OPTIMIZATION OF ROBUST AERODYNAMIC LOADING FOR INVERSE AIRFOIL DESIGN

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Abstract. *The procedures for aerodynamic shape design can be divided in two main streams: the direct and the inverse design methods. Direct methods work directly on the geometry of the body to perform the calculations, while inverse methods rely on obtaining the geometry from a prescribed aerodynamic load distribution. The associated computational costs tend to favor the later approach, but both methods have their advantages, and can achieve highly optimized shapes that meet the design goals. The concept of robust design, on the other hand, is being used over the last years in the majority of the engineering fields. A robust product can be defined as one that is capable of achieving not only the design objectives, but also attaining a performance that is less sensitive to variations on the design parameters or operating conditions. In the context of aerodynamics, this concept has been extensively applied using the direct optimization approach; apparently, no investigation has been published yet applying the concept for inverse methods. In this paper, we investigate the applicability of the robust design concept to the optimization of velocity distribution for inverse airfoil design. The aerodynamic load is parameterized using B-spline curves. For the performance objective, a second objective associated to the robustness of the shape is defined. This parameterization is optimized via a multi-objective genetic algorithm. An example of drag minimization for an isolated airfoil is presented to show the applicability of this method.*

Keywords: *Inverse methods; robust design; airfoil design*

1. INTRODUCTION

The development of high speed digital data processing brought a whole new set of tools to the engineer in the present days. Old techniques that were never fully implemented due to the lack of computing speed are now in use. And new techniques were developed. This is true in the optimization field, now coupled with FEA, CFD, and more. Genetic, evolutionary and simulated annealing methods mimic nature towards evolution of state in the observed system. These techniques have led the engineering fields to exciting new unexplored boundaries, but not without questions.

These questions can be summarized in two aspects: processing cost, as the cost related to CPU time (with all the implicit costs: software and programming costs, energy-related environmental issues), and the applicability of the results. Optimization searches for the best solution among the ones present in the search space, but, unassisted, can lead to solutions that have poor off-design performance, or that operate poorly under slight changes of environmental conditions (temperature, pressure, humidity, to mention a few general ones). In aerodynamic design, optimization coupled to the latest advances in CFD should lead to the best airfoil suited to a particular application, either high payload, endurance, maneuverability, lift, and others. This is not always the case, and it is often seen that the deterministic optimization techniques lead to unexpected problems, and often unacceptable results (Huyse, 2001). To overcome this limitation, in the recent years, "robust" analysis has appeared as a new trend used in the design of products and processes.

As "robust" design, we consider a set design variables that produce a result less sensitive to environmental variations during its life cycle. In the case of an aerodynamic shape, these design variables are the ones related to the shape itself. Also, during its useful life, this designed shape will have to cope with manufacturing and assembly errors, fluid properties variations, corrosion and erosion, or different operating conditions, just to mention a few.

Classical approaches to measure the robustness of a set of design variables rely on signal-to-noise ratios (SNR) for determining which variables provoke the higher variations in the response (Taguchi, 1990). This requires though a full factorial number of evaluations of the performance for that set of design variables (or, at least, a representative Latin hypercube-contained number of evaluations), and that can have a very high computational cost for most of the practical problems. Gradient optimization methods in the presence of noisy variables could be used, but the convergence is guaranteed only to local optima (Beyer, 2007).

Evolutionary algorithms allow a broader search in the design variables space, and many works employing evolutionary techniques to robust analysis are cited in (Beyer, 2007). There, it is noted by that author that re-sampling of noisy variables was used in most of the cases, evaluating the fitness functions several times, and from that obtaining a measure of robustness. This can also be computationally costly, and surrogate modeling and kriging methods are commonly employed to lower the number of evaluations of the fitness function (for example see Kontoleontos, 2005).

