ON FRICTION FACTORS FOR PSEUDOPLASTIC FLUIDS IN TURBULENT PIPE FLOW

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Abstract. *This work deals with the formulation of friction-factor equations for turbulent flows of fluids with rheological behavior described by Sisko and Cross models. The formulation assumes a logarithmic behavior of the turbulent mean-velocity profiles near the wall and consider an approach that uses friction-factor equations valid for power law fluids established in literature. The friction factors computed with the proposed relationships showed values lower than those obtained with the equation of Prandtl-von Kármán, which is valid for Newtonian fluids. The goal of this work is to establish relationships for the prediction of friction factor that take into account the rheological behavior of the non-Newtonian fluids aforementioned, which describe variations of viscosity over a spectrum more large of shear rates than power law rheology.*

Keywords: *Non-Newtonian Fluids, Turbulent Flows, Friction Factor, Drag Reduction.*

1. INTRODUCTION

The drag reduction that occurs in turbulent flow of non-Newtonian fluids is a very important phenomenon that has been extensively studied and used in many engineering applications. Among such applications are the increase of water flow rate in firefighting, hydropower and irrigation systems and the reduction of costs of pumping in materials transport as waste water, ore slurry and oil in pipelines. The subject was first reported in the literature by Toms (1949) as a phenomenon observed during experimental studies on the degradation of polymers. The results of this paper was presented in 1948 during a congress on rheology, when Toms revealed that a small amount (the order of tens of parts per million by weight) of a long chain polymer (polymethylmethacrylate) added to a Newtonian solvent (monochlorobenzene) under turbulent flow in a pipe reduced the pressure drop substantially below that of solvent alone at the same flow rate.

After Toms' work, several studies related to polymer drag reduction, the so-called "Toms' effect", have been reported. It should be emphasized that the drag also can be reduced by using surfactants agents, fiber suspensions, microbubbles, compliant coating over solid surfaces, wall oscillations, and modifications of wall as riblets. Effects on heat transfer and cavitation are also observed in polymer and surfactant solutions. Some studies that review and analyze the methods for promoting the drag reduction listed here can be found in the works of Wang, Yu, Zakin and Shi (2011), Choi (2000) and García-Mayoral and Jiménez (2011).

Despite the large amount of research and publications on drag reduction observed since its discovery in the final of forties, the topic has not yet been exhausted with respect to the accuracy of prediction models and the theories to explain the occurrence of the phenomenon. The verification of this fact was the motivation for this work, whose scope is to suggest a methodology to obtain the friction factor using more elaborate rheological models, defined on a wider range of shear rates, for use in the quantification of polymer drag reduction.

2. THE CROSS AND SISKO RHEOLOGICAL MODELS

For general flows, the apparent viscosity of the Generalized Newtonian Fluid (GNF) model, obtained from the Reiner-Rivlin fluid, is a scalar function of the 2nd and 3rd invariants of $\boldsymbol{D}$, where $\boldsymbol{D}$ (s^{-1}) is the shear rate tensor (see Macosko, 1994 for more details). For incompressible, steady, simple shear flows, simplifications are observed, and in this case, the apparent viscosity of GNF model reduces to a function of the shear rate magnitude~~,~~ only. Thus, for this class of flows, the Cross and Sisko models are generalizations of the Newtonian model which allow variations of viscosity with the shear rate. Both models have four rheological parameters, the minimum number of parameters to predict the shape of the general flow curve (Barnes, 1989). With these constraints, the Cross and Sisko models are defined, respectively, as follows:

$$\mu = \mu_\infty + \left(\mu_0 - \mu_\infty\right)\left[1 + (\lambda\dot{\gamma})^n\right]^{-1} \tag{1}$$

$$\mu = \mu_\infty + \mu_{ref}\,(\lambda\dot{\gamma})^{n-1} \tag{2}$$

The parameters μ_∞ (Pa·s) and μ_0 (Pa·s) refer to the asymptotic values of viscosity at very low and very high shear rates respectively (Barnes, 1989); μ_{ref} (Pa·s) has the same meaning of μ_0; λ^{-1} (s^{-1}) represents a characteristic shear rate at which the viscosity of the system is the mean of the two limiting values μ_0 and μ_∞ (Cross, 1965); n (dimensionless) is a value that defines the degree of pseudoplasticity – or dilatancy – of the material (Barnes, 2000). In other words, n defines the inclination of the curve between the asymptotic values of the viscosities μ_0 and μ_∞. If the material is pseudoplastic (or shear-thinning), $n < 1$, if it is dilatant (or shear-thickening), $n > 1$. $\dot{\gamma}$ (s^{-1}) is the single component of tensor $\boldsymbol{D}$. This description of the rheological parameters makes it unnecessary a display of the general flow curve, i.e., a chart ($\mu \times \dot{\gamma}$). Macosko (1994), Barnes (2000), Barnes *et al*. (1989), and Chhabra and Richardson (2008) discuss the two models, among others, and describe several inter-relationships between all models, from limiting cases.

