SCREW-BASED RELATIVE JACOBIAN FOR MANIPULATORS COOPERATING IN A TASK USING ASSUR VIRTUAL CHAINS

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Abstract. *A current trend in Robotics is the analysis and the design of cooperative manipulation system, i.e. systems composed of multiple manipulator units interacting one another in a coordinated way. A cooperative system is characterized by higher manipulability and load capacity with respect to single-manipulator systems. In this paper, the concept of the screw-based relative Jacobian is used in a novel method to resolve a trajectory generation in the joint space, for two robots cooperating to perform a specified task. This paper also proposes an alternative method to derive screw-based relative Jacobian This method is an extension of the Davies method and is based on the concept of Assur virtual chain to calculate the direct and inverse differential kinematics for each serial manipulator. A Cartesian-space tool path, defined relatively to the workpiece, is represented by an Assur virtual chain, using the screw representation of differential kinematics. This approach generalizes the concept of the screw-based relative Jacobian and presents a new systematic method to calculate it in a compact, direct and simple form. The presented method is specially suitable when the geometry of manipulators becomes more general and for systems with spatial manipulators.*

Keywords: *Multiple manipulators systems, kinematics, screw-based relative Jacobian, Assur virtual chain, screw theory*

1. INTRODUCTION

Compared with a single manipulator, a system with two or more manipulators working in a cooperative way could increase its available workspace and its load capability. A significant improvement of the system reachability, manipulability, and productivity can also be achieved (Huang and Lin, 2003).

A key question is how the inverse kinematics of the whole system is calculated. The approaches used can be broadly categorized into two classes. The first approach is an extension of the single robot kinematics in which the joint equations for each manipulator are solved separately. The master-slave paradigm is an example of this type of approach in which the master manipulator joint positions are solved according to the specification of the task and then the slave manipulator joint positions are solved to satisfy the constraint equations resulting from the closed kinematic chain.

In the second approach, the robot system is described by a single set of equations. The whole system results redundant and its inverse kinematics are solved by calculating its differential kinematics and, subsequently, by integrating the differential equations to obtain the joint positions. This approach allows to solve the system differential inverse kinematics by optimizing a performance measure along the trajectory (Owen et al., 2003, 2004, 2005).

The system differential kinematics could be formulated using the concept of relative Jacobian introduced in Lewis (1996). In that work, the author defines the relative Jacobian of two manipulators as the matrix that relates the velocities of the tool (attached to one of the manipulators) relative to the blank (attached to another manipulator) as a function of the manipulators joint velocities.

Lewis (1996) derives the relative Jacobian differentiating the position equation of the closed kinematic chain. We outline that this procedure may be cumbersome whenever the system has more than two manipulators. In order to overcome this difficulty, we propose a new method to derive the relative Jacobian.

An alternative is to use the screw-based relative Jacobian concept (Ribeiro et al., 2007) and the related method, which is simpler and systematic, especially in case the system has more then two manipulators.

The objective of this paper is (a) generalize the concept of the screw-based relative Jacobian and (b) to present a systematic method to calculate it, using Davies method with Assur virtual chain to express the velocities of a tool relative to a blank, using the screw representation of differential kinematics.

This paper is organized as follows. Section 2, shortly presents the fundamental kinematic tools employed. Section 3 describes the relative Jacobian. Section 4 describes the method to calculate screw-based relative Jacobian using the Davies method with Assur virtual chain. Section 5 outlines the main conclusions.

2. FUNDAMENTAL KINEMATIC TOOLS

Our approach is based on the method of successive screw displacements and on the screw representation of differential kinematics. Both techniques are shortly described in this section.

2.1 Method of Successive Screw Displacements

The method of successive screws displacements provides a representation of the location of a link in a serial kinematic chain with respect to a coordinate frame based on displacements along a series of screws in an appropriate order (successive screws). To describe the method of successive screw displacements, we first present the transformation matrix associated with a screw displacement. Next, we describe the concept of the resultant screw of two successive screw displacements.

