

# A METHOD TO ESTIMATE PARAMETERS OF LONGITUDINAL AND LATERAL DYNAMICS OF GROUND VEHICLES

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Abstract. On ground vehicles, there is a variation of their dynamic parameters. These variations produce saturation in the actuators, even on maneuvers of moderately demand, becoming a challenge to maintain its stability and security. In order to maintain vehicle's performance, it is important to estimate their dynamic parameters. In this paper, parameters of longitudinal and lateral dynamic of vehicles are estimated using Genetic Algorithms (GA). The GA has been applied directly in two mathematical models of vehicle, the first model is developed with scripts in MATLAB with 6 degree of freedom (DOF) and the second is the CarSim multi-degree of freedom vehicle model. The simulation in CarSim with the reference parameters, is considered as the real dynamic behavior of the vehicle. In the method of estimation, we define a vector containing the parameters to be estimated, and this evolves in each generation according to an evolution function that depends on the difference between simulated and real behavior of the vehicle. The estimated vehicle dynamic parameters are verified with different maneuvers and comparated with the parameters of reference to check the robustness of the method.

Keywords: longitudinal dynamics, lateral dynamics, parameter estimation, genetic algorithms.

# **1. INTRODUCTION**

For ground vehicles, of four wheels, there are many types of control such as electronic stability control, brake control, traction control, among others. These controls need to know the behavior of the vehicle in certain maneuvers in order to act properly through the control signal. This dynamics behavior can be approximated by mathematical models. Through mathematical models, we can get a general behavior of the system and with a data of the acquisition data system, we can make a mathematical model approximated to the behavior of this particular system.

In general, for any mathematical model that characterizes any real dynamic system, they need to know some of its parameters. Some of them are obtained directly by data acquisition systems, and others have to be estimated via optimization techniques or other methods. Some of the most important dynamic parameters of vehicles are the sprung mass, inertia moments, the position and height of the center of gravity, the coefficients of stiffness and damping of the suspensions, etc. Consequently, these parameters are critical in the control systems of ground vehicles, since through them and the mathematical model, we can characterize a dynamic, similar to the real dynamics of the vehicle.

In recent years, various types of estimation methods for the dynamic parameters of ground vehicles have been developed. The thesis work (Ryu, 2004), show an estimation of dynamic parameters of vehicle using a Global Position System (GPS) for data acquisition and as optimization method a Least Mean Squares. In the article (Vahidi *et al.*, 2005) used the method of recursive least squares estimation to make a real-time vehicle mass and profile of the track, using for this a global positioning system (GPS) mounted on the vehicle, getting errors of up to 5% in the estimation. The work (Rozyn and Zhang, 2010) developed a new method for estimation of inertial parameters of the vehicle such as the sprung mass, moments of inertia and location of center of gravity, this method measures the response of the sprung mass when the vehicle is driving under an unknown and random track. Then, according to the equivalent response of free decay and using a modal analysis technique, the natural frequency of the sprung mass, damping ratios and mode shapes are estimated. This information is combined with a simplified model of the vehicle, and using an optimization algorithm based on least squares, inertial parameters are obtained from the vehicle. Recently (Jaiganesh and Kumar, 2012), developed a method based in Genetic Algorithm for estimation of vehicle inertia parameters, they used a CarSim multi-degree of freedom as model of the vehicle, obtaining small errors. Exits other method for parameter estimation based in Kalman Filter, for example, (Wenzel *et al.*, 2006) and (Winstead and Kolmanovsky, 2005), but the difficult for this method is the accuracy of estimation, its error can exceed 10%.

This work develops a method for estimating the dynamic parameters of a ground vehicle for four wheels, through optimization methods based on Genetic Algorithms (GA). This estimation system is applied directly to a dynamic of 6 degrees of freedom (DOF) developed in MATLAB and CarSim multi-degree of freedom vehicle model. Our references for the dynamic behavior of the actual vehicle are obtained by the simulation package CarSim vehicle with the nominal values of parameters. The estimation is realized in two steps a first step for step steer angle and the second for sinusoidal steer angle to check the consistency and robustness of the estimation method.