3. THE FRICTION-FACTOR FORMULATION

The procedure followed here to derive the friction-factor equations is the same as described in Andrade (2002) and Andrade *et al*. (2007). For turbulent pipe flow, using any rheological model according to the GNF constitutive model, in the viscous sublayer the approximate mean momentum equation can be written as follows:

$$\frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = 0 \tag{3}$$

where $\mu := \mu(\dot{\gamma})$ (Pa·s) is the non-Newtonian dynamic viscosity, u (m·s^{-1}) is the mean velocity and y (m) is the normal coordinate from the pipe wall. Integrating and applying the boundary condition at the wall (no slip), it follows that

$$\tau_w = \mu \frac{\partial u}{\partial y} \tag{4}$$

Dividing Eq. (4) by ρ (kg·m^{-3}) and introducing the definition of friction velocity $u_* := (\tau_w/\rho)^{1/2}$ (m·s^{-1}), where τ_w (Pa) is the shear stress at the wall, with substitution of a rearranged form of the Eq. (1) into Eq. (4), the result obtained is

$$u_*^2 = \nu_0 \left(\mu_\infty \mu_0^{-1} + \left(1 - \mu_\infty \mu_0^{-1}\right)\left(1 + \left(\lambda \frac{\partial u}{\partial y}\right)^n\right)^{-1} \right) \frac{\partial u}{\partial y} \tag{5}$$

where $\nu_0 := \mu_0/\rho$ (m^2·s^{-1}) is a kinematic viscosity at low shear rates and $\dot{\gamma} := \partial u/\partial y$ for simple shear flows.

Admitting that in the viscous sublayer the velocity profile is nearly linear and defining a characteristic scale length near wall suitable δ (m), the following approximation for $\partial u/\partial y$ can be written (Andrade, 2002; Andrade *et al*., 2007):

$$\frac{\partial u}{\partial y} \cong \frac{u_*}{\delta} \tag{6}$$

The approximation for $\partial u/\partial y$ in the viscous sublayer given by Eq. (6) was used by considering that (*a*): the experimental profiles in turbulent pipe flow of shear thinning Carreau-Yasuda fluids obtained from measures with Laser Doppler Anemometry (LDA) by Japper-Jaafar *et al*. (2009) suggest an approximately linear behavior and (*b*): the Cross, Sisko, and the Carreau-Yasuda fluids have the same pattern of general flow curve, with two Newtonian plateaus, at low and high shear rates, and an intermediate region, between them.

Inserting Eq. (6) into Eq. (5) it follows that

$$u_*^2 = \nu_0 \left(\mu_\infty \mu_0^{-1} + \left(1 - \mu_\infty \mu_0^{-1}\right)\left(1 + \left(\lambda u_* \delta^{-1}\right)^n\right)^{-1} \right) u_* \delta^{-1} \tag{7}$$

The Introduction of U^2, with U (m·s^{-1}) being the bulk velocity, and D (m), with D being the pipe diameter, into Eq. (7) leads to appearing of the Fanning friction factor $f := 2\,(u_*/U)^2$ and the Reynolds number $Re := D\,U/\nu_0$, and the final result for the Cross model is given by:

$$2^{-1} f = Re^{-1} D\, \delta^{-1} \left(2^{-1} f\right)^{1/2} \left(\mu_\infty \mu_0^{-1} + \left(1 - \mu_\infty \mu_0^{-1}\right) \left(1 + \left(\lambda\, Re^{-1} U^2 \nu_0^{-1} D\, \delta^{-1} \left(2^{-1} f\right)^{1/2}\right)^n\right)^{-1} \right) \tag{8}$$

Applying a methodology similar to the Eq. (2), the final result for the Sisko model is:

$$2^{-1} f = Re^{-1} D\,\delta^{-1} \left(2^{-1} f\right)^{1/2} \left(\mu_\infty \mu_{ref}^{-1} \left(\lambda\, Re^{-1} U^2 \nu_{ref}^{-1} D\,\delta^{-1} \left(2^{-1} f\right)^{1/2} \right)^{n-1} \right) \tag{9}$$

