ADAPTIVE DIFFERENTIAL EVOLUTION BASED ON THE CONCEPT OF POPULATION DIVERSITY APPLIED TO SIMULTANEOUS ESTIMATION OF RADIATION PHASE FUNCTION, ALBEDO AND OPTICAL THICKNESS

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Abstract. *Differential Evolution Algorithm (DE) has shown to be a powerful evolutionary algorithm for global optimization in a variety of real world problems. DE differs from other evolutionary algorithms in the mutation and recombination phases. Unlike some meta-heuristic techniques such as genetic algorithms and evolutionary strategies, where perturbation occurs in accordance with a random quantity, DE uses weighted differences between solution vectors to perturb the population. Although the efficiency of DE algorithm has been proven in the literature, studies indicate that the efficiency of the DE methods is sensitive to its control parameters (perturbation rate and crossover rate) and there is not any guarantee that premature convergence will be avoided. To overcome this problem, the present work proposes an Adaptive Differential Evolution as based on the concept of population diversity aiming at dynamically updating the control parameters. The methodology proposed is applied to the simultaneous estimation of the radiation phase function of anisotropic scattering, albedo and optical thickness in an inverse radiative transfer problem. The results show that the procedure represents a promising alternative for the type of problem presented above.*

Keywords*: Adaptive Differential Evolution, Population Diversity, Inverse Problems, Radiative Transfer.*

1. INTRODUCTION

The inverse analysis of radiative transfer in participating media has numerous practical applications, such as the one-dimensional plane-parallel (Silva Neto and Özişik, 1995; Alvarez Acevedo *et al.*, 2004, Lobato *et al.*, 2008, Lobato *et al.*, 2009) and two-dimensional media (Carita Montero *et al.,* 2001; Carita Montero *et al.,* 2004), and radiative transfer in composite layer media (Siegel and Spuckler, 1993; Wang *et al.*, 2002), which are devoted to applications in scientific and technological areas that are related to environmental sciences (Hanan, 2001), parameter estimation (Sousa et al., 2007), and tomography (Kim and Charette, 2007).

Traditionally, three research lines are proposed for the solution of parameter identification problems by using optimization techniques: the Deterministic, the Non-Deterministic and the Hybrid Approach (Wang *et al.*, 2001; Silva Neto and Soeiro, 2002; Silva Neto and Soeiro, 2003a, Silva Neto and Silva Neto, 2003b; Chalhoub et al., 2007).

The main difficulty found in the so-called non-deterministic approach is the high number of objective function evaluations needed to solve optimization problems. Besides, in spite of the performance and the number of applications encompassed when fixed parameters are used by the algorithm, there is no guarantee that premature convergence will be avoided (Coelho and Mariani, 2006). In addition, the DE algorithm is sensitive to control parameters (Storn *et al.*, 2005; Gämperle *et al.*, 2002) and it is highly problem dependent (Zaharie, 2002; Qin and Suganthan, 2005; Brest *et al.*, 2006), thus claiming for ad-hoc configurations. According to Nobakhti and Wang (2006), because of the special mutation mechanism used in DE, if for any reason (such as an incorrect choice of the perturbation rate - F) the DE population looses diversity, then the search will completely stop as mutation becomes zero.

To overcome this difficulty, many methodologies have been proposed. Zaharie (2003) proposes a feedback update rule for F that is designed to maintain the diversity of the population at a given level (and thus stop the search procedure prematurely). Recently, chaotic search models have been used for the adaptation of parameters in non-deterministic approach due to its ability in escaping premature convergence (Coelho and Mariani, 2006). In Tavazoei and Haeri (2007), a study about the performance of different chaotic search models when they are incorporated to classic optimization is addressed. In this context, Coelho and Mariani (2007) have used the Ant Colony algorithm with logistic maps in engineering problems.

Another research line to make the number of objective function evaluations to decrease is using special strategies to update the population size. In this context, Lobato and Steffen (2009) defined the convergence rate concept based on the homogeneity of the population and applied this concept to solve optimal control problems.