2.1.1 Homogeneous transformation screw displacement representation

Chasles's theorem states that the general spatial displacement of a rigid body is a rotation about an axis and a translation along the same axis. Such a combination of translation and rotation is called a screw displacement (Bottema and Roth, 1979). In what follows, we derive a homogeneous transformation that represents a screw displacement (Tsai, 1999).

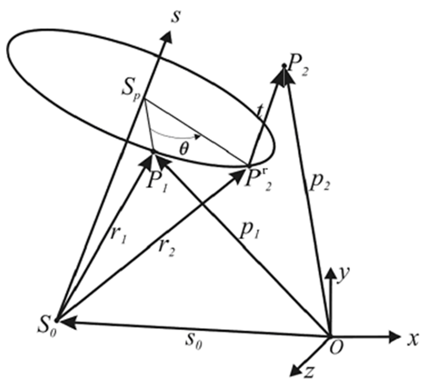

Figure 1. Vector diagram of a spatial displacement

Figure 1 shows a point P of a rigid body that is displaced from a first position P_1 to a second position P_2 by a rotation θ about a screw axis followed by a translation of t along the same axis. The rotation brings P from P_1 to P_2^r , and the translation brings P from P_2^r to P_2 . In Fig. 1, $s = [s_x\ s_y\ s_z]^T$ denotes a unit vector along the direction of the screw axis, and $s_0 = [s_{0x}\ s_{0y}\ s_{0z}]^T$ denotes the position vector of a point lying on the screw axis. The rotation angle θ and the translational distance t are called the screw parameters. These screw parameters together with the screw axis completely define the general displacement of a point attached to a rigid body. So, they completely define the general displacement of a rigid body.

Representing the first position P_1 by the vector $p_1 = [p_{1x}\ p_{1y}\ p_{1z}]^T$ and the second P_2 by $p_2 = [p_{2x}\ p_{2y}\ p_{2z}]^T$, the general screw displacement for a rigid body can be given by the Rodrigues's formula as follows:

$$p_2 = R(\theta)p_1 + d(t) \tag{1}$$

where $R(\theta)$ is the rotation matrix corresponding to the rotation θ about the screw axis and $d(t)$ is displacement vector corresponding to the translation of t along the screw axis.

Considering the augmented vectors $\hat{p}_1 = [{p_1}^T\ 1]$ and $\hat{p}_2 = [{p_2}^T\ 1]$ the general displacement of a rigid body (Eq. (1)) can be represented by a homogeneous transformation given by:

$$\hat{p}_2 = A(\theta, t)\ \hat{p}_1 \tag{2}$$

where

$$A(\theta, t) = \begin{bmatrix} R(\theta) & d(t) \\ 0 & 1 \end{bmatrix} \tag{3}$$

and the elements of $R(\theta)$ and of $d(t)$ (see Tsai (1999) for details):

$$R(\theta) = \begin{bmatrix} \cos\theta + s_x^2\,(1-\cos\theta) & s_y s_x\,(1-\cos\theta) - s_z \sin\theta & s_z s_x\,(1-\cos\theta) + s_y \sin\theta \\ s_x s_y\,(1-\cos\theta) + s_z \sin\theta & \cos\theta + s_y^2\,(1-\cos\theta) & s_z s_y\,(1-\cos\theta) - s_x \sin\theta \\ s_x s_z\,(1-\cos\theta) - s_y \sin\theta & s_y s_z\,(1-\cos\theta) + s_x \sin\theta & \cos\theta + s_z^2\,(1-\cos\theta) \end{bmatrix} \quad (4)$$

$$d(t) = t\; s + [I - R(\theta)]\;\; s_0 \quad (5)$$

2.1.2 Successive screw displacements

We now use the homogeneous transformation screw representation to express the composition of two or more screw displacements applied successively to a rigid body.

Figure 2 shows a rigid body which corresponds to a second moving link and is moved by two successive screw displacements: a first one, called the fixed joint axis , applied to the joint axis situated between the ground (fixed base) and the first link (first link screw axis), and a second one, called the moving joint axis , applied to the joint axis between the first and the second link (second link axis).