In the airfoil design field, there are two main streams of design techniques: the direct methods, where the analysis is performed upon the geometry generated by the design variables set, and the inverse methods, where the main effort is to obtain the optimal velocity distribution around the immersed body, and only then calculating the resulting shape. Since the final geometrical form is not available a priori, all the analysis is concentrated in the boundary layer behavior, at a fraction of the computational cost of the direct methods. The drawback is that some characteristics of the resulting airfoil can only be estimated with moderate confidence, like the thickness of the airfoil, and the lift and moment coefficients. And it requires a second step, the inversion of the velocity distribution into the final airfoil. Nevertheless, the technique was used by Obayashi et all (Obayashi, 1996) for isolated transonic airfoils, using a genetic algorithm, but with no attempt to asses the robustness of the results in that work. Robust analysis are often coupled with direct design methods, and at present, no publications were found by the authors about robust analysis techniques applied to inverse design of airfoils.

In this work, an attempt was made to investigate the multi-objective optimization via genetic algorithm of inverse 2D airfoils, with one of the objectives minimized being the *vulnerability* (inverse of the robustness) of the airfoil to errors in its final geometry. Starting from a baseline airfoil, the pressure (velocity) distribution over its suction side is allowed to vary within boundaries, and form a population that will evolve towards the optimum according to a desired fitness function. Selected results are then transformed in airfoils, and these are directly analyzed to assess if the robustness values are comparatively similar. Strengths and weaknesses of the technique are commented by the authors.

2. APPROACH

2.1 The Multi-objective Genetic Algorithm

Genetic algorithms simulate evolution by selection. An initial population of design candidates is created satisfying the boundaries of the design variables space, and these candidates are ranked by means of a user supplied fitness function, with no derivatives needed. Each candidate, or individual, is characterized by its various design variables, or its genotype, represented as a string of parameters. The number of parameters describing the velocity distribution over an airfoil depend on the parameterization scheme chosen. In the present work, we use the B-spline scheme, since it has several properties that are important for our purposes (Rogers, 2001):

- The curve generally follows the shape of the control polygon (the polygonal lines, from one point to the next);
- The curve lies within the convex hull formed by its control polygon.
- The first and last points of the curve coincide with the first and last control points, respectively.
- The first derivative of the curve formed in its first and last points coincide with the inclination of the line between the first and second point, and last and second-last points, respectively.
- The curve is differentiable, and the derivatives are continuous up to one degree less than the order of the B-spline.

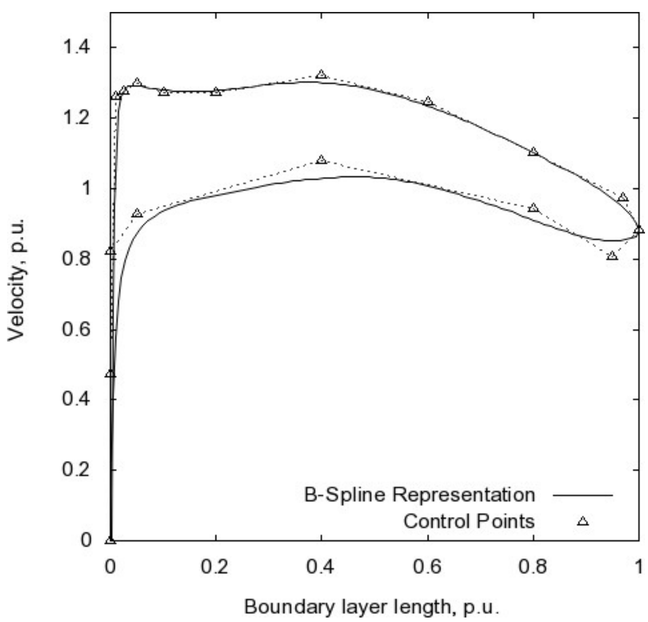

Figure 1. B-spline parameterization of the velocity distributions, and the corresponding control points.

We can see in Fig. 1 the parametric representation of the velocity distribution over an airfoil. Moving the control points, the curve will follow the resulting convex hull, maintaining its smoothness. Using the control points as genotype representation of the velocity distribution thus provides a straightforward method to characterize each individual of the population.