The characteristic scale length near wall δ, for Eqs. (8) and (9), is calculated from the relationship written below:

$$D\,\delta^{-1} = 2^{(n-2)/(2n)} (3n+1)(4n)^{-1} 8^{(n-1)/n} 10^{\left(f^{-1/2} + 0{,}4 n^{1{,}2}\right) 4^{-1} n^{-0{,}25}} \tag{10}$$

In the derivation of Eq. (10) were used the solution of Eq. (6) and a relationship based on the velocity profile for power law fluids in the viscous sublayer (cf. Skelland, 1967) achieved after introducing the variables D and U, leading to the relationship $(D \cdot \delta^{-1})^n\, 2^{(2-n)/2} = Re'\, f^{(2-n)/2}$ that is applied on the formulation of Dodge and Metzner (1959) for power law fluids in turbulent pipe flows. The consistency index K (Pa $\cdot$ s^n) of the Reynolds number modified of Metzner and Reed $Re' = \rho\, D^n\, U^{2-n}/K$ (cf. Dodge and Metzner, 1959) is replaced in each case (for the Cross and Sisko models) by products $\mu_0\, \lambda^{-1}$ and $\mu_{ref}\, \lambda^{-1}$, both with the same dimension of K if used in the power law constitutive model, as described by Bird, Armstrong and Hassager (1987). Such substitution was used to obtain a Reynolds number modified similar (to that of Metzner and Reed) Re'' in each case, as well as the corresponding formulations similar to that of Dodge and Metzner (1959). In this way, to obtain the Eq. (10) it was assumed that the turbulent mean velocity profile has a logarithmic behavior out of the viscous sublayer. This assumption also is supported by results of Japper-Jaafar *et al*. (2009), which have noted a logarithmic behavior for the turbulent experimental profiles measured in this region of the flow. A more detailed description on how to obtain Eq. (10) can be found in Andrade (2002).

4. NUMERICAL RESULTS

Rheological data from Pereira and Pinho (1999) and Escudier and Smith (2001) were applied to extract friction factors from the set of Eqs. (8)-(10) and Eqs (9)-(10), thereafter called Cross Model Formulation of Friction-Factor (CMFFF) and Sisko Model Formulation of Friction-Factor (SMFFF), respectively. The aqueous polymeric solution used by Pereira and Pinho (1999) was xanthan gum (XG), grade Keltrol TF from Kelco Division of Merck and Co. Inc., at weight concentrations (w/w) of 0.050%, 0.100%, 0.125%, 0.150%, 0.200%, 0.250%. To all solutions 0.020% by weight of kathon LXE from Rohm and Haas was added to prevent against bacteriological degradation. Escudier and Smith (2001) used an aqueous polymeric solution of 0.100% w/w high viscosity grade sodiumcarboxymethyl-cellulose (CMC), supplied by the Aldrich Chemical Co., blended with 0.100% w/w Keltrol TF, a xanthan gum (XG) as aforementioned. In this way, XG holds for aqueous solutions of xanthan gum whose rheology was fitted by the Sisko model (Pereira and Pinho, 1999), and XG-CMC holds for the blended aqueous solution of xanthan gum with sodiumcarboxymethyl cellulose, where the rheology was adjusted by the Cross Model (Escudier and Smith, 2001). The density ρ of all solutions was assumed as being that of water, since all the solutions considered were in the low concentration regime. All physical properties of water were taken at Standard Ambient Temperature and Pressure (SATP), namely, for the density and dynamic viscosity, respectively, $\rho = 1000$ kg $\cdot$ m^{-3} and $\mu = 0.001002$ Pa $\cdot$ s. The rheological parameters according to the Sisko and Cross models of the aqueous solutions used in the simulations with FFFECM and FFFESM, are shown in Tab. 1.

Table 1. Rheological parameters of the Sisko model* (XG solutions) and the Cross model† (XG-CMC solutions).