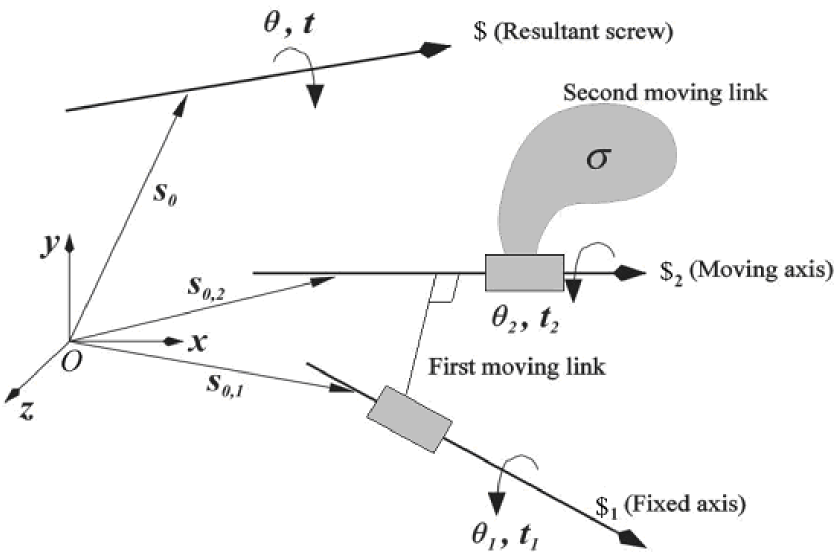

Figure 2. Two-link chain and its associated screw displacements.

As the rigid body rotates about and/or translates along these two joint axes, the best way to obtain its resultant displacement is to displace the rigid body about/along the fixed axis and, in what follows, displace the body about/along the moving axis. In this way, the initial location of the moving joint axis can be used for derivation of transformation matrix , which represents the screw displacement while the fixed joint axis is used for derivation of matrix , which represents the screw displacement (see details in Tsai (1999)). Consequently, the resulting transformation matrix is given by a premultiplication of the two successive screw displacements,

$$A_r(q_1, q_2) = A_1(q_1)A_2(q_2) \quad (6)$$

By generalizing this procedure, the resulting homogeneous matrix can be calculated by

$$A_r(q_1, \ldots, q_{i-1}) = \prod_{j=1}^{i-1} A_j \quad (7)$$

2.2 Screw representation of differential kinematics

The Mozzi theorem states that the general spatial differential motion of a rigid body consists of a differential rotation about, and a differential rotation along an axis named instantaneous screw axis (see Cecarelli (2000)). In this way, the velocities of the points of a rigid body with respect to an inertial reference frame $O{-}xyz$ may be represented by a differential rotation w about the instantaneous screw axis and a simultaneously differential translation τ about this axis. The complete movement of the rigid body, combining rotation and translation, is called screw movement or twist and is here denoted by . Figure 3 shows a body "twisting" around the instantaneous screw axis. The ratio of the linear velocity and the angular velocity is called pitch of the screw $h = ||\tau||/||w||$.

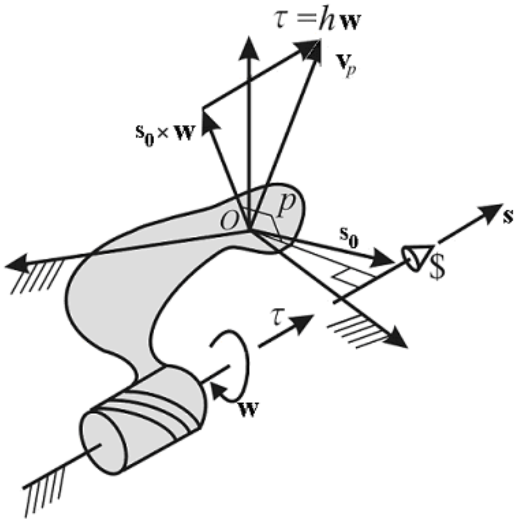

Figure 3. Screw movement or twist.

