ANALYSIS AND OPTIMIZATION OF A VIBROACOUSTIC SYSTEM

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Abstract. The study of coupled problems considering the existence of some fluid-structure interaction, as the vibroacoustic problem, involves many open questions. The analysis and numerical optimization in vibroacoustic models, subject of this work, under defined boundary conditions, is important not only for understanding the physical phenomenon, but also to acquire sensitivity relative to the factors that influence the vibroacoustic response. The objectives of this work are: the exposition of a simple formulation for the modal analysis of vibroacoustic systems, the computational implementation of this formulation for comparison with results obtained from a commercial program, and to apply optimization. This work uses a system discretization with finite elements, by means of a non-symmetric matrix formulation u-p, in displacement u of the structure and pressure p of the fluid. After the modal analysis is realized, it is evaluated the numerical optimization of the structural mass. Some results are concerned with the pressure fringes in the fluid that match the structural deformation shape, which serve to verify if the structure or the fluid is predominant in the coupled mode. These results help to control the modal behaviour of a vibroacoustic system.

Keywords: Vibroacoustic, Coupled System, Fluid-Structure Interaction, Nonlinear optimization.

1. INTRODUCTION

In a fluid-structure coupled system under dynamic excitation, the acoustic medium influences the behavior of the structure and vice versa. The vibration of the structure is influenced by the variation of the fluid pressure and the acoustic waves are sensible to the variation of the structural displacement. In the context of the free vibration, the natural frequencies and modes of the coupled system are different with reference to the uncoupled systems.

The energy in a coupled mode is divided between the structure and the fluid. Usually the largest amount of energy stays in the structure or in the fluid, from which the coupled system is classified as dominated by the structure or by the fluid (De Mello, 2003).

Usually, a mode dominated by the structure is originated by a structural uncoupled mode that induces an acoustic mode in the fluid. Equally, a mode dominated by the fluid is an acoustic mode that induces a mode in the structure. The fluid influences the movement of the structure through the pressure in the interface surface, as well as the movement of the interface surface modifies the acoustic domain.

The effect of the fluid pressure in the surface interface can be approximate through the term $f_{s\Gamma_l}$, that is part of the excitement in the structural dynamic equation, constituted by the vectors of surface force and structural volume $(f_{s\Gamma_l} + f_{sB})$, according to Eqs. (1) and (2):

$$\boldsymbol{K}_{ss}\boldsymbol{u} + \boldsymbol{M}_{ss}\boldsymbol{\ddot{u}} = \boldsymbol{f}_{s\boldsymbol{\Gamma}_{I}} + \boldsymbol{f}_{sB} \tag{1}$$

$$f_{s\Gamma_{I}} = \int_{0}^{L} n_{s} q \, dx \tag{2}$$

where n_s is the vector of structural shape functions. The forces in the interface surface of the structure are originated by the fluid action, being associated the normal component of the surface force q to the pressure distribution on the interface.

Considering the finite element method, it is possible to replace q by the expression of nodal polynomial approach for each finite element $\tilde{p} = \mathbf{n}_f^T \mathbf{p}$, where \tilde{p} is the approach for the scalar field of local pressures, \mathbf{n}_f is the vector of fluid shape functions, and \mathbf{p} is the vector of nodal pressures of the element, and resulting the Eq. (3):

$$\boldsymbol{f}_{s\boldsymbol{\Gamma}_{I}} = \int_{0}^{L} \boldsymbol{n}_{s} \boldsymbol{n}_{f}^{T} \, dx \, \boldsymbol{p} \tag{3}$$

that represents the equilibrium condition in the interface. Substituting Eq. (3) into Eq. (1) we obtain Eqs. (4) and (5):

$$\boldsymbol{K}_{ss}\boldsymbol{u} + \boldsymbol{M}_{ss}\boldsymbol{\ddot{u}} + \boldsymbol{K}_{sf}\boldsymbol{p} = \boldsymbol{f}_{s} \tag{4}$$

$$\boldsymbol{K}_{sf} = -\int_{0}^{L} \boldsymbol{n}_{s} \boldsymbol{n}_{f}^{T} \, d\boldsymbol{x}$$
⁽⁵⁾

The coupling of the structural domain with the fluid domain is imposed in the normal direction \hat{n} of the interface surface, through an identity that guarantees the cinematic compatibility:

$$\dot{\bar{\boldsymbol{v}}}_{\hat{\boldsymbol{n}}} = \ddot{\boldsymbol{u}}_{\hat{\boldsymbol{n}}} \tag{6}$$

that represents an slipping condition in the tangential direction to the interface.