Solution	μ_{r0} (Pa s)	μ_{ref} (Pa s)	μ_∞ (Pa s)	λ (s)	n (–)	$\dot{\gamma}$ (s^{-1})
0.050% XG	–	0.016743	0.000334	2.500	0.6986	10-1600
0.100%XG-CMC	0.68100	–	0.00328	3.210	0.552	0.1-1500

*data from Pereira and Pinho (1999) measured at 25°C
†data from Escudier and Smith (2001) measured at 19°C

The global flow parameters (experimental input data) applied to the FFFECM and FFFESM were used in Pereira and Pinho (1999) for measurements computed using aqueous polymeric solution made with tylose grade MH 10000K from Hoechst. The rheology of these solutions was fitted by the Carreau-Yasuda model. The global flow parameters were kindly provided by prof. F. T. Pinho and used in Andrade (2002) and Andrade *et al*. (2007). The full set of experimental input data include the pipe diameter, bulk velocities, pressure drops, wall viscosities, wall Reynolds numbers, experimental friction factors, among others variables. In this study were used as experimental input data the pipe diameter $D = 0{,}026$ m and the bulk velocities U (m $\cdot$ s^{-1}), the last, as given in Tab. 2.

Table 2. Bulk velocities† used in the simulations FFFECM and FFFESM.

$U\,(\mathrm{m \cdot s^{-1}})$	1.049	1.424	1.792	2.164	2.436	2.911	3.154	3.586	3.970	4.339	4.621

†data from Pereira and Pinho (1999) measured at 25°C used in measurements computed for the tylose aqueous solution 0.1% w/w fitted by the Carreau-Yasuda model

The FFFECM and FFFESM were solved for the Fanning friction factor (f) by numerical codes built in FORTRAN language using the Newton-Raphson method. The values of Re'' were computed for use as single input data in the solution of Prandtl-von Kármán equation (cf. Skelland, 1967) for the sake of providing a reference of the magnitude of friction factors computed at the same Reynolds number. The providing of reference values among data computed is a common practice. Representative examples are given by data computed from relationships as maximum drag reduction asymptote (MDRA) of Virk, Blasius equation, Dodge and Metzner equation, Dodge and Metzner equation based on wall Reynolds number, Prandtl-von Kármán equation and experimental friction factors. Such examples can be found in Dodge and Metzner, 1959, Escudier and Presti, 1996, Pereira and Pinho, 1994 and Cruz and Pinho, 2003. After computation of f from FFFECM, FFFESM and Prandtl-von Kármán equation the relationship $f_D = 4f$ was used to compute the values of Darcy friction factor f_D, and display plots of f_D against Re'' in Fig. 1.

Similar plots depicting the friction factors computed from a Formulation of Friction-Factor Equation for the Carreau-Yasuda Model (FFFEC-YM), the experimental friction factors measured by Pereira and Pinho (1999) with tylose solutions, and the corresponding values calculated from the Prandtl-von Kármán equation against a Reynolds number Re'' based on parameters from the Carreau-Yasuda fitting (Pereira and Pinho, 1999), are shown in the Fig. 2 using $D = 0{,}026$ m and two sets of bulk velocities U according to Tab. 3. The rheological parameters from the Carreau-Yasuda fittings (Pereira and Pinho, 1999) are shown in Tab. 4.

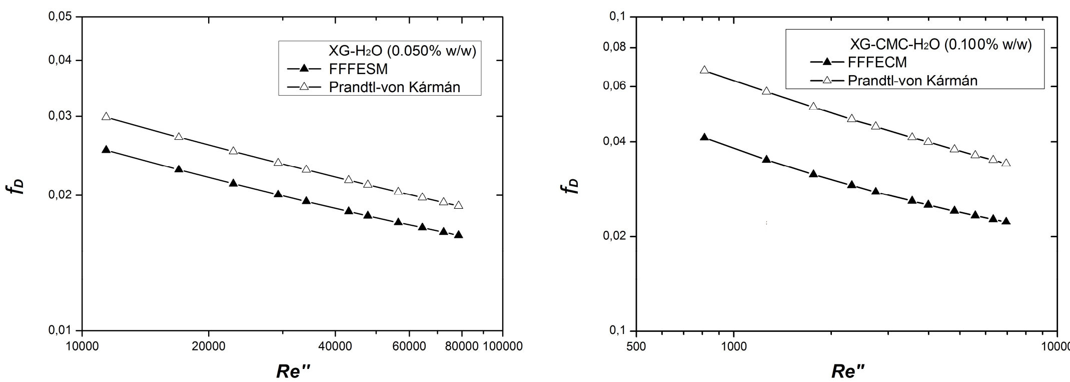

Figure 1. Darcy friction factors *versus* Re''. Visualization of the friction factors given by FFFESM, FFFECM, and Prandtl-von-Kármán equation: (a) XG-H_2O 0.050% w/w ; (b) XG-CMC-H_2O 0.010% w/w.