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In this paper, we first develop dynamics of 6DOF vehicle that we use to estimate the parameters. Then, the optimization method that was used to estimate the dynamic parameters is described. Finally, the estimated parameters are now checked through different maneuvers demonstrating the robustness of the estimation method developed.

# 2. MAGIC FORMULA TYRE MODEL

The tires are responsible for generating the forces that move the vehicle, so, these forces are the inputs that excite our vehicle. Due to the great importance of tires, its modeling should be done properly according to the required application.

There are two main types of tires modeling, empirical methods and physical methods. The empirical methods widely, developed by (Bakker *et al.*, 1987), (Sharp, 2004) and (Pacejka, 2006), are mainly functions that approximate the actual behavior of both, tire forces and moments. Physical methods, for example, (Gipser, 2005) explain the mechanisms that determine the behavior of the tire; this modeling is typically more accurate; however, involve a high computational cost.

The model studied in this paper is the semi-empirical model of the Magic Formula tire (Pacejka, 2006). This model is used to calculate forces and moments in a stable state that will be used in studies of the dynamics of the vehicle. In the next sections, a short overview of the model and the fit results will be presented.

## 2.1 Preliminary Concepts

#### 2.1.1 Effective Rolling Radius

When the tire is rotating freely, i.e. without traction or braking torque, the effective rolling radius is the ratio between longitudinal velocity of the tire and its angular velocity, and can be calculated as follows:

$$R_e = \frac{v_x}{\omega} \tag{1}$$

where  $v_x$  is the longitudinal velocity of the wheel center, and  $\omega$  its angular velocity. Figure 1 illustrates the coordinate system of the wheel.



## 2.1.2 Longitudinal Slip

To generate forces while the tire is rolling, the tire must slip. Slip occurs in different planes of the tire's motion. A tire exerts longitudinal force only if a longitudinal slip is present, and it is developed when a driving or braking torque is applied to the wheel. The longitudinal slip  $\kappa$  could be defined as follows:

$$\kappa = -\frac{v_{slip\,velocity}}{v_r} \tag{2}$$

The slip velocity  $v_{slip velocity}$  is the relative velocity of the tire contact point with the ground, and is given by  $v_x - v_c$ where  $v_c = R_e \omega$  is the circumferential or tangential velocity of the tire.

During acceleration, the actual longitudinal velocity  $v_x$  is less than the circumferential velocity  $v_c$ , and therefore,  $\kappa > 0$ . However, during braking, the longitudinal velocity  $v_x$  is higher than the circumferential velocity  $v_c$  and therefore,  $\kappa < 0$ .



#### 2.1.3 Side Slip Angle

The side slip angle  $\alpha$  is the ratio of lateral velocity and longitudinal velocity of the tire. The lateral velocity  $\alpha$  could be defined as follows:

$$\alpha = -\arctan\frac{v_y}{v_x} \tag{3}$$

#### 2.2 Model description

The magic formula model was introduced by Pacejka et al. in [3] and is defined as:

$$y(x) = D\sin[C\arctan(Bx - E(Bx - \arctan(Bx)))]$$
(4)

with:

$$Y(X) = y(x) + S_V$$

$$x = X + S_H$$
(5)

where Y is the output of the Magic Formula and is defined as the longitudinal force  $F_x = y(\kappa)$ , lateral force  $F_y = y(\alpha)$ and the aligning moment  $M_z = y(\alpha)$ . X is the input of model, this input can be the longitudinal slip  $\kappa$  or side slip angle  $\alpha$ . The other parameters are defined as:

B: is the stiffness factor C: the shape factor D: is the peak value E: is the curvature factor  $S_H$ : horizontal shift  $S_V$ : vertical shift