The twist may be expressed by a pair of vectors, i.e. $\$ = [w^T \ v_p^T]^T$, where w represents the angular velocity of the body with respect to the inertial frame, and v_p represents the linear velocity of a point P attached to the body which is instantaneously coincident with the origin O of the reference frame. A twist may be decomposed into its magnitude (the terms amplitude and intensity are also found in the literature) and its corresponding normalized screw. The twist magnitude, denoted as $\dot{q}$ in this work, is either the magnitude of the angular velocity of the body, $||w||$, if the kinematic pair is rotative or helical, or the magnitude of the linear velocity, $||v_p||$, if the kinematic pair is prismatic. The normalized screw, $\hat{\$}$, is a twist in which the magnitude is factored out, i.e.

$$\$ = \hat{\$}\ \dot{q} \tag{8}$$

The normalized screw coordinates (Davidson and Hunt, 2004) may be given by,

$$\hat{\$} = \begin{bmatrix} s \\ s_0 \times s + h\, s \end{bmatrix} \tag{9}$$

where, as above, the vector denotes a unit vector along the direction of the screw axis, and the vector denotes the position vector of a point lying on the screw axis.

So, the twist given in Eq. (8) expresses the general spatial differential movement (velocity) of a rigid body with respect to an inertial reference frame O–xyz. The twist could also represent the movement between two adjacent links of a kinematic chain. In this case, the twist $\$_i$ represents the movement of link i^{th} with respect to link $(i-1)$.

In Robotics, generally, the movement between a pair of bodies is determined by either a rotative or a prismatic joint. For a rotative joint, the pitch of the twist is null $(h = 0)$, and the normalized screw of the i^{th} joint is expressed by:

$$\hat{\$}_i = \begin{bmatrix} s_i \\ s_{0_i} \times s_i \end{bmatrix} \tag{10}$$

For a prismatic joint, the pitch of the twist is infinite $(h = \infty)$ and the normalized screw of the i^{th} joint reduces to:

$$\hat{\$}_i = \begin{bmatrix} 0 \\ s_i \end{bmatrix} \tag{11}$$

2.3 Davies method

Davies method is a systematic way to relate the joint velocities in closed kinematic chains. Davies (Davies, 1981, 2000) derives a solution to the differential kinematics of closed kinematic chains from the Kirchhoff circulation law for electrical circuits. The resulting Kirchhoff-Davies circulation law states that "The algebraic sum of relative velocities of kinematic pairs along any closed kinematic chain is zero" (Davies, 1981).

We use this law to obtain the relationship among the velocities of a closed kinematic chain as in Campos et al. (2005) and in Santos et al. (2006). In this way, considering that the velocity of a link with respect to itself is null, the circulation law could be expressed as

$$\sum_{i=1}^{n} \$_i = 0 \tag{12}$$

where 0 is a vector which dimension corresponds to the dimension $\$_i$. The Eq. (12) can be rewritten as

$$\sum_{i=1}^{n} \hat{\$}_i \dot{q}_i = 0 \tag{13}$$

Equation (13) is the constraint equation which, in general could be written as

$$N\dot{q} = 0 \tag{14}$$

where $N = [\hat{\$}_1 \ \hat{\$}_2 \ \ldots \ \hat{\$}_n]$ is the network matrix containing the normalized screws which signs depend on the screw definition in the circuit orientation, and $\dot{q} = [\dot{q}_1 \ \dot{q}_2 \ \ldots \ \dot{q}_n]$ is the magnitude vector.

A closed kinematic chain as actuated joints, here named primary joints, and passive joints, here named secondary joints. The constraint equation, Eq. (14), allows calculating the secondary joint velocities as functions of the primary joint velocities. To this end the constraint equation is rearranged highlighting the primary and secondary joint velocities. So, Eq. (14) can be written as follows:

$$[N_p \ N_s] \begin{bmatrix} \dot{q}_p \\ \dot{q}_s \end{bmatrix} = 0 \tag{15}$$

where N_p and N_s are the primary and secondary network matrices, respectively, and $\dot{q}_p$ and $\dot{q}_s$ are the corresponding primary and secondary magnitude vectors, respectively.

