CHAOTIC MOTIONS OF A SIMPLE PORTAL FRAME

Reyolando M.L.R.F. Brasil
José Manoel Balthazar
Departamento de Engenharia de Estruturas e Fundações, Escola Politécnica da Universidade de São Paulo, C P 61548, CEP 05424-930, São Paulo, S P, Brasil. e-mail: rmlrdfbr@usp.br

Abstract

This paper presents a simple portal frame structure of nonlinear behavior under internal resonance conditions. The equations of motion of a simplified two degree of freedom model are obtained via a Lagrangian approach. Free undamped and resonant forced damped vibrations are analyzed for several energy levels to show the onset of chaotic motions in both cases. Poincaré Maps and Lyapunov exponents are obtained and commented upon. Possible application to real civil engineering structures is suggested.

Key words: Nonlinear Dynamics, Structures, Chaos

1. INTRODUCTION

This paper is concerned with the study of free and forced nonlinear vibrations of structures under internal and external resonance conditions for several energy levels. We search for examples of practical civil engineering problems for which chaotic motions may occur.

The nonlinear vibrations of frames have been investigated by a number of researchers. As early as 1970, Barr and McWannell (1970) studied a frame under support motion, but nonlinear elastic forces were not taken into account. Yet, these are extremely important and affect qualitatively and quantitatively the analysis. Brasil and Mazzilli (1990) studied the related problem of a framed machine foundation of similar geometry. They recast the problem, considering both inertial and elastic nonlinear effects, including that of the geometric stiffness of the columns and geometric imperfections, such as the elastic deformations of the frame, before the excitation would come into action. Some other studies of nonlinear oscillations of other portal frames under a single ideal harmonic excitation will be found in several papers by Brasil and Mazzilli (1993, 1995). Another study of a structure under several ideal harmonic loads is presented by Brasil (1999). An extension to the non ideal case (limited power supply) will be found in Brasil and Mook (1994).

Here, another related problem of considerable practical importance is presented. A simple portal frame with two vertical columns and a horizontal pinned beam is considered. If linear behavior should be adopted, the two first vibration modes, anti-symmetrical (sway) and symmetrical, respectively, would be uncoupled. In our model, consideration of the shortening of the bars due to bending render a set of two coupled nonlinear equations of motion derived via a Lagrangian approach. Next, the physical and geometrical characteristics of the frame are chosen to tune the natural frequencies of these two modes into a 1:2 internal resonance.

Modal saturation and energy transference due to internal coupling and external resonance are observed at certain levels of excitation.

To study the possible onset of chaotic motions, two cases are considered.

a) Undamped free vibrations with initial conditions exciting each one of mode separately. As the system level of energy increases, the other mode is set into motion due to the internal coupling with regular interchange of energy between the modes. Further increase of energy will cause irregular motions (chaos).
b) Damped forced vibrations due to ideal harmonic excitation resonant with each one of the modes separately. As the system level of energy increases, the other mode is set into motion due to the internal coupling with regular interchange of energy between the modes. Further increase of energy will cause irregular motions (chaos).

2. PROBLEM DEFINITION

2.1 The Model

The portal frame in Fig. 1 is considered in the analysis. It has two columns clamped at their bases with height h and cross section moment of inertia I_c , with concentrated masses m at their tops. The horizontal beam is pinned to the columns with length L and cross section moment of inertia I_b . A linear elastic material is considered whose Young modulus is E . The structure will be modeled as a two-degree-of-freedom system. q_1 is related to the horizontal displacement in the sway mode (with natural frequency ω_1) and q_2 to the mid-span vertical displacement of the beam in the first symmetrical mode (with natural frequency ω_2). The stiffness related to these modes are k_c and k_b, respectively. Geometric nonlinearity comes from shortening due to bending of the columns and beam, given by Δh and ΔL . In the forced vibration case, an unbalanced motor, with total mass M, is placed at the mid-span of the beam. The angular velocity of its rotor is given by Ω rendering an harmonic excitation. Coefficients of modal linear viscous damping c_1 and c_2 may be adopted.