# 2.3 Estimating the Factors

In this work, the real data of tire behavior is provided by CarSim for the reference tire used in this paper. In order to characterize the behavior of our tire, we need to tune the parameters (factors) of the magic formula. To determine the Magic Formula Tyre parameters, an optimization routine is used to minimize the error between the measurement (CarSim data) and the Magic Formula Tyre model. In this works (Cabrera *et al.*, 2004), (Ortiz *et al.*, 2006) develop methods for estimating parameters of magic formula used optimization algorithms. The optimization routine applied in this article is the *fminsearch* function of MATLAB. In this optimization, the following tire characteristics are optimized:

- Pure longitudinal force fit, pure kappa sweeps.
- Pure lateral force fit, pure alpha sweeps.
- Pure aligning torque fit, pure alpha sweeps.

The optimization problem is modeled as follows:

$$Min f(p_1, p_2, ..., p_n)$$

$$subject to:$$

$$g(p_1, p_2, ..., p_n) \le 0$$
(6)

Where f is the fitness function,  $p_1, p_2, ..., p_n$  are the parameters of magic formula and g is the constraint that defining the searching space. All these characteristics are minimized with the fitness function defined as:

$$fitness\,function = \sqrt{\frac{\sum_{i=1}^{n} [Y_{fit} - Y_{CarSim}]^2}{\sum_{i=1}^{n} Y_{CarSim}^2} \times 100\%}$$
(7)

The coefficients for each tire factor are expressed as follows, and their meaning is presented at (Pacejka, 2006). Longitudinal Force  $F_x$  (pure longitudinal slip):

$$B_{x} = \frac{C_{f\kappa}}{C_{x}D_{x}}$$

$$D_{x} = \mu F_{z}$$

$$C_{f\kappa} = c_{1} \sin[2 \arctan(\frac{F_{z}}{c_{2}})]$$
(8)

The parameters for the optimization of the longitudinal force are  $[C_x, E_x, \mu, c_1, c_2]$ . Lateral Force  $F_y$  (pure lateral slip):

$$B_{y} = \frac{C_{f\alpha}}{C_{y}D_{y}}$$

$$D_{y} = F_{z}(b_{13}F_{z} + b_{14})$$

$$C_{f\alpha} = b_{4}\sin[2\arctan(\frac{F_{z}}{b_{5}})]$$
(9)

The parameters for the optimization of the lateral force are  $[C_y, E_y, b_4, b_5, b_{13}, b_{14}]$ . Aligning Moment  $M_z$  (pure lateral slip):

$$B_m = \frac{C_{m\alpha}}{C_m D_m}$$

$$D_m = F_z (b_{15}F_z + b_{16})$$

$$C_{m\alpha} = F_z (\frac{b_6F_z + b_7}{e^{b_8F_z}})$$
(10)

The parameters for the optimization of the aligning moment are  $[C_m, E_m, b_6, b_7, b_8, b_{15}, b_{16}]$ .

The table 1 show the results of estimation of Magic Formula parameters using *fminsearch* function of MATLAB.

Table 1. Parameters estimated of magic formula.

Parameters	$F_x$ [N]	$F_y$ [N]	$M_z$ [N-m]	
C [-]	-1.52365	1.4425	2.1194	
E [-]	-0.456989	$-6.2444 \times 10^{-1}$	-1.4247	
μ[-]	1.39337	-	-	
$c_1$ [N/rad]	$3.36991 \times 10^{6}$	-	-	
$c_2$ [N]	268415	-	-	
$b_4$ [N/rad]	-	$2.2167 \times 10^5$	-	
b <sub>5</sub> [N]	-	$1.4189 \times 10^4$	-	
<i>b</i> <sub>6</sub> [1/N-rad]	-	-	$-3.2686 \times 10^{-5}$	
b <sub>7</sub> [1/rad]	-	-	$5.3413 \times 10^{-1}$	
b <sub>8</sub> [1/N]	-	-	$-1.9542 \times 10^{-4}$	
b <sub>13</sub> [1/N]	-	$-5.4576 \times 10^{-6}$	-	
b <sub>14</sub> [-]	-	1.0067	-	
b <sub>15</sub> [1/N]	-	-	$2.7659 \times 10^{-6}$	
b <sub>16</sub> [-]	-	-	$3.8895 \times 10^{-3}$	

Simulations with the parameters of Table 1 are compared with the test data of the CarSim for longitudinal force  $F_x$ , lateral force  $F_y$  and aligning moment  $M_z$  as shown in Figure 2. This optimization was done with different normal loads  $F_z$ .