In the present contribution the Adaptive Differential Evolution algorithm (ADE) is used for the solution of the inverse radiative transfer problem related to the simultaneous estimation of the optical thickness, single scattering albedo, diffuse reflectivies and anisotropic scattering phase function of a one-dimensional homogeneous participating media. The results obtained with this methodology are compared with the standard Differential Evolution (DE) algorithm with fixed parameters. This work is organized as follows. The mathematical formulation of the direct and inverse problems is presented in Sections 2 and 3, respectively. A review of the Differential Evolution method and the strategy for the dynamic adapting of parameters is presented in Section 4. The results and discussion are described in Section 5. Finally, the conclusions and suggestions for future work conclude the paper.

2. MATHEMATICAL FORMULATION OF THE DIRECT PROBLEM

A plane-parallel, gray, anisotropically scattering slab of optical thickness τ_o, with diffusely reflecting boundaries is subjected to external isotropic irradiation at both boundaries, $\tau = 0$ and $\tau = \tau_o$ as shown in Figure 1.

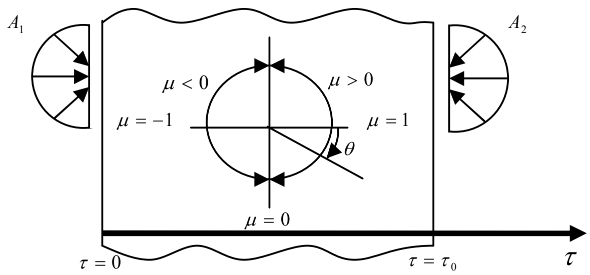


Figure 1: Scheme of the one-dimensional participating medium.

It is assumed that the emission of radiation by the medium due to its temperature is negligible in comparison to the intensity of the external incoming radiation. Also the effects of possible differences on the refractive indices of the participating medium and surrounding environment are not taken into account.

The mathematical formulation of the direct radiative transfer problem is given by (Özişik, 1973);

$$\mu \frac{\partial I(\tau,\mu)}{\partial \tau} + I(\tau,\mu) = \frac{\omega}{2}\int_{-1}^{1} p(\mu,\mu') I(\tau,\mu') d\mu', \quad 0 < \tau < \tau_o, \ -1 \le \mu \le 1 \tag{1}$$

$$I(0,\mu) = A_1 + 2\rho_1 \int_0^1 I(0,-\mu')\mu' d\mu', \quad \mu > 0 \tag{2}$$

$$I(\tau_o,\mu) = A_2 + 2\rho_2 \int_0^1 I(\tau_o,\mu')\mu' d\mu', \quad \mu < 0 \tag{3}$$

where $I(\tau, \mu)$ is the dimensionless radiation intensity, τ is the optical variable, μ is the direction cosine of the radiation beam with the positive τ axis, ω is the single scattering albedo, ρ_1 and ρ_2 are the diffuse reflectivities at boundaries $\tau = 0$ and $\tau = \tau_o$, respectively, A_1 and A_2 are the strength of the external irradiation at these boundaries, and $p(\mu,\mu')$ is the phase function of anisotropic scattering which is represented in terms of a series of Legendre polynomials as

$$p(\mu,\mu') = \sum_{m=0}^{M} (2m+1) f_m P_m(\mu) P_m(\mu') = \sum_{m=0}^{M} b_m P_m(\mu) P_m(\mu') \tag{4}$$

with $b_0 = 1$, where the coefficients b_m, $m = 1, 2, \ldots, M$ are tabulated by Chu et al. (1957).

In the direct problem defined by Eqs. (1) to (3) the radiative properties and boundary conditions are considered known, then the problem becomes the one of determining the radiation intensity $I(\tau,\mu)$. For that purpose a Collocation Method (Villadsen and Michelsen, 1978; Wylie and Barrett, 1985) was used together with a Gauss-Legendre quadrature for the terms given on the right hand sides of Eqs. (1) to (3).