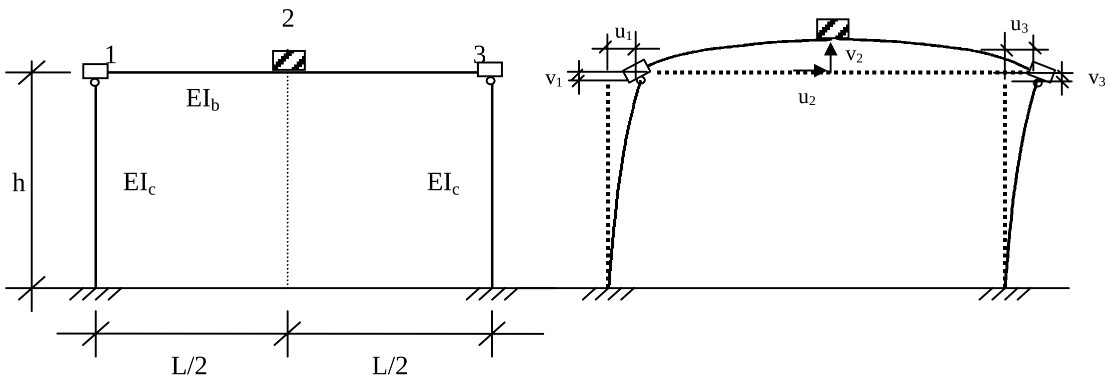

Figure 1. The model portal frame

The two non dimensional generalized coordinates of this model are chosen to be

$$q_1 = \frac{u_2}{h}, q_2 = \frac{v_2}{L} \tag{1}$$

where $\boldsymbol{u}_2$ is the lateral displacement of M in the sway mode, and $\boldsymbol{v}_2$ is its vertical displacement in the first symmetric mode.

The linear stiffness of the columns and of the beam associated to these modes can be evaluated by a Rayleigh-Ritz procedure, rendering:

$$k_c = \frac{3EI}{h^3} \qquad k_b = \frac{48EI}{L^3} \tag{2}$$

The geometric nonlinearity is introduced by considering the shortening due to bending of the columns and of the beam as:

$$\Delta h = \frac{1}{2}C(hq_1)^2 \qquad \Delta l = \frac{1}{2}B(h\ q_2)^2 \tag{3}$$

were, by the same Rayleigh-Ritz consideration as before,

$$C = \frac{6}{5h} \qquad B = \frac{24}{5L} \tag{4}$$

2.2 The Equations of Motion

To derive the equations of motion, the generalized Lagrange's Equations will be used. The kinetic energy and the total potential energy (including the work of the conservative forces) are, respectively,

$$T = \frac{1}{2}\{(2m+M)h^2\dot{q}_1^2 + (ML^2)\dot{q}_2^2\} \tag{5}$$

$$V = MgLq_2 + (k_c - mgC)h^2q_1^2 + (\frac{1}{2}k_bL^2)q_2^2 + (\frac{1}{2}k_bCh^2L)q_1^2q_2 \tag{6}$$

rendering, for damped forced vibrations the following two equations

$$\ddot{q}_1 + \omega_1^2 q_1 = -2\mu_1\dot{q}_1 - \alpha_1 q_1 q_2 + \frac{E_1(t)}{(2m+M)h^2} \tag{7}$$

$$\ddot{q}_2 + \omega_2^2 q_2 = -2\mu_2\dot{q}_2 - \alpha_2 q_1^2 - \frac{g}{L} + \frac{E_2(t)}{ML^2} \tag{8}$$

where

$$\omega_1^2 = \frac{2(k_1 - mgC)}{2m+M},\ \omega_2^2 = \frac{k_2}{M},\ \mu_1 = \frac{c_1}{(2m+M)h},\ \mu_2 = \frac{c_2}{ML},\ \alpha_1 = \frac{k_2CL}{2m+M},\ \alpha_2 = \frac{k_2Ch^2}{2ML}$$

$E_1(t)$ and $E_2(t)$ are time ideal forcing functions to be defined in each case.