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(c)  $M_z$  against the side slip angle  $\alpha$ .

Figure 2. Real data CarSim (solid lines) versus optimized Magic Formula (red circles).

## 3. VEHICLE MODELING

In this work, we develop similar mathematical model developed in (Schofield, 2008) as a rigid double-track ground vehicle model including roll and pitch dynamics, using a Newton-Euler modeling approach. The suspension system is modeled as a rotational spring and damper system about x axis (roll motion) and y axis (pitch motion), where the spring and damper constants for each wheel have been grouped to two constants, one for each degree of freedom (x and y).

# 3.1 Coordinate Systems

To facilitate the derivation of the dynamics equations, it is useful to define certain coordinate systems. This allows a more systematic approach for modeling, which is particularly important when dealing with more complex models. Figure 3 illustrates the coordinates systems utilized for the study of vehicle motion. Roll, pitch and yaw are defined as rotations around the x, y and z axes respectively.

Three moving coordinate systems are employed in the derivation of vehicle's motion. The equations of motion are derived in the vehicle-fixed frame  $S_v$ , which rotate with angle  $\psi$  (yaw angle) about z axis of the inertial system  $S_I$ , the matrix of rotation of the system frame  $S_v$  is:

$$R_{v}^{I} = \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0\\ \sin(\psi) & \cos(\psi) & 0\\ 0 & 0 & 1 \end{pmatrix}$$
(11)

Then, the pitch angle is defined as a rotation about y axis of the vehicle frame  $S_v$ , giving the chassis system  $S_c$ , the matrix of rotation that defined the frame  $S_c$  is:

$$R_c^v = \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix}$$
(12)

Finally, the roll angle is defined as a rotation about x axis of the chassis system  $S_c$ , giving the body frame  $S_b$ , its

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matrix of rotation is:

$$R_b^c = \begin{pmatrix} 1 & 0 & 0\\ 0 & \cos(\phi) & -\sin(\phi)\\ 0 & \sin(\phi) & \cos(\phi) \end{pmatrix}$$
(13)

The global rotation from the inertial system  $S_I$  to the body frame system  $S_b$  can be expressed as the product of the above rotation matrices:

$$R_I^b = R_c^b(\phi) \times R_v^c(\theta) \times R_I^v(\psi) \tag{14}$$

When dealing with moving coordinate systems, the expressions for velocities and accelerations become more complex due to the need to express these quantities in inertial frames when they are to be used in equations of motion. Consider a point described by the vector P relative to a body-fixed frame. Assume that the origin of the body system is translated by a vector R from the origin of an inertial frame, and that the body frame rotates with angular velocity  $\omega$  relative to the inertial frame. The expression for the velocity of the point P in the inertial frame is given by:

$$\frac{dP}{dt}|_{i} = \frac{dR}{dt} + \frac{dP}{dt}|_{b} + \omega_{i}^{b} \times P$$
(15)

Then, we get the expression of the acceleration by differentiating the equation 15:

$$\frac{d^2P}{dt^2}|_i = \frac{d^2R}{dt^2} + \frac{d^2P}{dt^2}|_b + \dot{\omega}_i^b \times P + 2\omega_i^b \times \frac{dP}{dt}|_b + \omega_i^b \times (\omega_i^b \times P)$$
(16)

#### 3.2 Two-Track Model

For the analysis and the derivation of equation motion of vehicle dynamics of four wheels, we must set certain coordinate systems to be able to study all the vehicle movements, these coordinate systems are defined in Figure 3.



Figure 3. Reference Systems in Vehicle Dynamics.