3. MATHEMATICAL FORMULATION OF THE INVERSE PROBLEM

In the inverse problem considered here the optical thickness τ_o, the single scattering albedo ω, the diffuse reflectivities ρ_1 and ρ_2, and the coefficients of phase function of anisotropic scattering, b_m, $m = 1, 2, \ldots, M$ are considered unknown. Note that M is also unknown and from now on it will be referred to as M^*. Measured exit intensities at both surfaces of the plate $\{Y_i\}$, at different polar angles corresponding to $i = 1, 2, \ldots, K$, are considered available, where K is the total number of measured data. Therefore, the inverse problem can be stated as: utilizing the measured data $\{Y_i\}$, i=1, 2, ..., K, determine the M*+ 4 elements of the vector of unknowns Z defined as:

$$\vec{Z} = \{\tau_o,\ \omega, \rho_1, \rho_2,\ b_1,\ b_2,\ ...,\ b_{M*}\}^T \tag{5}$$

Considering that the number of measured data, K, is larger than the number of parameters to be estimated, M*+ 4, an implicit formulation based on an optimization problem is used for the inverse radiation problem at hand, in which it is required the minimization of the least square norm as given below:

$$Q(\vec{Z}) = \sum_{i=1}^{K}\left[I_i(\tau_o,\ \omega, \rho_1, \rho_2,\ b_1,\ b_2,\ ...,\ b_{M*}) - Y_i\right]^2 = \vec{G}^T\vec{G} \tag{6}$$

where I_i and Y_i are computed and measured exit intensities, respectively, and the elements of the vector of residues are

$$G_i = I_i(\tau_o,\ \omega, \rho_1, \rho_2,\ b_1,\ b_2,\ ...,\ b_{M*}) - Y_i, \quad i = 1,\ 2,\ ...,\ K \tag{7}$$

As real experimental data are not available, the measured exit intensities, $Y_{i,}$ were obtained from simulation. For this aim, random error E (with normal distribution and standard deviation σ) was added to the exact intensities, I_{exact}, obtained from the solution of the direct problem.

$$Y_i = I_{exact_i} + \sigma E_i, \quad i = 1,\ 2,\ ...,\ K \tag{8}$$

4. SOLUTION OF THE INVERSE PROBLEM

4.1. Differential Evolution Algorithm

Differential Evolution (DE) is a recent optimization technique in the family of evolutionary computation proposed by Storn and Price (1995), which differs from other evolutionary algorithms in the mutation and recombination phases. According to several authors, DE has as main advantages the conceptual simplicity and faster convergence. However, the main difficulty with the technique appears to be in the slowing down of convergence as the region of global minimum is approached and stagnation of the population (Lampinen and Zelinka, 2000).

This methodology consists in generating trial parameter vectors by adding the weighted difference between two population vectors to a third vector. The control parameters in DE are: NP, the population size, CR, the crossover rate, and, F, the weight applied to random differential (perturbation rate). According to Storn *et al.* (2005), NP should be about 5 to 10 times the problem dimension (number of parameters in a vector), F should be in the range 0.1 to 2.0 and CR in the range 0.01 to 1.0.

The procedure of standard DE is shown in Figure 2 and summarized as follows (Storn *et al.*, 2005):

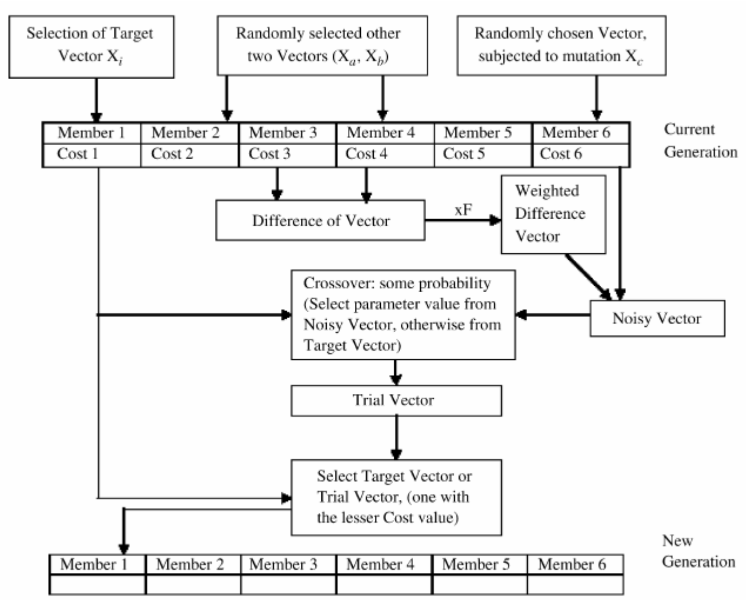


Figure 2. Differential Evolution Structure.