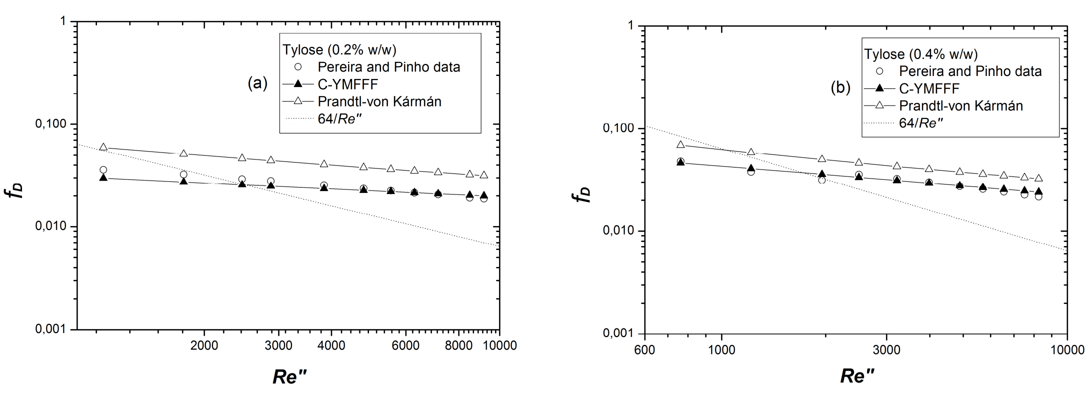

Figure 2. Darcy friction factors *versus* Re''. Friction factors from FFFEC-YM, Prandtl-von-Kármán equation and Pereira and Pinho data: (a) tylose 0.2% w/w; (b) tylose 0.4% w/w. Dotted lines corresponds to the friction factors in laminar flows.

The complete data set from Pereira and Pinho (1999) for the tylose solutions at weight concentrations of 0.1% to 0.6% was used in a similar approach to the derived in this work for Carreau-Yasuda fluids (FFFEC-YM) and show good agreement with the experimental friction factors (Andrade, 2002; Andrade *et al*., 2007).

As the global flow parameters applied to the FFFECM and FFFESM are measurements computed using aqueous solutions of tylose with viscosities adjusted according to Carreau-Yasuda model, the data computed for the plots displayed in Fig. 1 has a character essentially qualitative. Until final preparation of this paper was not possible to use the global flow parameters actual related to rheology of the polymeric solutions used in the FFFECM and FFFESM to provide more accurate data. Due to this fact was not computed a more extensive list of data for analysis.

Table 3. Bulk velocities[†] used in the simulations of FFFEC-YM.

U $(\mathrm{m \cdot s^{-1}})$ 0.2% tylose	1.047	1.434	1.804	2.019	2.487	2.905	3.233	3.546	3.892	4.405	4.656
U $(\mathrm{m \cdot s^{-1}})$ 0.4% tylose	0.817	1.143	1.603	1.912	2.293	2.672	3.094	3.456	3.821	4.217	4.511

[†]data from Pereira and Pinho (1999)

Table 4. Rheological parameters of the Carreau-Yasuda model[*] (tylose solutions).

	μ_0 (Pa s)	μ_∞ (Pa s)	λ (s)	n (–)	a (–)	$\dot{\gamma}$ $(\mathrm{s^{-1}})$
0.2% tylose	0.00608	0.00100	0.0001200	0.61110	0.3008	20-4000
0.4% tylose	0.02276	0.00100	0.0030000	0.60510	0.7432	10-4000

[*]data from Pereira and Pinho (1999) measured at 25°C

5. CONCLUSIONS

Relationships for the Cross and Sisko models were derived for the prediction of friction factors in turbulent pipe flows of non-Newtonian fluids. The friction factors calculated from these relationships, for polymeric solutions, were displayed with the ones computed from Prandtl-von Kármán equation, for Newtonian fluids, all at the same Reynolds number Re''. As expected for the non-Newtonian fluids considered here, the results showed coherence in terms of reducing the friction factors in comparison with the corresponding values computed via Prandtl-von Kármán resistance equation for turbulent flow in smooth pipes, which holds for Newtonian fluids.

Additional simulations with other databases and comparisons with experimental data and numerical results from other formulations need to be carried out for a better assessment of the degree of robustness and accuracy of the proposed formulations.

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