To incorporate the effects of the individual tire forces, as well as suspension and a more accurate representation of the roll dynamics, a two-track model can be used, as in Figure 4. The suspension is modeled as a torsional spring and damper system acting around the roll axis, illustrated in Figure 5. For this paper, the pitch dynamics of the vehicle are ignored. The resulting model has 4 DOF, with translational motion along the x and y axis, as well as rotational motion about the x axis (roll  $\phi$ ) and the z axis ( yaw  $\psi$ ).

The equations of total forces on axis x, y and the total moment about z is derived as follows:

$$\begin{aligned} F_{xT} &= F_x^{rl} + F_x^{rr} + (F_x^{fl} + F_x^{fr})\cos\delta - (F_y^{fl} + F_y^{fr})\sin\delta \\ F_{yT} &= F_y^{rl} + F_y^{rr} + (F_y^{fl} + F_y^{fr})\cos\delta + (F_x^{fl} + F_x^{fr})\sin\delta \\ M_{zT} &= (F_y^{fl} + F_y^{fr})a\cos\delta - (F_y^{rl} + F_y^{rr})b + (F_x^{fl} + F_x^{fr})a\sin\delta + (F_x^{rr} + F_x^{fr}\cos\delta + F_y^{fl}\sin\delta - F_x^{rl} \\ &- F_x^{fl}\cos\delta - F_y^{fr}\sin\delta)l \end{aligned}$$
(17)

where:

 $\delta$ : is the road wheel steer angle.

a, b, c: are the geometric parameters of the vehicle shown in the Figure 4.

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Figure 4. Two-Track Model.

 $F_x^{ij}, F_y^{ij}$ : are the longitudinal and lateral force obtained of fit Magic Formula, and  $ij = \{fl, fr, rl, rr\}$ .

The Newton-Euler approach is used to solve equations for translational and rotational motion. This approach will be used for deriving a model based on the assumption of a roll axis coinciding with the vehicle x axis. This gives a model on ordinary differential equation (ODE) form that will be resolved in environment of programming MATLAB. Model validation is performed with CarSim vehicle simulation solved.



Figure 5. Vehicle Roll Model, suspension are modeled as a torsional spring and damper.

# 3.2.1 Translational Motion

The Newton-Euler equations of translation, relative to B frame are defined:

$$\int_{B} \overline{a}_{p} dm_{p} = m \overline{a}_{G} = \overline{F}_{ext}$$
(18)

To derivate the translation equations we assumed that non-inertial frame  $S_v$  is translating with velocity  $\overline{v}$  relative to the inertial frame  $S_I$ . Due to the center mass of the vehicle is attach with the origin of  $S_v$ , the equations are derived with the vehicle frame  $S_v$ .

The equations for translation motion can be obtained for formulation of Newton-Euler, and can be found combining

the equations 16, 17 and 18:

$$\dot{v_y} = \frac{F_{yT}}{m} - v_x \dot{\psi} + h[-\sin(\theta)\cos(\phi)\ddot{\psi} - \sin(\phi)\dot{\psi}^2 - 2\cos(\theta)\cos(\phi)\dot{\theta}\dot{\psi} + \sin(\theta)\sin(\phi)\dot{\phi}\dot{\psi} - \sin(\phi)\dot{\phi}^2 + \cos(\phi)\ddot{\phi}]$$
(19)

# 3.2.2 Rotational Motion

The Euler equations for angular motion are defined for external torques acting on the system and are given by the rate of change of angular momentum:

$$\tau = \frac{d(I^v \omega^s)}{dt} \tag{20}$$

where:

 $\tau$ : is the external torque applied to the system.

 $I^{v}$ : is the inertia tensor of the system relative to the frame in which the equations are derived.

 $\omega^s$ : is the spacial angular velocity of the system.