Equation (15) could be rewritten as

$$N_p\dot{q}_p + N_s\dot{q}_s = 0 \tag{16}$$

and

$$\dot{q}_s = -N_s{}^{-1} \ N_p\dot{q}_p \tag{17}$$

The primary and secondary matrices columns are the screws corresponding to the respectively primary and secondary joints. By Eqs. (10) and (11) it is clear that to compute theses matrices, the directions (s) and the locations of the joints axes (s_0) relative to a reference frame should be determined first. This could be done using the successive screw displacement method as presented in the sequel.

Consider the augmented vectors $\bar{s}_i = [s_{ix} \ s_{iy} \ s_{iz}]^T$ and $\bar{s}_{0i} = [s_{0ix} \ s_{0iy} \ s_{0iz}]^T$, corresponding to the direction and location of the i^{th} joint, and let $\bar{s}_{i_{ref}}$ and $\bar{s}_{0i_{ref}}$, with ref index, be the vectors $\bar{s}_i$ and $\bar{s}_{0i}$ at the reference position, i.e., in case θ_i and t_i are null. As the vectors defining the i^{th} joint axis direction and location depend on the movement of the (i–1) preceding joints, the augmented vectors are calculated by:

$$\bar{s}_i = A_r(q_1, q_2, \ldots, q_n) \ \ \bar{s}_{i_{ref}} \tag{18}$$

$$\bar{s}_{0i} = A_r(q_1, q_2, \ldots, q_n) \ \ \bar{s}_{0i_{ref}} \tag{19}$$

where $A_r(q_1, q_2, \ldots, q_n)$ is the resulting matrix given in Eq. (7)

2.4 Assur virtual chain

The concept of Assur virtual chain, virtual chain for short, is essentially a tool to obtain information about the movement of a kinematic chain or to impose movements on a kinematic chain (Campos et al. (2005)).

This concept was first introduced by Bonilla (2004) which defines the virtual chain as a kinematic chain composed of links (virtual links) and joints (virtual joints) satisfying the following three properties: (a) the virtual chain is open; (b) it has joints whose normalized screws are linearly independent; and (c) it does not change the mobility of the real kinematic chain.

A virtual chain useful to describe movements in three-dimensional space is the PPR orthogonal chain with two virtual links (C1 and C2) connected by three prismatic joints (PP), whose movements are in the x, y, and z orthogonal directions, and a rotative joint (R), see Fig. 4. The prismatic joints are called Px and Py, and the rotative joint is called Rz .

The first prismatic joint (Px) and the last rotative joint (Rz) are attached to the chain to be analyzed (real chain). Joint connects real link R1 with virtual link C1, joint connects virtual link C1 with virtual link C2 and rotative joint connects virtual link C2 with real link R2 (see Fig. 4).

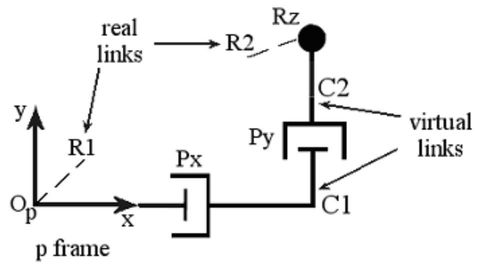

Figure 4. PPR Assur virtual chain.

Let the twist represent the movement of link C1 in relation to link R1, twist represent the movement of link C2 in relation to link C1, twist represent the movement of link R2 in relation to link C2. Therefore, the movement of real link R2 in relation to real link R1 may be expressed by $\$^P_{Px} + \$^P_{Py} + \$^P_{Rz}$.

The normalized screws corresponding to the virtual joints represented in the p frame are

$$\$^p_{P_x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad \$^p_{P_y} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad \$^p_{R_z} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ P_y \\ -P_x \\ 0 \end{bmatrix} \tag{20}$$

Notice that the orthogonal PPR Assur virtual chain represents relative movements in a planar Cartesian system. Other Assur virtual chains can be found in Bonilla (2004) and Campos et al. (2005).