- Step 1: Randomly initialize the population of individuals for DE, where each individual contains n variables;
- Step 2: Evaluate the objective values of all individuals, and determine the individual that has the best objective value;
- Step 3: Perform mutation operation for each individual to obtain each individual's mutant counterpart;
- Step 4: Perform crossover operation between each individual and its corresponding mutant counterpart to obtain each individual's trial individual;
- Step 5: Evaluate the objective values of the trial individuals;
- Step 6: Perform selection operation between each individual and its corresponding trial counterpart to generate the new individual for the next generation;
- Step 7: Determine the best individual of the current new population with the best objective value. If the objective value is better than the objective value of X_{best}, then update X_{best} and its objective value with the value of the current best individual;
- Step 8: If a stopping criterion is met, then output X_{best} and its objective value; otherwise go back to Step 3.

Storn and Price (1995) proposed various mutation schemes for the generation of new vectors (candidate solutions) by the combining the vectors that are randomly chosen from the current population as shown in Table 1.

Table 1. Mutation Schemes used in the Differential Evolution Algorithm (r_1, ..., r_5 are random integer indexes and mutually different, x_0 is the trial vector (individual), x_{best} is the best individual of the current population and x is the current individual).

Strategy	Updating Equation
rand/1	$x' = x_{r1} + F\left(x_{r2} - x_{r3}\right)$
rand/2	$x' = x_{r1} + F\left(x_{r2} - x_{r3} + x_{r4} - x_{r5}\right)$
best/1	$x' = x_{best} + F\left(x_{r2} - x_{r3}\right)$
best/2	$x' = x_{best} + F\left(x_{r2} - x_{r3} + x_{r4} - x_{r5}\right)$
rand/best/1	$x' = x_{r1} + F\left(x_{best} - x_{r1} + x_{r1} - x_{r2}\right)$
rand/best/2	$x' = x_{r1} + F\left(x_{best} - x_{r1}\right) + F\left(x_{r1} - x_{r2} + x_{r3} - x_{r4}\right)$

Applications of the above technique are found in various fields of science and engineering, such as: digital filter design (Storn, 1999), batch fermentation process (Chiou and Wang, 2001), parameter estimation in fed-batch fermentation process (Wang et al., 2001), parameter estimation in biofilter modeling (Bhat *et al.*, 2006), economic load dispatch problem (Coelho and Mariani, 2007), engineering system design applied to a multi-objective context (Lobato and Steffen, 2007), apparent thermal diffusivity estimation of fruits drying (Mariani *et al.*, 2008), solution of inverse radiative transfer problems in two-layer participating media (Lobato *et al.*, 2008), optimal control problems (Lobato and Steffen, 2009), multi-objective optimization (Lobato *et al.*, 2009), and other applications (Storn *et al.*, 2005).

4.2. Self-Adaptive Differential Evolution Algorithm

In this work, parameter updating is performed according to the previous work from (Zaharie, 2003). This methodology is based on the evolution of the population variance (viewed as a measure of the diversity population) given by:

$$Var\left(x\right) = x^2 - \overline{x}^2 \tag{9}$$

where $\overline{x} = \sum_{i=1}^{NP} x_i \big/ NP$.