Relative to the fluid domain, the fluid-structure coupling is described in terms of pressure variations in the neighborhood of the structural domain according to the interface boundary condition of Eq. (7), after using Eq. (6):

$$\frac{\partial p}{\partial \hat{n}} = -\rho_f \ddot{u}_{\hat{n}}, \quad \text{em}\,\Gamma_I \tag{7}$$

where ρ_f is the fluid density. Having substituted the component in the normal direction $\overline{v}_{\hat{n}}$ by $\ddot{u}_{\hat{n}}$, and considering the expression $\ddot{\vec{u}} = \mathbf{n}_s^T \ddot{\vec{u}}$ to approximate the value of $\ddot{u}_{\hat{n}}$ by $\ddot{\vec{u}}_{\hat{n}}$, or in discretized way by $\mathbf{n}_s^T \ddot{\vec{u}}$, it results the Eq. (8):

$$\boldsymbol{M}_{ff} \ddot{\boldsymbol{p}} + \boldsymbol{K}_{ff} \boldsymbol{p} + \boldsymbol{M}_{fs} \ddot{\boldsymbol{u}} = \boldsymbol{f}_f \tag{8}$$

being the matrix with the interface terms expressed by Eq. (9):

$$\boldsymbol{M}_{fs} = \int_{\boldsymbol{\Gamma}_I} \boldsymbol{n}_f \boldsymbol{n}_s^T d\boldsymbol{\Gamma}_I \tag{9}$$

that allows to write the coupled system in its semi-discretized way. Rewriting Eqs. (4) and (8) together:

$$\boldsymbol{K}_{ss}\boldsymbol{\mu} + \boldsymbol{M}_{ss}\ddot{\boldsymbol{\mu}} + \boldsymbol{K}_{sf}\boldsymbol{p} = \boldsymbol{f}_{s} \tag{10}$$

$$\boldsymbol{M}_{ff} \ddot{\boldsymbol{p}} + \boldsymbol{K}_{ff} \boldsymbol{p} + \boldsymbol{M}_{fs} \ddot{\boldsymbol{u}} = \boldsymbol{f}_f \tag{11}$$

that allocated in a matrix, it generates the coupled formulation *u-p* in structural displacement and fluid pressure:

$$\begin{bmatrix} \boldsymbol{M}_{ss} & \boldsymbol{0} \\ \boldsymbol{M}_{fs} & \boldsymbol{M}_{ff} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{u}} \\ \ddot{\boldsymbol{p}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_{ss} & \boldsymbol{K}_{sf} \\ \boldsymbol{0} & \boldsymbol{K}_{ff} \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{p} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_s \\ \boldsymbol{f}_f \end{bmatrix}$$
(12)

For free vibrations, the second term of Eq. (12) is zero. The nonsymmetrical aspect of this formulation is its principal disadvantage, because it is not possible to use several efficient algorithms developed for symmetrical cases. The main advantage of this formulation is its reduced number of degrees of freedom to model the fluid domain, especially when compared with models based with more variables for the fluid domain.

2. STRUCTURAL OPTIMIZATION IN VIBROACOUSTIC SYSTEMS

This work looks for the dimensional optimization in vibroacoustic coupled systems, with the purpose of reducing the structural mass. The system is discretized with the finite element method, and the sequential quadratic programming SQP algorithm available in the software MSC.Nastran[©] is used for the optimization.

The structural domain is formed by plates, where the thickness is smaller with reference to the other dimensions. It is studied the optimization of the structural mass, and the thickness is chosen as the variable for the non linear optimization with constraints, although other choices are also possible, for example some properties of the fluid domain and characteristics associated to the connectivity.