Considering the PPR Assur virtual chain, let be the real link R2 angular velocity relative to the p frame(attached to the real link R1). Let be the linear velocity of the real link R2 that is instantaneously coincident with the origin of the p frame. According to the twist definition given before, and considering that the resulting movement of the real link R2 relative to the link R1 is obtained adding linearly the joint twists (Tsai, 1999), the twist expressing the velocity of real link R2 relative to real link R1 is given by:

$$\dot{x}^p = \begin{bmatrix} w \\ v_0 \end{bmatrix} = \hat{\$}^p_{P_x} \dot{q}_{P_x} + \hat{\$}^p_{P_y} \dot{q}_{P_y} + \hat{\$}^p_{R_z} \dot{q}_{R_z} \tag{21}$$

3. RELATIVE JACOBIAN

The relative Jacobian was first defined by (Lewis, 1996) for two manipulators operating in kinematic cooperation, with the movement of the tool (attached to the end-effector of one of the manipulators) relative to the blank (attached to another manipulator). More specifically the relative Jacobian gives the velocities of the tool relative to the blank as a function of the manipulators joint velocities. The same definition is used in Owen et al. (2003) (2004) (2005).

In this work we use the frame, vector and rotation matrix notation as in Sciavicco Siciliano (2000), in which O_i is the i^{th} frame and $r^k_{i,j}$ is the vector from the i^{th} frame to the j^{th} frame as seen from the k^{th} frame, and R^i_j denotes the rotation matrix of frame j with respect to frame i .

Let the velocity vector of the tool relative to the blank's frame given by:

$$\dot{x}_{p,t} = \begin{bmatrix} \dot{r}^p_{p,t} \\ \\ w^p_{p,t} \end{bmatrix} \tag{22}$$

where, $\dot{r}^p_{p,t}$ and $w^p_{p,t}$ are the vectors of the linear and angular velocities of the tool (point t), in the blank's frame (p).

The relative Jacobian is defined by (Lewis, 1996):

$$\dot{x}_{p,t} = J_R \, \dot{q} \tag{23}$$

where, J_R is the relative Jacobian, and $\dot{q}$ is the vector of the manipulators joint velocities, obtained by combining the tool manipulator joint velocities ${}^1\dot{q}$ and ${}^2\dot{q}$ the blank manipulator joint velocities :

$$\dot{q} = \begin{bmatrix} {}^1\dot{q} \\ \\ {}^2\dot{q} \end{bmatrix} \tag{24}$$

In Lewis (1996) the relative Jacobian is calculated for a planar case of two manipulators by differentiating the relative position vector. This procedure is based on visualization of that vector, which is easy only in the planar case. Moreover, the differentiation of that relative position vector is cumbersome and can be difficult when we have more than two manipulators or spatial robots.

An alternative is to use the screw-based relative Jacobian concept (Ribeiro et al., 2007) and the related method, which is simpler and systematic, especially in case the system has more then two manipulators.

In the next section we present a new method based on screw representation of the movements and using the Assur virtual chain concept. This new method beside the same characteristics of the screw-based method, allows, addicionally, to introduce kinematic constraints to the kinematic chain.

4. SCREW-BASED RELATIVE JACOBIAN USING ASSUR VIRTUAL CHAINS

In this section, the screw-based relative Jacobian is derived using the screw representation of differential kinematics, the Assur virtual concept and the Davies method. As in the relative Jacobian definition (Eq. (23)), we intend to calculate the velocity of a point t attached to the tool with respect to the p frame attached to the blank (see Fig. 5).

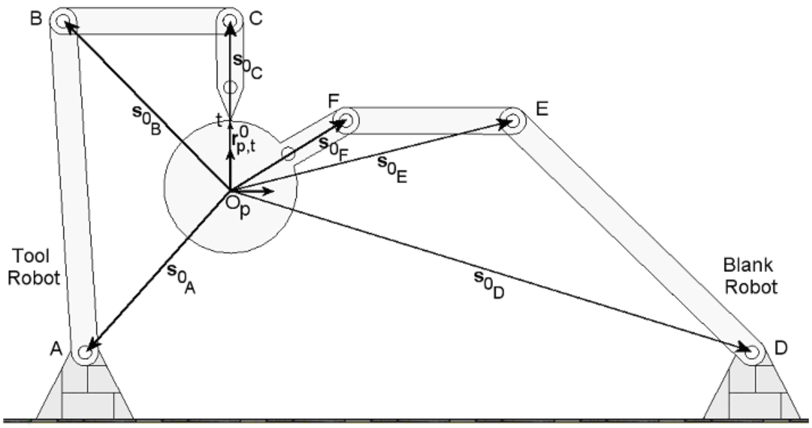

Figure 5. Two 3-dof robotic planar system.