According to Zaharie (2002; 2003) the expected value of the variance of population obtained after recombination if the best element of the population is not taken into consideration is:

$$E\left(Var\left(x\right)\right) = \left(2F^2CR + 1 - \frac{2CR}{NP} + \frac{CR^2}{NP}\right)Var\left(x\right) \tag{10}$$

Consider that $x(g)$ is the population obtained at generation g-1 (initial population). During the g-th generation the vector x is transformed into x' (recombination); then in x'' (selection). x'' will represent the starting population for the next generation, $x(g+1)$. Defining γ as

$$\gamma \equiv \frac{Var\left(x\left(g+1\right)\right)}{Var\left(x\left(g\right)\right)} \tag{11}$$

information about the variance tendency is provided: if $\gamma<1$ we can compensate an increase of the variance, thus we could accelerate the convergence but with the risk of inducing premature convergence and if $\gamma>1$ we can compensate a high decrease of the variance, thus we can avoid premature convergence situations. The controlling idea is to choose the parameter F such that the recombination applied in generation g compensates the effect of the previous application of recombination and selection.

The idea of the parameter adaptation is to solve, with respect to F:

$$1+2F^2 CR - \frac{2CR}{NP} + \frac{CR^2}{NP} = c \tag{12}$$

where $c \equiv \gamma \dfrac{Var\left(x\left(g+1\right)\right)}{Var\left(x\left(g\right)\right)}$.

Equation (12) can be solved with respect to F:

$$F = \begin{cases} \sqrt{\dfrac{1}{NP}}\sqrt{\dfrac{\eta}{2CR}} & if \quad \eta \geq 0 \\ F_{min} & \text{otherwise} \end{cases} \tag{13}$$

where $\eta \equiv NP\left(c-1\right)+CR\left(2-CR\right)$ and F_{min} is the minimal value for F. A sufficient condition for increasing the population variance by recombination is that $F > \sqrt{1/NP}$, thus $F_{min} = \sqrt{1/NP}$ should be used. An upper bound for F can also be imposed as suggested by Storn *et al.* (2005) ($F_{max} = 2$). By solving Eq. (12) with respect to CR one obtains the following adaptation rule for CR:

$$CR = \begin{cases} -\left(NPF^2-1\right)+\sqrt{\left(NPF^2-1\right)^2 - NP\left(1-c\right)} & if \quad c \geq 1 \\ CR_{min} & \text{otherwise.} \end{cases} \tag{14}$$

with $CR_{min}=0.01 \leq CR \leq 1$.

5. RESULTS AND DISCUSSION

In the inverse radiative transfer problem described before, the goal is the simultaneous estimation of the optical thickness, τ_o, the single scattering albedo ω, the diffuse reflectives ρ_1 and ρ_2, and the coefficients of the anisotropic scattering phase function b_m, m = 1, 2, ..., M*. Due to space limitation, the present contribution will focus on the estimation of the phase functions represented by PF-1 and PF-2, whose coefficients are listed in Table 2. Therefore, even though the other four parameters are also estimated, only test cases with one set of exact values $\tau_o = 1.0$, $\omega = 0.5$ and $\rho_1 = \rho_2 = 0.2$ were considered.

Table 2. Phase function expansion coefficients (Chu *et al.*, 1957).

Coefficient	Forward Scattering PF-1 (M=4) m=1.4, α=1	Backward Scattering PF-2 (M=5) m=∞, α=1
b_1	0.57024	-0.56524
b_2	0.56134	0.29783
b_3	0.11297	0.08571
b_4	0.01002	0.01003
b_5	0.00000	0.00063
b_6	-	0.00000

m is the index of refraction of the particle relative to the surrounding media and α is $\pi D/\lambda$, D is the particle diameter and λ is the wavelength of incident radiation.

For evaluating the methodology proposed in this work, some practical points should be emphasized:

- To compare the results obtained by the proposed methodology the standard DE algorithm is used with the following parameters: DE-1 {F=0.5 and CR=0.5}, DE-2 {F=0.5 and CR=0.8} and DE-3 {F=0.8 and CR=0.5}. Parameters used in all algorithms tested: 10 individuals, 1000 generations and DE/rand/1/bin strategy for the generation of potential candidates;

- To perform the ADE algorithm γ equal to 1 was taken;