Being N the order of certain frequency, the mass optimization is written so that two constraints are satisfied, the first is concerned with limits of the frequency N in Hz, and the second is concerned with minimum and maximum values of

the plates thickness, so that the solution must be technological and mathematically possible, because the thickness must have physical meaning.

The description of the optimization problem is defined by,

Optimize $f = \text{Weight}(\mathbf{x})$ $\mathbf{x} \in \mathfrak{R}^r$ subject to: $\omega_N^L \le \omega_N(\mathbf{x}) \le \omega_N^U$ $\mathbf{x}^{\min} \le x \le \mathbf{x}^{\max}$

where *r* is the number of structural elements, $\omega_N(\mathbf{x})$ is the N^{th} natural frequency exposed as function of the thicknesses vector \mathbf{x} with lower ω_N^L and upper ω_N^U bounds respectively, \mathbf{x}^{\min} is the vector of minimum thicknesses and \mathbf{x}^{\max} is the vector of maximum thicknesses.

An optimization without constraints of the thickness can lead to singular points of value zero. Also, the constraint $\mathbf{x} \le \mathbf{x}^{m \acute{a} x}$ can be ignored after choosing a sufficiently large value of $\mathbf{x}^{m \acute{a} x}$, so that the inequality is always verified. During the optimization process, there is no explicit control over the modes crossing.

3. VIBROACOUSTIC SYSTEM: HEXAHEDRICAL CAVITY OVER A PLATE

Considering the model described in Msc.Software (1996), this system is studied for showing the coupling of modes when the frequencies of the decoupled structural and acoustic systems are closed for the interval of analyzed frequencies, as well as for optimizing the structural mass. The rectangular cavity has a height five times larger than the side of the base.

The software MSC.Nastran[©] is used for simulation and optimization of the coupled system, while the software MSC.Patran[©] is used for pre/post-processing the obtained results.

3.1. Square plate

The structure is a square plate for bending of side 20 in (0.508 m) and thickness 0.2 in (5,08 mm) supported in the four vertexes. The structural material is aluminum with the following properties: module of elasticity $E=1.0 \times 10^7$ psi (68.948×10⁹ N/m²), Poisson's ratio $\nu=0.3$ and density $\rho=2.54 \times 10^{-4}$ lb-s²/in⁴ (2700 kg/m³). Experimental and numerical studies with similar properties were conducted by Petyt and Mirza (1972) and Reid (1965).

The plate is discretized with a mesh (8x8), counting 64 two-dimensional elements *CQUAD4* of four nodes each and totaling 81 nodes. The boundary conditions are $u=v=r_z=0$ entirely in the plate. Additionally, it is considered the displacement of the vertexes equal to zero, z=0, as shown in Fig. 1.



Figure 1. Mesh of the square shaped plate with indication of the boundary conditions

The natural frequencies are compared, in Tab. 1, with analytical solutions. The average variation of the results of MSC.Nastran \odot in relation to the analytical ones for the first two frequencies is 0.69%, value considered appropriate if observed that the maximum variation is only 0.39 Hz.

Mode	Analytical	MEF	Variation (%)	Variation Hz
	(Blevins, 1995)	MSC.Nastran©	MSC.Nastran [©] / analytical	MSC.Nastran© / analytical
1	34.02	34.41	1.15	0.39
2	75.50	75.67	0.23	0.17
3		75.67		
4		94.78		
5		191.56		
6		214.55		
7		252.84		
8		252.84		
9		343.86		
10		404.14		

Table 1.	Predicted	frequencies	in Hz of a	square plate	supported i	n the vertexes
		1		1 1	11	

It is observed in Fig. 2 two modes, the second and the third one, with different modal patterns but with identical frequencies because of the symmetry.



Figure 2. Modal patterns of the square plate supported in the vertexes

3.2. Hexahedrical acoustic cavity

The dimensions of the system are: sides of base equal to 20 in (0.508 m) and height equal to 100 in (2.54 m). The internal fluid is air with density $\rho_{f}=1.21 \times 10^{-7}$ lb-s²/in⁴ (1.29 kg/m³) and sound velocity $c_{s}=13000$ in/s (330.2 m/s).