3. NUMERICAL SIMULATIONS

In order to search for possible irregular oscillations in this model, numerical simulations are carried out and their results presented in this section. To that end, Equations 7 and 8 are transformed into a set of four first order differential equations. Next, they are numerically integrated using a Runge-Kutta algorithm and Poincaré Maps (PM) are presented to characterize the geometry of the dynamics of this model. Lyapunov Exponents (LE) are also calculated to confirm possible chaotic motions.

3.1 Free Undamped Vibrations

Here, damping is neglected and no forcing functions are considered, leading to free vibrations resulting of several initial conditions. These change the level of energy imparted to the system and may excite directly only one of the two modes. Nevertheless, due to internal resonance, this energy may be passed back and forward between the modes.

First, initial conditions are set to directly excite only the first (sway) mode at a relatively low level of energy. Figure 2a presents the related PM in the q_1 x q_1' plane (with $q_2=0$ and $q_2' > 0$). If the energy imparted to the system is increased, a certain point is reached so that this same PM, shown in Fig. 2b, displays a chaotic character, confirmed by the LE calculation.

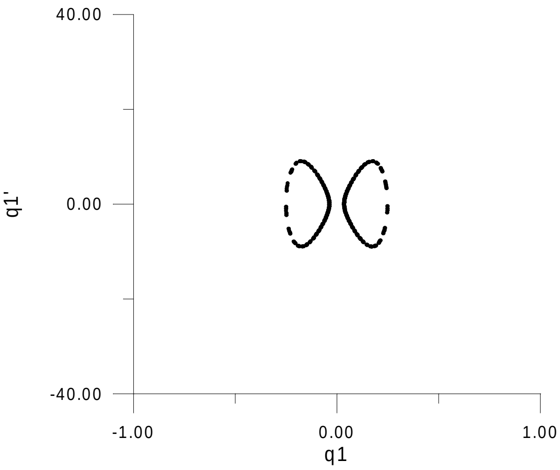

Figure 2a

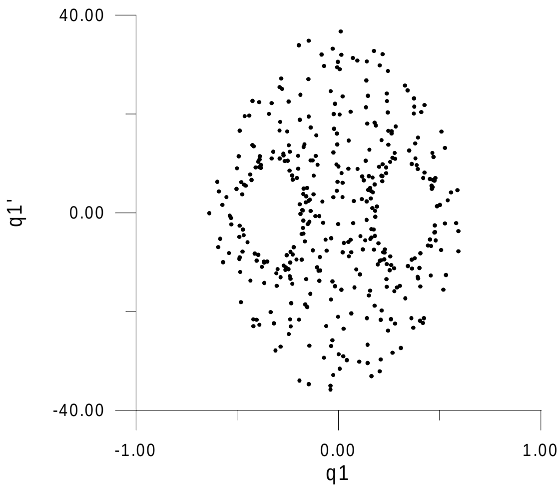

Figure 2b

It is interesting to note that Poincaré Maps in the q_2 x q_2' plane (with $q_1=0$ and $q_1' > 0$) for this case, shown in Figures 3a and 3b, for the same two levels of energy, present chaotic motions too.

Next, initial conditions are set to directly excite only the second (symmetric) mode at a relatively low level of energy. Figure 4a presents the related PM in the q_1 x q_1' plane (with $q_2=0$ and $q_2' > 0$). If the energy imparted to the system is increased, a certain point is reached so that this same PM, shown in Fig. 4b, displays a chaotic character, confirmed by the calculation of the LE.