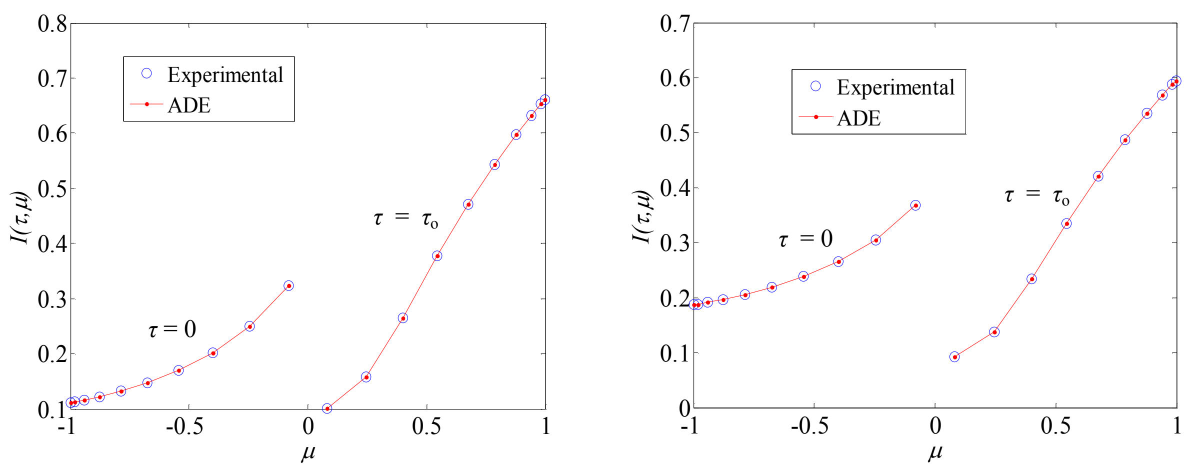
- In all test cases it is also considered the following external illumination: A_1=1 and A_2=0;

- Finally, the stopping criterion for all the algorithms is associated to the difference between the best and the worst values of the objective function; this difference should be smaller than 10^{-9}. All the algorithms were executed 10 times to obtain the average values presented.

In order to examine the accuracy of the inverse methodology of analysis considered, test cases with noise (σ =0.02, i. e., corresponding to 5% error) or without noise (σ =0) have been studied.

Table 3 presents the results obtained by DE and ADE algorithms considering the Phase Function PF-1 and Phase Function PF-2. In this table it is important to observe that, considering noiseless data, both DE and ADE were able to estimate the parameters satisfactorily as shown by the values obtained for the objective function. However, the ADE algorithm leads to a smaller number of objective function evaluations as compared with the original DE algorithm (a reduction of 32%, 34% and 22% in the number of objective function evaluations, respectively). Good estimates are obtained when noise is taken into account.

Figure 3 presents the radiation intensities profiles at boundaries τ=0 and τ=τ_o by using the ADE algorithm without noise (σ =0) and with exact parameters showed in Table 3.



(a) Phase Function 1. (b) Phase Function 2.

Figure 3. Radiation intensities profiles at boundaries τ=0 and τ=τ_o.

In theses figures it is possible to observe that the ADE algorithm is able to perform satisfactorily at the boundaries τ=0 and τ=τ_o for all cases considered without noise. It is important to emphasize that similar behavior can also be observed when noise is considered, however larger deviations with respect to the profiles obtained by using the exact values of the parameters appear.

Table 3. Results obtained using the DE and ADE methods, where OF is the objective function, Eq. (6), and NEVAL is the number of function evaluations.