The cavity is discretized through a mesh (8x8x8) of hexahedrical solid elements CHEXA of 8 nodes each one, totaling 512 elements and 729 nodes. The boundary conditions correspond to rigid walls, Fig. 3.



Figure 3. Mesh of the rectangular cavity with boundary conditions of rigid wall on faces

The obtained natural frequencies are compared, in Tab. 2, with analytical solutions according to Eq. (13), (Blevins, 1995); observing that the walls of the cavity are considered as rigid with infinite impedance.

$$f = \frac{c_s}{2} \sqrt{\left(\frac{i}{L_x}\right)^2 + \left(\frac{j}{L_y}\right)^2 + \left(\frac{k}{L_z}\right)^2} \qquad i = 0, 1, 2, \dots; j = 0, 1, 2, \dots; k = 0, 1, 2, \dots$$
(13)

where,

f : natural frequency in Hz; c_s : sound velocity at medium, in this case air, equal to 343.3 m/s; L_x, L_y, L_z : length, width and height of the cavity.

The variation of the MSC.Nastran results relative to the analytical ones is 3.92% for the first twenty frequencies. The maximum difference is 38.2 Hz (11.52%) for the frequency of order 15. These set of results supplies sufficient conformity, especially for the first frequencies.

Table 2. Predicted frequencies of the hexahedrical acoustic cavity in Hz

Mode	Notation	Analytical	MEF	Variation (%)	Variation Hz
	(i,j,k)	(Blevins, 1995)	MSC.Nastran	MSC.Nastran / analytical	MSC.Nastran / analytical
1	(0,0,0)	0.00	0	0	0
2	(0,0,1)	65.00	65.41	0.63	0.26
3	(0,0,2)	130.00	133.36	2.58	2.14
4	(0,0,3)	195.00	206.37	5.83	7.27
5	(0,0,4)	260.00	286.69	10.27	17.24
6	(0,0,5)	325.00	327.09	0.64	1.33
7	(0,1,0)	325.00	327.09	0.64	1.33
8	(1,0,0)	325.00	333.56	2.63	7.80
9	(0,1,1)	331.44	333.56	0.64	1.36
10	(1,0,1)	331.44	353.23	6.58	20.63
11	(0,1,2)	350.04	353.23	0.91	2.03
12	(1,0,2)	350.04	374.88	7.10	8.32
13	(0,1,3)	379.01	386.75	2.04	4.93
14	(1,0,3)	379.01	386.75	2.04	4.93
15	(0,0,6)	390.00	434.94	11.52	38.20
16	(0,1,4)	416.20	434.94	4.50	12.00
17	(1,0,4)	416.20	462.57	11.14	29.70
18	(0,0,7)	455.00	465.88	2.39	6.51
19	(0,1,5)	459.62	467.18	1.64	6.48
20	(1,0,5)	459.62	481.41	4.74	20.43

3.3. Hexahedrical acoustical cavity over a square plate

The system is modeled as a fluid-structure system. The plate is modeled through two-dimensional elements CQUAD4, with dimensions and material specified in section 3.1. The cavity is modeled with hexahedrical solid elements CHEXA, with dimensions and fluid properties specified in section 3.2.

The vertexes displacements of the plate are equal to zero. Then, the boundary conditions of the entire plate are $u=v=r_z=0$ and still in its vertexes z=0. The boundary conditions of the cavity are equivalent to rigid walls.

The adopted discretization results in a mesh of 64 two-dimensional elements of plate *CQUAD4* of four nodes each and 512 hexahedrical solid elements *CHEXA* of eight nodes each one, totaling 576 elements and 810 nodes.

The coupled frequencies obtained with the MSC.Nastran are shown in Tab. 3, where they are compared relative to the uncoupled ones. The average variation of the first thirty frequencies with predominance structural is -1.34Hz (-0.006%), for the frequencies with the fluid as the predominant medium is 0.92Hz (0.004%), and for all the frequencies is 0.091Hz (0.001%). The maximum value of the variation is -9.93 Hz (-0.03%).