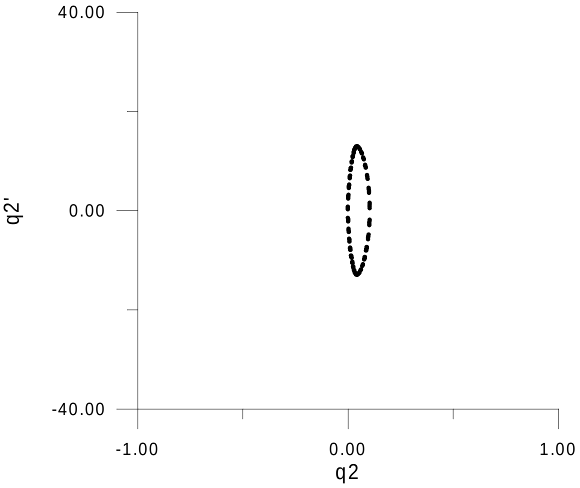

Figure 3a

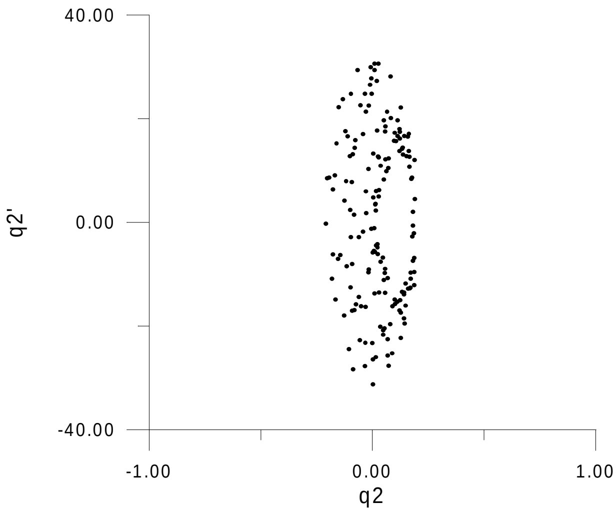

Figure 3b

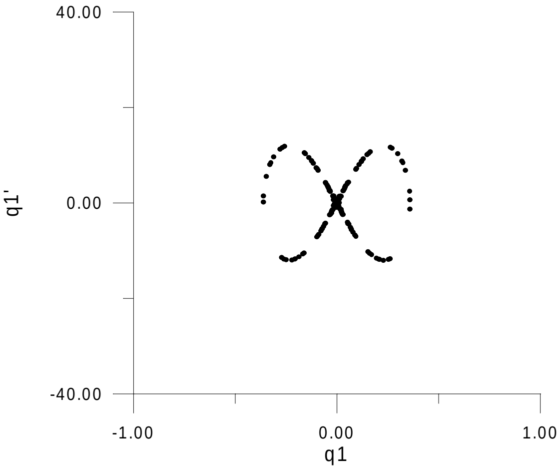

Figure 4a

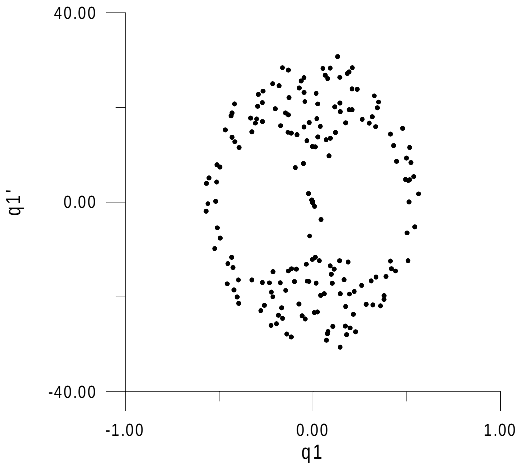

Figure 4b

It is surprising to note that Poincaré Maps in the q_2 x q_2' plane (with $q_1=0$ and $q_1' > 0$) for this case, shown in Figures 5a and 5b, for the same two levels of energy, do not present chaotic motions.