Then, the coordinate frame of the vehicle is rotating, so, it is not an inertial frame and the equation 20 must be modified as:

$$\tau = \frac{d(I^v \omega^s)}{dt}|_v + \omega_i^v \times I^v \omega^s \tag{21}$$

where  $\omega_i^v$  is the angular velocity of the vehicle coordinate system ( $S_{CoG}$ ) relative to the inertial system, the vehicle only experiment rotation about z axis, so:

$$\omega_i^v = \begin{pmatrix} 0\\ 0\\ \dot{\psi} \end{pmatrix} \tag{22}$$

We know there is a rotation about x to the body respect to the vehicle reference system  $(S_{CoG})$ , then the inertia tensor of body in the vehicle frame is given by:

$$I^{v} = R^{b}_{c}(\theta)R^{v}_{b}(\phi)I^{b}R^{vT}_{b}(\phi)R^{bT}_{c}(\theta)$$

$$\tag{23}$$

where  $I^b$  is the inertia matrix in the body frame, defined by:

$$I^{b} = \begin{pmatrix} I_{x} & 0 & 0\\ 0 & I_{y} & 0\\ 0 & 0 & I_{z} \end{pmatrix}$$
(24)

Then, we obtained:

$$I^{v} = \begin{pmatrix} I_{1} & I_{2} & I_{3} \\ I_{2} & I_{4} & I_{5} \\ I_{3} & I_{5} & I_{6} \end{pmatrix}$$
(25)

Where:

$$\begin{split} I_1 &= \sin(\theta)(I_y \sin(\theta) \sin^2(\phi) + I_z \sin(\theta) \cos^2(\phi)) + I_x \cos^2(\theta) \\ I_2 &= I_y \sin(\theta) \sin(\phi) \cos(\phi) - I_z \sin(\phi) \cos(\phi) \\ I_3 &= \sin(\theta)(I_y \cos(\theta) \sin^2(\phi) + I_z \cos(\theta) \cos^2(\phi)) - I_x \sin(\theta) \cos(\theta) \\ I_4 &= I_z \sin^2(\phi) + I_y \cos^2(\phi) \\ I_5 &= \sin(\phi) \cos(\phi) \cos(\theta)(I_y - I_z) \\ I_6 &= I_x \sin^2(\theta) + \cos^2(\theta)(I_y \sin^2(\phi) + I_z \cos^2(\phi)) \end{split}$$

The spacial angular velocity  $\omega_s$  is produced by the simultaneous rotation of the body frame about x respect to the vehicle frame, and the vehicle frame about z respect the inertial frame, and is given by:

$$\omega_s = \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} \tag{26}$$

The components of the torque about x and z are obtained for the free body diagram shown in the Figure 5, and are given by:

$$\tau_x = h(F_{yT}\cos(\phi)\cos(\theta) + mg\sin(\phi)) - C_{\phi}\phi - K_{\phi}\dot{\phi}$$
  

$$\tau_y = h(mg\sin(\theta)\cos(\phi) - F_{xT}\cos(\theta)\cos(\phi)) - C_{\theta}\theta - K_{\theta}\dot{\theta}$$
  

$$\tau_z = M_{zT} - h(F_{xT}\sin(\phi) - F_{yT}\sin(\theta)\cos(\phi))$$
(27)

Using the equation 21, 22, 25, 26 and 27, we can define the dynamic of rotation about x, y and z:

 $\ddot{\phi} = [h(F_{yT}\cos(\phi)\cos(\theta) + mg\sin(\phi)) - C_{\phi}\phi - K_{\phi}\dot{\phi} + \dot{\psi}(I_y - I_z)(\dot{\psi}\sin(\phi)\cos(\phi)\cos(\theta) + \dot{\phi}\sin(\theta)\sin(\phi)\cos(\phi)) + \dot{\psi}\dot{\phi}(I_y\cos^2(\phi) + I_z\sin^2(\phi))]/[I_x\cos^2(\theta) + I_y\sin^2(\theta)\sin^2(\phi) + I_z\sin^2(\theta)\cos^2(\phi)]$ 