Plate	Cavity MSC.	Decoupled	Coupled	Coupled	Coupled	Variation (%)	Variation Hz
MSC.Nastran	Nastran	mode	mode	MSC.Nastran	MEFLAB	coupled	coupled
						MEFLAB /	MEFLAB -
						MSC.Nastran	MSC.Nastran
	0	F1	1	0	0	0	0
34.41		S1	2	34.22	34.26	0.12	0.04
	65,41	F2	3	67.36	64.9	-3.65	-2.46
75,67		S2	4	75.22	75.66	0.58	0.44
75,67		S3	5	75.22	75.66	0.58	0.44
94,78		S4	6	94.55	93.79	-0.80	-0.76
	133,36	F3	7	134.17	133.16	-0.75	-1.01
191,56		S5	8	190.73	185.58	-2.70	-5.15
	206,37	F4	9	206.65	206.36	-0.14	-0.29
214,55		S 6	10	214.4	210.63	-1.76	-3.77
252,84		S7	11	252.18	243.19	-3.56	-8.99
252,84		S 8	12	252.18	243.19	-3.56	-8.99
	286,69	F5	13	287.23	286.56	-0.23	-0.67
	327,09	F6	14	327.24	327.05	-0.06	-0.19
	327,09	F7	15	327.24	327.05	-0.06	-0.19
343,86		S9	16	333.90	331.96	-0.58	-1.94
	333,56	F8	17	339.90	333.49	-0.12	-0.41
	333,56	F9	18	343.48	333.49	-2.91	-9.99
	353,23	F10	19	353.56	353.16	-0.11	-0.4
	353,23	F11	20	353.56	353.16	-0.11	-0.4
	374,88	F12	21	375.36	374.76	-0.16	-0.6
	386,75	F13	22	387.03	380.97	-1.57	-6.06
	386,75	F14	23	387.03	380.97	-1.57	-6.06
404,14		S10	24	403.54	386.61	-4.20	-16.93
404,14		S11	25	403.54	386.61	-4.20	-16.93
	434,94	F15	26	435.34	434.85	-0.11	-0.49
	434,94	F16	27	435.34	434.85	-0.11	-0.49
	462,57	F17	28	462.68	447.78	-3.22	-14.9
	465,88	F18	29	466.29	462.55	-0.80	-3.74
		F19	30	467.40	465.73	-0.36	-1.67

Table 3. Predicted frequencies of the hexahedrical cavity over a square plate in Hz

It is also verified in Table 3 the average variation of the first thirty coupled frequencies obtained by our finite element program MEFLAB relative to the decoupled ones obtained by the software MSC.Nastran; for frequencies with structural predominance it results -1,83%, for frequencies with predominance of the fluid medium it results -0,85% and for all of the frequencies it results-1,21%. The maximum absolute value of the variation is 16,93 Hz (4,20%) for the mode 24. This set of results displays the correspondence between MEFLAB and MSC.Nastran.

3.4. Structural mass minimization of the vibroacoustic system

Table 4 shows the natural frequencies before and after the mass optimization, where the thicknesses of the eight rows of the plate are the variables of the problem, as indicated in Tab. 5. The two constraints are relative to the minimum and maximum limits for the thickness, 0.01 in (0,0254 mm) and 1 in (25,4 mm) respectively, and the lower limit of 25 Hz for the second coupled natural frequency. As consequence, it is observed that the second coupled natural frequency decreases 9.21 Hz (26.9%).

Table 5 shows that thicknesses reduced on average 0.066 in (1.676 mm), equivalent to 33%. The maximum reduction is 0.091 in (2.314 mm) or 45.5%.