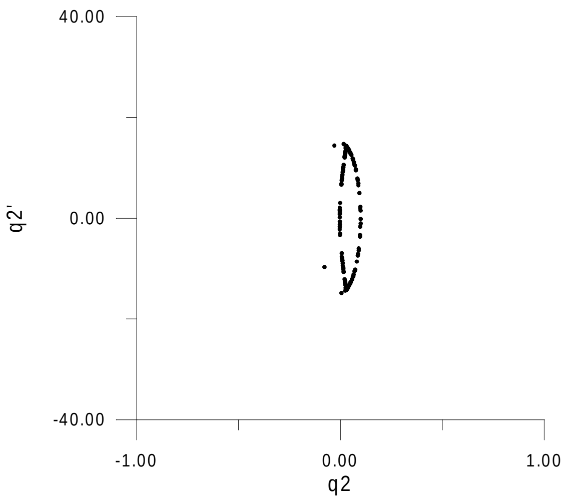

Figure 5a

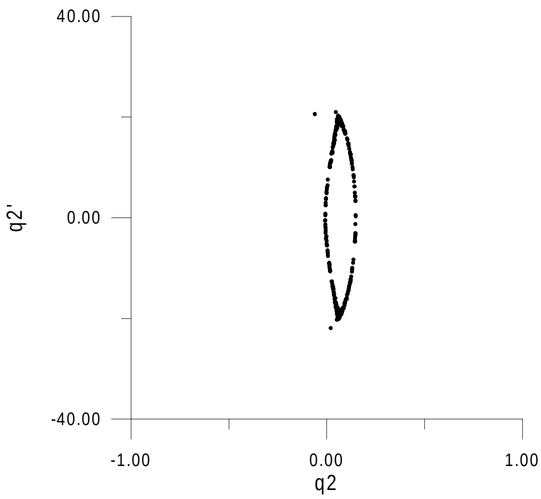

Figure 5b

3.2 Damped Forced Vibrations

Now linear viscous damping is adopted and harmonic forcing functions are considered with fixed Ω frequency and may be made to be resonant with only one of the two modes. Nevertheless, due to internal resonance, energy may be passed back and forward between the modes. Amplitude of these functions are also increased gradually, searching for chaotic motions.

First, we set $\Omega \cong \omega_1$ to directly excite only the first (sway) mode at a relatively low amplitude. Figure 6a presents the related PM in the q_1 x q_1' plane (with $q_2=0$ and $q_2' > 0$). If the amplitude is increased, a certain point is reached so that this same PM, shown in Fig. 6b, displays a chaotic character, confirmed by the calculation of the LE.

