional and local quantities.

The analysis of these figures yields information about the system stability in the neighborhood of the fixed point (or equilibrium solution) (0,0,0,0).

In the cases where $\mu_2 < 0$, no equilibrium solution is reached. The same don't occur in the cases where $\mu_2 = 0$ and in the cases where $\mu_2 > 0$, according to the Fig. 2 and Fig. 3, respectively.

The inicial conditions for the following simulations are : u2 = -0.30/$\in$ and u3 = -0.30/$\in$.

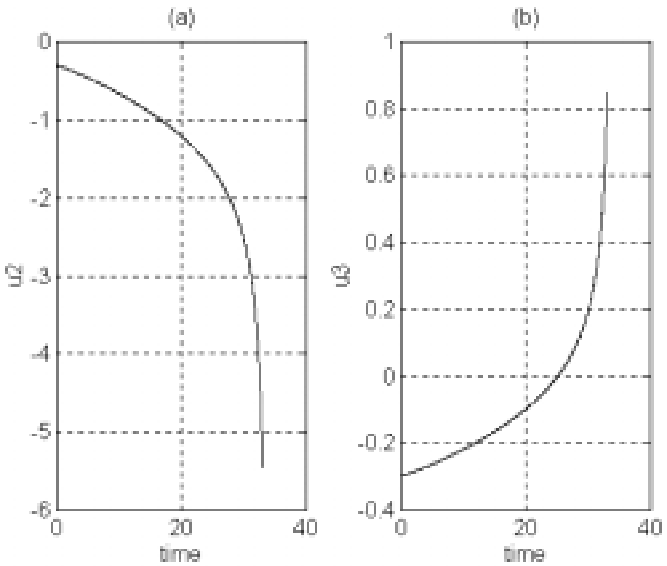

Figure 1 – Situation 1: $\mu_2 = -0.050\,\mu_{1C}$: (a) behaviour of the variable u_2 in the center manifold ; (b) behaviour of the variable u_3 in the center manifold.

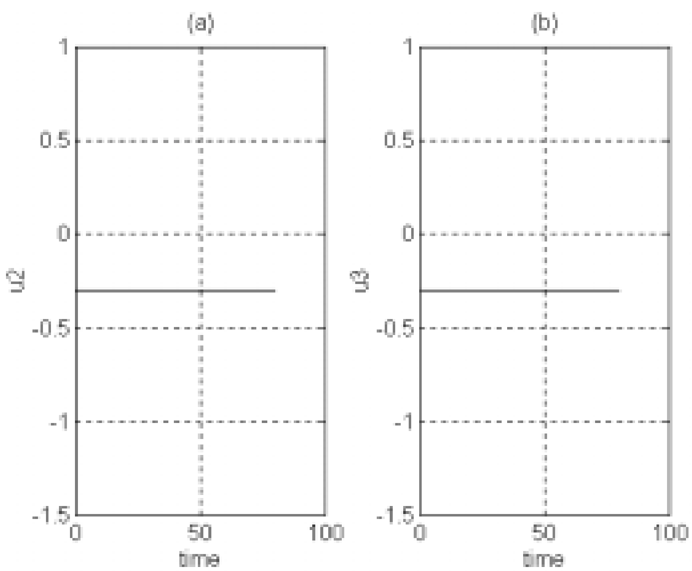

Figure 2 – Situation 2: $\mu_2 = 0$: (a) behaviour of the variable u_2 in the center manifold ; (b) behaviour of the variable u_3 in the center manifold.

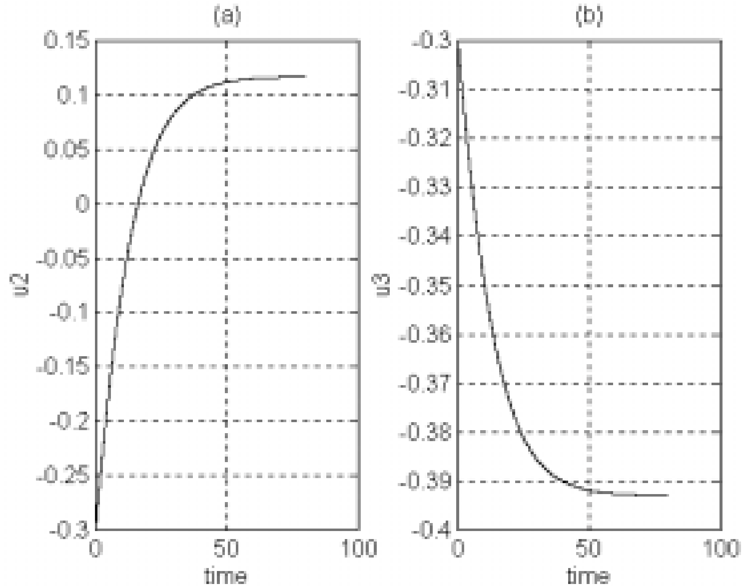

Figure 3 – Situation 3: $\mu_2 = 0.050\,\mu_{1C}$: (a) behaviour of the variable u_2 in the center manifold ; (b) behaviour of the variable u_3 in the center manifold.

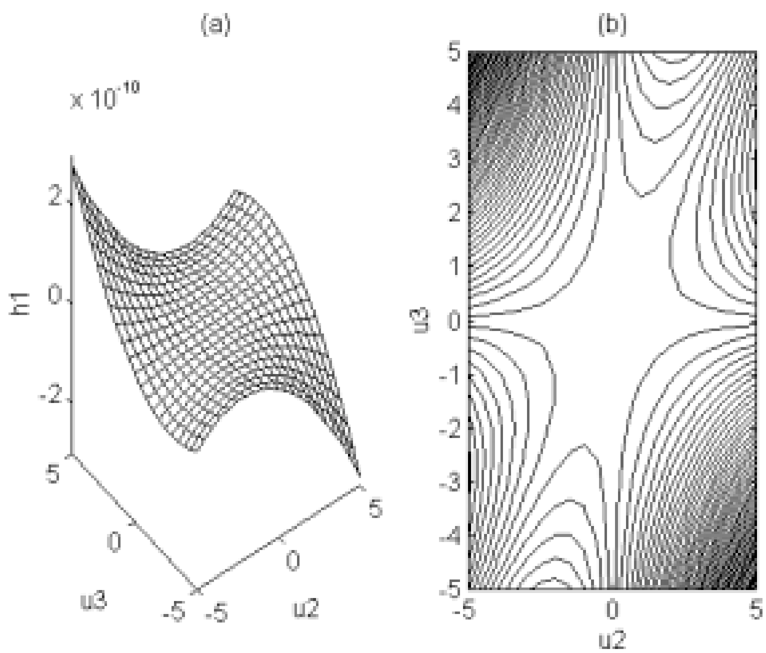

Figure 4 – (a) Center manifold (function h_1) ; (b) level curves

6. CONCLUSIONS

The local analysis in the neighborhood of an equilibrium solution for the complete nonlinear and nonideal dynamical system studied gives informations about the stability of the system maneuvers when one varies the structural damping coefficient. The stability of the system changes of unstable to stable when this control parameter is altered around a critical value in a considered fixed point solution (or equlibrium solution). The expression for the center manifold is not unique. New considerations for Eq. (14) can be proposed and the results compared with the ones obtained here.

7. ACKNOWLEDGEMENTS

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