$$\ddot{\theta} = [h(mg\sin(\theta)\cos(\phi) - F_{xT}\cos(\theta)\cos(\phi)) - C_{\theta}\theta - K_{\theta}\dot{\theta} + \dot{\psi}(\dot{\psi}\sin(\theta)\cos(\theta)(I_x - I_y + \cos^2(\phi)(I_y - I_z)) - \dot{\phi}\cos^2(\theta)I_x + \sin^2(\phi)\sin^2(\theta)I_y + \sin^2(\theta)\cos^2(\phi)I_z) - \dot{\theta}(\sin(\theta)\sin(\phi)\cos(\phi)(I_y - I_z))/[I_y\cos^2(\phi) + I_z\sin^2(\phi)]$$

$$\ddot{\psi} = [M_{zT} - h(F_{xT}\sin(\phi) + F_{yT}\sin(\theta)\cos(\phi))] / [I_x\sin^2(\theta) + \cos^2(\theta)(I_y\sin^2(\phi) + I_z\cos^2(\phi))]$$
(28)

#### 4. ESTIMATING PROCEDURE

The problem of estimating the dynamic parameters of the vehicle can be treated as an optimization problem of multiple inputs and outputs. In this article, the inputs are the angle of the steering wheel and the longitudinal velocity, and the outputs are state variables that describe the behavior of the vehicle. These state variables are the speed and lateral acceleration, the yaw rate, roll angle and body side slip angle. In dynamic systems, vehicular modes of excitation (state variables), depend on the type of input system, especially the angle of the steering wheel, brake pedal and accelerator pedal. In this work, the input type to the angle of the steering wheel are two, step and sinusoidal steering, its features are shown in table 2.

Table 2.	Types	of ma	neuvers	for the	inputs	of s	vstems.
	21						

Type of maneuver	Velocity $\left[\frac{km}{h}\right]$	Steering angle [deg]		
Step steer	75	40		
Sinusoidal steer	60	60 (max)		

The dynamic parameters of the vehicle estimated in this article are, sprung mass, the position and height of the center of gravity, moments of inertia, roll stiffness, roll damping, pitch stiffness and pitch damping. The multiple inputs and outputs problem of the estimation can be expressed as follows:

$$x = f(p, s, t) \tag{29}$$

where:

 $x = (a_y, \dot{\psi}, \phi, \beta, v_y)$ , state variables of the system.  $p = (a, h, m, I_x, I_y, I_z, C_{\phi}, K_{\phi}, C_{\theta}, K_{\theta})$ , inertia parameters of the vehicle. s: the input system, in this article only the steering angle. t: time.

The nominal values of the table 3 have been used to run the model of CarSim simulator and the state response values  $(a_y, \dot{\psi}, \phi, \beta, v_y)$  are used as real data. Unlike other vehicle parameter estimation methods, the evolutionary optimization procedure, Genetic Algorithm has been applied directly to the model of 6DOF and to the CarSim multi-degree of freedom vehicle model.

Vehicle Parameters	Nominal Values		
Vehicle model	D class Sedan		
Sprung mass	1370 kg		
Unsprung mass	80 kg (front and rear)		
Wheelbase	2.78 m		
Wheel track	1.55 $m$ (front and rear)		
CG location from front axle	1.11 m		
CG heigth from ground	0.54 m		
$I_{xx}$	$606.1 \ kg \times m^2$		
$I_{yy}$	4192 $kg \times m^2$		
Izz	4192 $kg \times m^2$		

Table 3. Nominal values for CarSim simulation.

## 4.1 Fitness function

The fitness function used for this estimation could be defined as follows:

$$Fitness function(p) = \sum_{i=1}^{n} [R_{Reference}(i) - R_{6DoF/CarSim Model}(i)]^2$$
(30)

where  $R_{Reference}(i)$  is the reference response of system (CarSim) at time *i*,  $R_{6DoF}(i)$  and  $R_{6DoF/CarSimModel}(i)$  are the response of the model of 6DoF (MATLAB) and the model CarSim multi-degree of freedom respectively. The main objective of the method of estimation is identify an approximate solution  $p_e$  to minimize the *Fitness function* expressed in the equation (30).