Mode	Frequency before optimization	Frequency after optimization
1	0.00	0.00
2	34.22	25.01
3	67.36	52.67
4	75.22	55.63
5	75.22	60.75
6	94.56	67.91
7	134.18	134.22
8	190.73	134.43
9	206.65	145.76
10	214.41	150.27

Table 5. Variables for	or mass optimization
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Variable	Original value	Optimized value	
P1	0.2 pol (5.08 mm)	0.17718 pol (4.500mm)	P2
P2	0.2 pol (5.08 mm)	0.10888 pol (2.765 mm)	P4
P3	0.2 pol (5.08 mm)	0.12651 pol (3.213 mm)	P3
P4	0.2 pol (5.08 mm)	0.12651 pol (3.213 mm)	P5
P5	0.2 pol (5.08 mm)	0.12352 pol (3.137 mm)	P8
P6	0.2 pol (5.08 mm)	0.10888 pol (2.765 mm)	+
P7	0.2 pol (5.08 mm)	0.17718 pol (4.500 mm)	ř.
P8	0.2 pol (5.08 mm)	0.12352 pol (3.137 mm)	2 × ×

Fig. 4 shows the history of variables and objective function during the five iterations required for the optimization.



Figure 4. Iterations during the mass minimization

Table 6 shows the mass before and after the structural optimization. The structural mass reduces from 3.537 kg to 2.3706 kg, characterizing a reduction of 1.1664 kg (33%).

Figs. 5 and 6 show the modal shapes of modes 2, 3 and 4 of the coupled model as non-optimized and optimized respectively, being observed that the fringes of fluid pressure match the structural deformation in both cases. This serves to understand if the structure or the fluid predominates in the mode. From this consideration, it is observed that the third optimized coupled mode has the structure as predominant instead of the fluid, as can be seen in the non-optimized case. The third coupled frequency reduces from 67.36 to 52.67 Hz, meaning a variation of 14.69 Hz (21.81%).

Table	6 Mass	of the	system	before a	and	after	the o	ptim	izatior	۱.
			~							

Subsystem	Original volume in^3 (m ³)	Optimized volume $in^3 (m^3)$	Density lb s^2/in^4 (kg/m ³)	Original mass lb s ² /in (kg)	Optimized mass $lb s^{2}/in (kg)$
Fluid	40000 (0.655)	40000 (0.655)	$1.27 \times 10^{-7} (1.29)$	0.057 (0.844)	0.057 (0.844)
Plate	$80(1.31 \times 10^{-3})$	53.609 (8.78 x 10 ⁻⁴)	$2.65 \times 10^{-4} (2700)$	0.021 (3.537)	0.014 (2.370)
Total	40080 (0.657)	40053.609 (0.656)		0.078 (4.381)	0.071 (3.215)



a) Structural deformation Mode 2: frequency 34.22 Hz, predominance of the structure.



a) Structural deformation

b) Fringes of fluid pressure





a) Structural deformation *b)* Fringes of fluid pressure Mode 4: frequency 75.22 Hz, predominance of the structure.

Figure 5. Coupled modes before the optimization



a) Structural deformation *b)* Fringes of fluid p Mode 2: frequency 25.01 Hz, predominance of the structure.



Mode 3: frequency 52.67 Hz, predominance of the structure.



Mode 4: frequency 55.63 Hz, predominance of the structure.

Figure 6. Coupled modes after the optimization

4. CONCLUSIONS

The coupled natural frequencies of the system composed by a hexahedrical cavity on a square plate are compared with the decoupled frequencies, through the simulation in MSC.Nastran©. The average variation of the first thirty frequencies predominantly structural is -1.34%, for the frequencies predominantly fluid is 0.92%, and for all frequencies is 0.091%. The maximum variation value is -9.93 Hz.

During the mass minimization of the coupled system the structural mass decreases from 3.537 kg to 2.370 kg, characterizing a reduction of 1.166 kg (33%).

In a coupled system, the pressure fringes in the fluid match the structural shape deformation. This information and the decoupled frequencies values serve to infer if the structure or the fluid prevails in the coupled mode. If the fluid pressure fringes show a discontinuous behavior, probably the coupled mode is originated from the structural mode. If the structural deformation shows a discontinuous behavior, probably the coupled mode is originated from the fluid mode.

Once identified the original domain that drives the optimized coupled mode, it is possible to control better the coupled mode behavior due to some excitement.

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