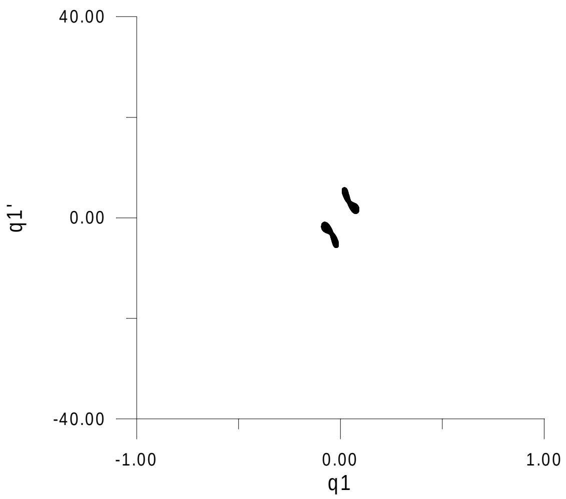

Figure 6a

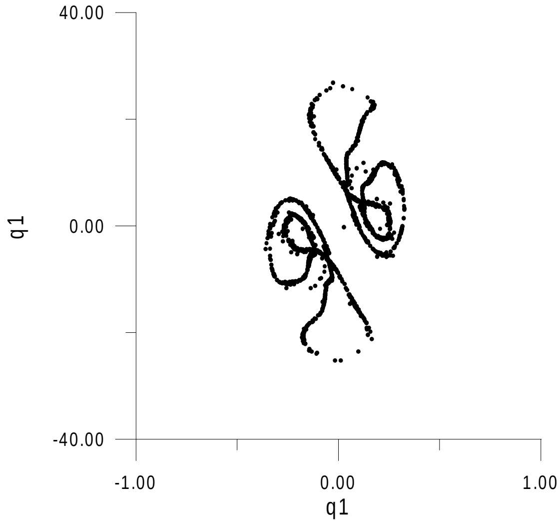

Figure 6b

It is interesting to note that Poincaré Maps in the q_2 x q_2' plane (with $q_1=0$ and $q_1' > 0$) for this case, shown in Figures 7a and 7b, for the same two amplitude levels, also present chaotic motions.

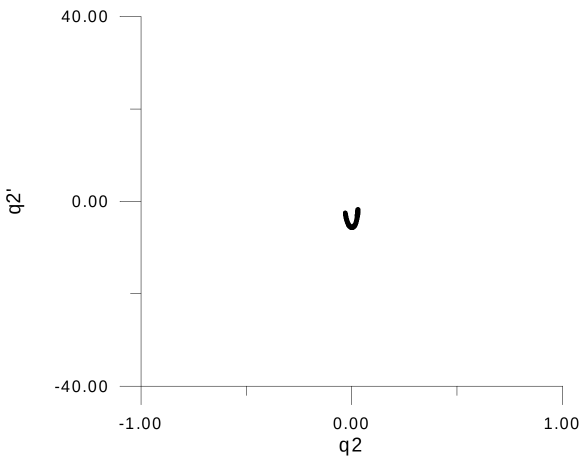

Figure 7a

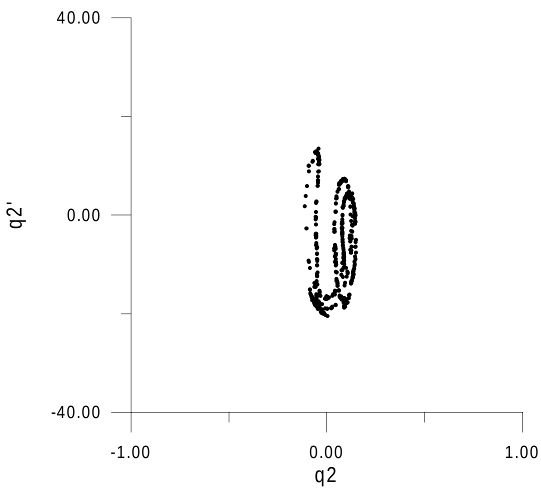

Figure 7b

Next, we set $\Omega \cong \omega_2$ to directly excite only the second (symmetric) mode at a relatively low amplitude. Figure 8a presents the related PM in the q_1 x q_1' plane (with $q_2=0$ and $q_2' > 0$), showing only two plotted points for the steady state regime. If the amplitude is increased, as shown in the MP of Fig. 8b, the same two point pattern is observed, with no chaotic motions.