			M=4 (n=1.4, α=1)					**M=5 (n=∞, α=1)**			
	Error (%)	*Exact*	DE-1	DE-2	DE-3	ADE	*Exact*	DE-1	DE-2	DE-3	ADE
τ_o	0	1.0	0.99996	0.99837	0.99999	1.00000	1.0	0.99920	0.99910	0.99901	0.99977
	5		0.97864	1.01950	0.98790	0.98677		1.00042	0.99444	0.99453	0.99271
ω	0	0.5	0.49998	0.49950	0.49999	0.50000	0.5	0.49964	0.49914	0.49974	0.49994
	5		0.49143	0.51085	0.49524	0.49558		0.49120	0.48871	0.48862	0.48368
ρ_1	0	0.2	0.19999	0.20001	0.20000	0.199999	0.2	0.20000	0.20000	0.19999	0.20000
	5		0.19983	0.19949	0.20035	0.20014		0.20467	0.20462	0.20467	0.20540
ρ_2	0	0.2	0.19503	0.19964	0.21000	0.199877	0.2	0.19806	0.19867	0.19977	0.19977
	5		0.22444	0.23073	0.18235	0.19885		0.220000	0.21404	0.20128	0.19487
b_1	0	0.57024	0.57011	0.56076	0.57020	0.57030	-0.56524	-0.57089	-0.57007	-0.57049	-0.57049
	5		0.49617	0.49240	0.48401	0.49325		-0.62175	-0.66698	-0.66698	-0.66698
b_2	0	0.56134	0.56114	0.55878	0.56107	0.56123	0.29783	0.29485	0.29589	0.29499	0.29688
	5		0.51042	0.53294	0.51173	0.53698		0.26804	0.26506	0.26506	0.26506
b_3	0	0.11297	0.11176	0.11019	0.11162	0.11320	0.08571	0.08485	0.08457	0.08455	0.08465
	5		0.09315	0.09907	0.10214	0.10414		0.07714	0.08826	0.09331	0.07945
b_4	0	0.01002	0.00991	0.01052	0.01052	0.00994	0.01003	0.00996	0.00996	0.00997	0.00999
	5		0.00820	0.01254	0.01071	0.00945		0.01103	0.01175	0.01183	0.01152
b_5	0	0.00000	0.00000	0.00001	0.00000	0.00000	0.00063	0.00063	0.00063	0.00062	0.00062
	5		0.00133	0.00122	0.00144	0.00101		0.00069	0.00064	0.00069	0.00061
b_6	0	-	-	-	-	-	0.00000	0.00001	0.00001	0.00000	0.00000
	5		-	-	-	-		0.00090	0.00071	0.00012	0.00015
OF	0	-	6.52E-10	2.51E-10	8.12E-10	4.30E-10	-	4.78E-10	5.71E-10	4.44E-10	3.77E-10
	5		1.29E-02	1.34E-02	1.33E-02	1.25E-02		1.24E-02	1.22E-02	1.24E-02	1.28E-02
NEVAL	0	-	2960	3050	2580	2010	-	2420	2880	2380	1920
	5		1220	1660	1480	1060		1200	1680	1540	1100

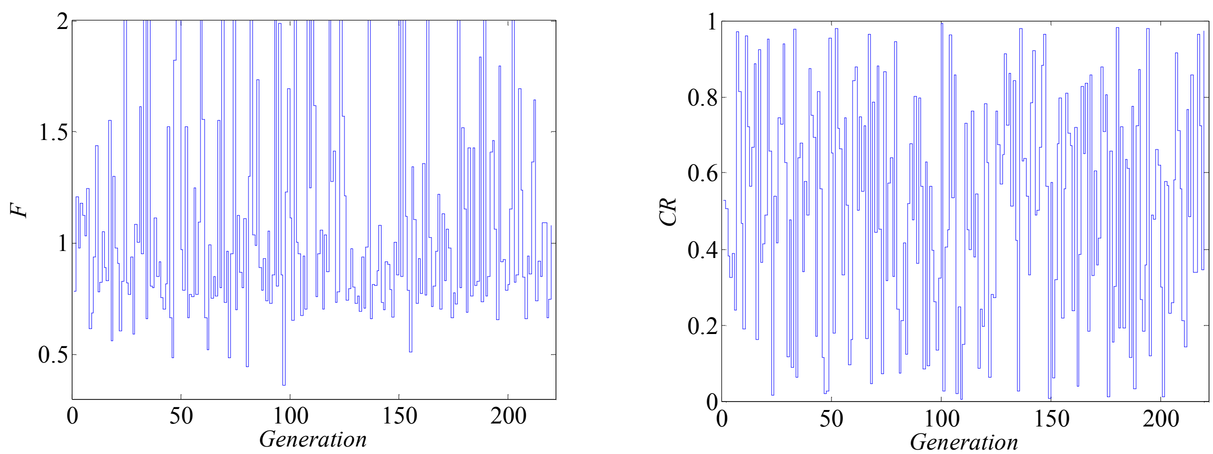
Table 4 presents the average errors obtained by DE and ADE algorithms considering the Phase Function PF-1 and Phase Function PF-2.