To this end we introduce an Assur virtual chain between the blank and the t point in the tool (see Fig. 6). The Assur virtual chain closes a kinematic chain formed by manipulator 1 (tool robot) kinematic chain and manipulator 2 (blank robot) kinematic chain.

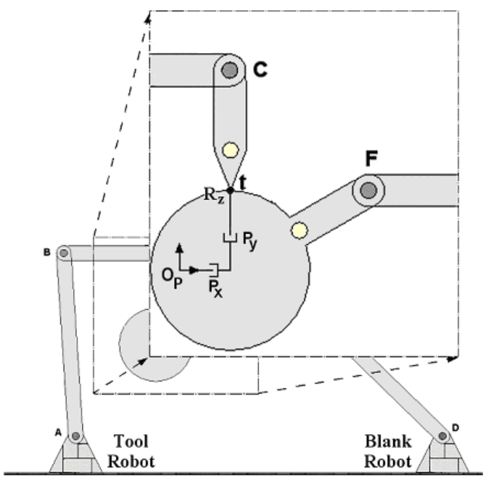

Figure 6. PPR Assur virtual chain in two 3-dof robotic planar system.

To use the systematic way to relate the joint velocities in closed kinematic chains, the cooperative manipulator system is modeled using the linear graph theory. A network diagram is drawn, including each manipulator and the PPPS Assur virtual chain (see Fig. 7), and is used for kinematic chain representation.

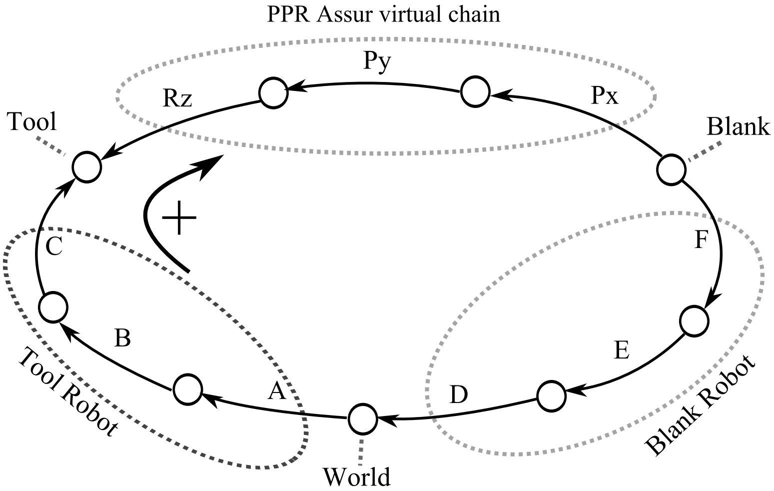

Figure 7. Network diagram of PPR Assur virtual chain in two 3-dof robotic planar system.

The graph of a closed kinematic chain is a tool to straightforwardly obtain the relation among the joint velocities in each single circuit. This allows to use the Kirchhoff-Davies circulation law, which can be written as the following constraint equation:

$$N_2\,{}^2\dot{q} + N_1\,{}^1\dot{q} - \hat{\$}^p_{P_x}\dot{q}_{P_x} - \hat{\$}^p_{P_y}\dot{q}_{P_y} - \hat{\$}^p_{R_z}\dot{q}_{R_z} = 0 \tag{25}$$

where N_1 is the matrix whole columns are the normalized screws corresponding to the manipulator 1 joints, ${}^1\dot{q}$ are the twist magnitudes of the manipulator 1 joints, N_2 is the matrix whole columns are the normalized screws corresponding to the manipulator 2 joints and ${}^2\dot{q}$ are the twist magnitudes of the manipulator 2 joints.