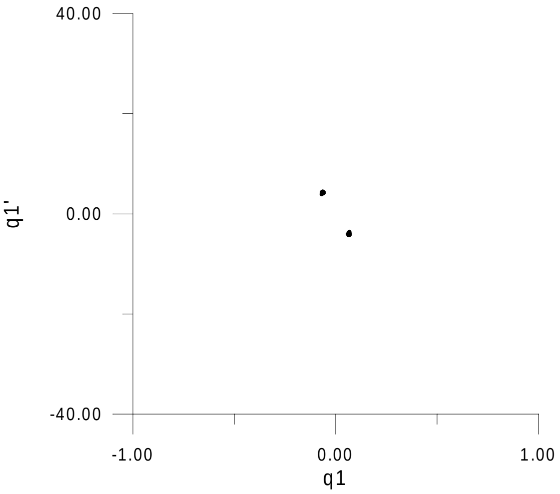

Figure 8a

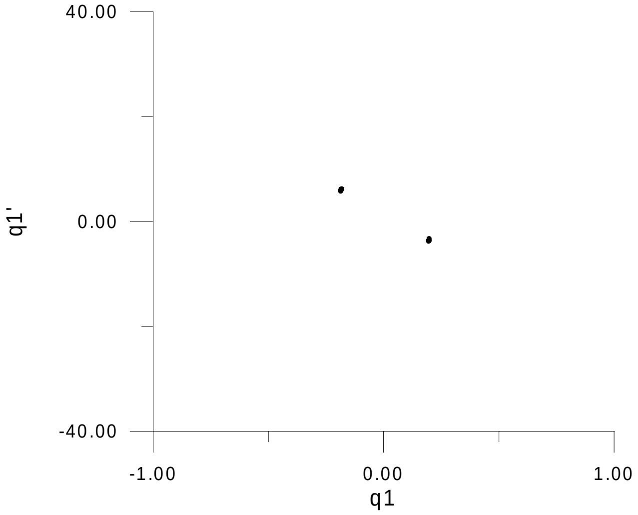

Figure 8b

Similarly, we note that Poincaré Maps in the q_2 x q_2' plane (with $q_1=0$ and $q_1' > 0$) for this case, shown in Figures 9a and 9b, for the same two amplitude levels, present only periodic motions.

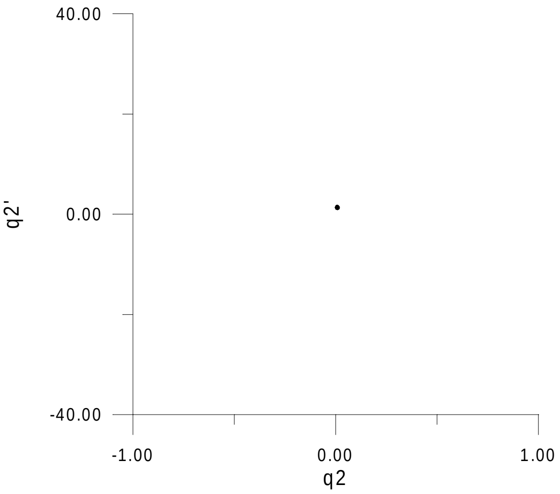

Figure 9a

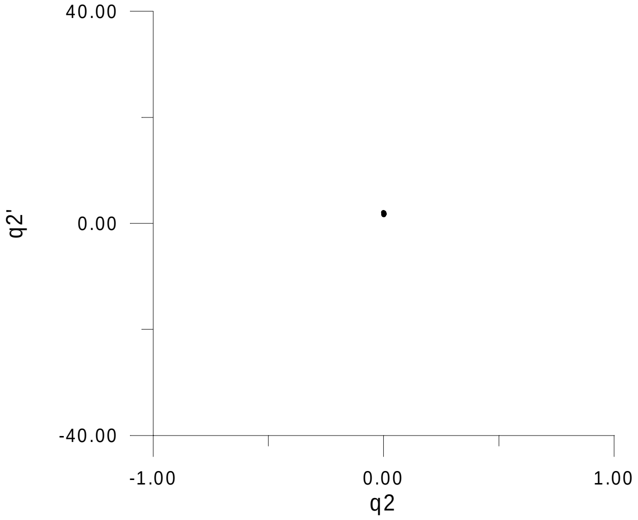

Figure 9b

4. CONCLUSIONS

A two degree of freedom model of a simple portal frame of geometric nonlinear behavior was studied for free and forced vibrations. Conditions of internal resonance were set, allowing for exchange of energy between the modes. The increase of the level of energy imparted to the system lead to chaotic motions in certain situations, as shown via Poincaré Maps and the calculation of Lyapunov exponents. Extension of this work for non ideal sources of energy is in its way and will be reported in the future.

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