Table 4. Average errors obtained using the DE and ADE methods.

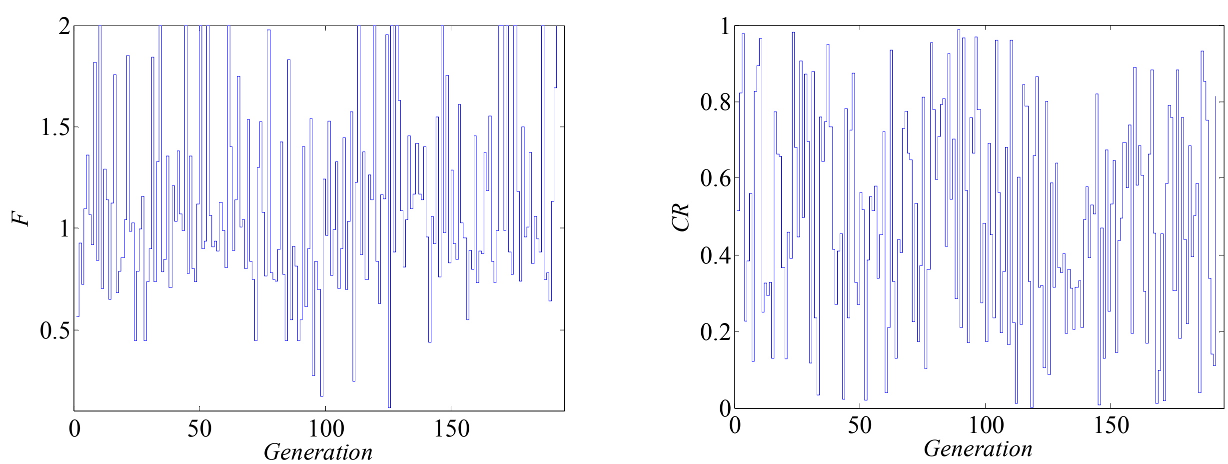
	M=4 (n=1.4, α=1)				**M=5 (n=∞, α=1)**			
Error (%)	DE-1 (%)	DE-2 (%)	DE-3 (%)	ADE (%)	DE-1 (%)	DE-2 (%)	DE-3 (%)	ADE (%)
0	0.0743	0.1981	0.1353	0.0067	0.1267	0.1108	0.1081	0.0783
5	2.2508	2.0608	2.0407	1.4522	1.7072	1.7501	1.7657	1.6689

In this table is possible to observe that both the algorithms are able to estimate satisfactorily the radiation phase function, albedo and optical thickness. However, it is important to emphasize that the ADE algorithm leads to a smaller error in all cases studied as compared with the standard DE algorithm (DE-1, DE-2 and DE-3).

Figure 4 shows the perturbation rate and crossover rate profiles obtained by using the ADE algorithm for noiseless data.



Best result obtained for Phase Function 1.



Best result obtained for Phase Function 2.

Figure 4. Perturbation rate (F) and crossover rate (CR) for the case without noise using ADE algorithm.

6. CONCLUSIONS

In this work, the Adaptive Differential Evolution (ADE) algorithm, which is based on the concept of population diversity to dynamically updating the control parameters, was applied to the simultaneous estimation of the radiation phase function, albedo and optical thickness of an inverse radiative transfer problem. The ADE has been found to be beneficial for adjusting control parameters during the evolutionary process, especially when compared with the standard Differential Evolution algorithm with fixed parameters.

The main characteristics of the proposed methodology are: dynamic updating of control parameters based on the diversity of the population and the easiness of incorporating this strategy to other evolutionary strategies. This first characteristic avoided the necessity of choosing these parameters and, as a consequence, the premature convergence of the evolutionary process was avoided. Finally, the results showed that the methodology conveyed represents a promising alternative for dealing with optimization problems.

Further research work will be focused on the influence of the parameter values required by ADE on the solution of others case studies.

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