Genetic algorithm (GA) toolbox used for this optimization is available in MATLAB [4]. For each iteration of GA, the generated (aleatory) vehicle parameters are fed into the mathematical model of 6DOF and CarSim model, then, its response is used to calculate the *Fitness function*. The method continued till the *Fitness function* is minimized to small value or the lower bound defined for the user in the GA method. Figure 6 shows the evolution of the Fitness function against the number of iteration of the method, for this work note that the method converges in about 45 iterations.



Figure 6. Fitness function vs number of iterations.

The table 4 shows the parameters of GA method used in this work. The flux of data for system estimation using GA optimization method is shown in the Figure 7.

## 5. SIMULATION AND RESULTS

All programming was done in MATLAB environment. This program applies the method of genetic algorithms to estimate the parameters of the vehicle. Once these parameters are estimated, they are directly applied to the model of 6DOF and CarSim model and their responses are compared to the reference responses. To properly estimate the parameters, we use maneuvers in Table 2 to make an estimation in two steps and can prove the accurate of the estimated parameters. The results of estimation for step steering and sinusoidal steering for the 6DOF model and CarSim model are shown in table 5.

GA parameters	Values		
Size of Population	80		
Inital range	-40% to +40%		
Number of iteration (max)	50		
Scaling function	Rank		
Selection	Reminder		
Crossover	0.85		
Mutation	0.02		
Type crossover	Single point		
Fitness limit	0.01		

Table 4. Values of parameters of GA method.





Table 5. Results of estimation method.

		CarSim Model		6DOF Model		
Parameter	Nominal Values	Step Steer	Sinusoidal	Step Steer	Sinusoidal	
a[mm]	1110	1113.8	1112.37	988.5	998	
$m\left[kg ight]$	1370	1358	1368.31	1399.9	1506.24	
h[mm]	540	538	540.56	598.5	589.7	
$I_x [kg.m^2]$	606.1	598.8	599.76	584.6	647.32	
$I_y [kg.m^2]$	4192	4170.9	4155	4648.8	4553.6	
$I_z [kg.m^2]$	4192	4142.7	4150.9	3817.4	3862.84	
$C_{\phi} \left[ N.m.s/rad \right]$	*	*	*	$3.81 \times 10^4$	$3.25 \times 10^4$	
$K_{\phi}\left[N.m/rad ight]$	*	*	*	1000	1000	
$C_{\theta} [N.m.s/rad]$	*	*	*	$4 \times 10^4$	$3.7  imes 10^4$	
$K_{\theta} [N.m/rad]$	*	*	*	$9 \times 10^3$	$8.7 \times 10^3$	

To validate the estimated parameters in this work, shows the comparison of some system state variables such as lateral velocity  $v_y$ , yaw rate  $\dot{\psi}$  and roll angle  $\phi$ . In the Figure 8 for step and for sinusoidal the Figure 9.

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Figure 8. Comparing between real data, 6DOF estimation and CarSim model estimation for step steer.



Figure 9. Comparing between real data, 6DOF estimation and CarSim model estimation for sinusoidal steer.

## 6. CONCLUSION

A method to estimate the dynamic parameters of ground vehicles with 4 wheels is presented. The estimation is done in two steps, for the first step we use the step steering and for the second we use sinusoidal steer, in order to confirm the validity of the estimated parameters. The simulations showed small errors for the CarSim model and high errors using the 6DOF model respect to the nominal values of parameters. The estimation technique development, based on genetic algorithms, can be applied to parameter estimation of dynamic vehicle from actual data obtained by data acquisition systems mounted on the vehicle, accordingly the method, can be used for parameter estimation in any dynamics system. The difference between the variables of states of the mathematical model of 6DOF and the reference (CarSim) is because the first model is not able to cover all the actual dynamic behavior of the vehicle. This difference can be reduced by adding degrees of freedom to mathematical model.

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