Using Eq. (21), the Eq. (25) results:

$$\dot{x}^p = N_2\,{}^2\dot{q} + N_1\,{}^1\dot{q} \tag{26}$$

or

$$\dot{x}^p = J^p\,\dot{q} \tag{27}$$

where,

$$J^p = [\,N_2\;\;N_1\,] \tag{28}$$

We intend to calculate the velocity of the t point attached to the tool with respect to the p frame attached to the blank, and this velocity can be expressed by the orthogonal PPR Assur virtual chain as defined in section 2, and is given by:

$$\dot{x}_{p,t} = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & \dot{q}_{R_z} & \dot{q}_{P_x} & \dot{q}_{P_y} & 0 \end{bmatrix}^T \tag{29}$$

Equation (21) can be rewritten, and is given by:

$$\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & \dot{q}_{R_z} & \dot{q}_{P_x} & \dot{q}_{P_y} & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & \dot{\$}^p_{R_z} & \dot{\$}^p_{P_x} & \dot{\$}^p_{P_y} & 0 \end{bmatrix}^{-1} \;\; \dot{x}^p \tag{30}$$

Substituting Eq. (26) and (30) in Eq. (29), the velocity of the t point attached to the tool with respect to the p frame attached to the blank ($\dot{x}^p_{p,t}$) can be expressed by:

$$\dot{x}_{p,t} = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & \dot{\$}^p_{R_z} & \dot{\$}^p_{P_x} & \dot{\$}^p_{P_y} & 0 \end{bmatrix}^{-1} [\,N_2\;\;N_1\,]\,\dot{q} \tag{31}$$

Comparing Eq. (31) and Eq. (23), we obtain the relative Jacobian as function of the matrices whole columns are the normalized screws corresponding to the manipulator 2 joints (N_2), the normalized screws corresponding to the manipulator 1 joints (N_1), and the normalized screws corresponding to the Assur virtual joints, as following:

$$J_R = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & \dot{\$}^p_{R_z} & \dot{\$}^p_{P_x} & \dot{\$}^p_{P_y} & 0 \end{bmatrix}^{-1} [\,N_2\;\;N_1\,] \tag{32}$$

Considering the skew-symmetric matrix, $\Omega(r^p_{p,t})$ given by (see Tsai, 1999):

$$\Omega(r^p_{p,t}) = \begin{bmatrix} 0 & -r^p_{p,t_z} & r^p_{p,t_y} \\ r^p_{p,t_z} & 0 & -r^p_{p,t_x} \\ -r^p_{p,t_y} & r^p_{p,t_x} & 0 \end{bmatrix} \tag{33}$$

Substituting Eq. (20), (23) and (28) in Eq. (32), the screw-based relative Jacobian can be expressed by:

$$J_R = \begin{bmatrix} -\Omega(r^p_{p,t}) & I \\ I & 0 \end{bmatrix} J^p \tag{34}$$

From Eqs. (34), (20), (14), (13), (10) or (11), and (7) the screw-based relative Jacobian using the Assur virtual chain may be calculated in a systematic way, entering the following input data:

i. the tool point t position with respect to p frame is expressed by PPR Assur virtual chain;

ii. the directions (s_i) and locations (s_{0i}) of the manipulators axes joints are given with respect to the p frame, in the manipulators reference position; and,

iii. the manipulators joint movements.

It should be outlined that the proposed screw-based relative Jacobian method using Assur virtual chain, this first requirement, is introduced in a direct and compact way. This makes it simpler, especially in case the system has more than two manipulators. Because of this, in a certain way, we could state that the screw-based relative Jacobian using Assur virtual chain generalizes the screw-based relative Jacobian concept.

5. CONCLUSIONS

The paper presents a new method to calculate the screw-based relative Jacobian for manipulators cooperating in a task. The method uses the Davies method with Assur virtual chains to express the velocities of a tool relative to a blank. We use the screw representation of differential kinematics and define the screw-based relative Jacobian as function the normalized screws corresponding to the manipulators joints and the normalized screws corresponding to the Assur virtual joints.

This new method generalizes the concept of the screw-based relative Jacobian and is a systematic procedure to calculate it in a compact, direct and simple form.

6. ACKNOWLEDGEMENTS

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