Each of the two curves in Fig.1 represent the suction side and the pressure side of the airfoil. In this work, we use a baseline airfoil as a starting point, and let vary, during the optimization process, only the velocity distributions over the suction side of the airfoil. The abscissa of the points is fixed, thus only the ordinates are allowed to vary. A number of 12 points are used to parameterize the curve, and, of these 12, the first point is fixed in the origin, the stagnation point of the airfoil, and the last point coincide with the velocity in the trailing edge of the original airfoil. Hence, a total of 10 variables are allowed to vary within the lower and upper limits. These limits can be seen in Fig. 2. The second control point in the upper curve is set within tight limits and close to the y-axis, to lower the risk of a discontinuity between the radius of the leading edge of the airfoil in the suction side from the one in the pressure side.

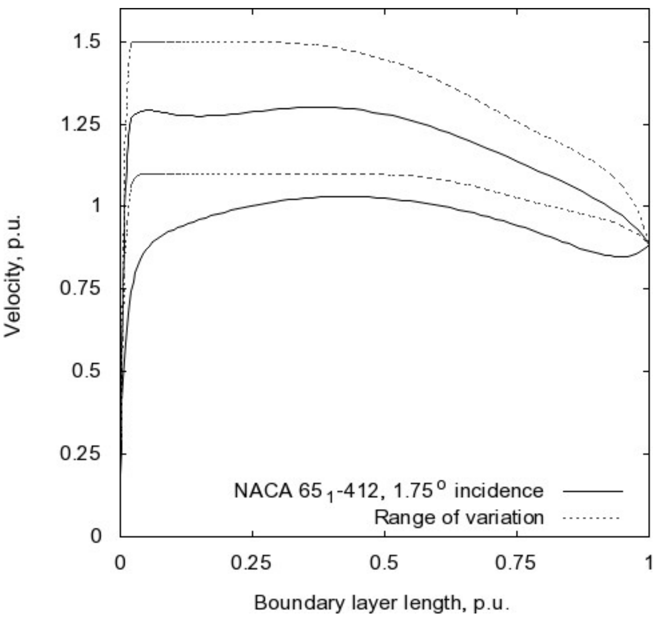

Figure 2. Range of variation allowed for the suction side of the population.

Unlike normal representations of velocity distributions, the abscissa of the graphs shown are not the percentage of the airfoil chord. Instead, it is used a distribution in terms of normalized curve length (boundary layer length), where the origin is at the stagnation point of the airfoil, increasing to the trailing edge over the curve, and mapping it into a straight line. This is a natural step, since the geometry of the body is still unknown.

In the multi-objective genetic algorithm used, the population is ranked by the fitness function, and the next generation is created based on the non-domination ranking (and distance measure to the other individuals, in case of individuals with equal rank). The concept of domination is the base in the optimality analysis via Pareto front of multi-objective optimization methods. Consider two individuals 'p' and 'q'. Individual 'p' dominates 'q' ('p' has a lower rank than 'q') if 'p' is strictly better than 'q' in at least one objective and 'p' is no worse than 'q' in all objectives. This is same as saying 'q' is dominated by 'p' or 'p' is non-inferior to 'q'. Two individuals 'p' and 'q' are considered to have equal ranks if neither dominates the other. Fig. 3 shows a set of non-inferior solutions, or Pareto optima (Mathworks, 2007).

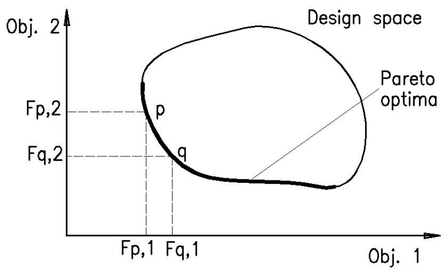

Figure 3. Schematic showing non-dominated solutions, forming the Pareto optima curve.

The elements in the Pareto front are the non-dominated individuals according to the fitness function, and one individual cannot be considered better than the other by the algorithm. A finer analysis is left for the engineer, who can weight each solution against the non-programmed constraints. One example would be the aesthetic appearance of the solution.

The fittest, or lower-ranked individuals in the population, have a higher chance to produce descendants. The selection is made using a tournament, i. e., the fittest within an arbitrary size grout of randomically chosen individuals is chosen as parent. The genotype of parents is combined via an arithmetic average of the strings of parameters of each parent. Mutation provides the population the ability to search a broader range of the design space. It is used the adaptative feasible type, i. e., it generates random directions of mutation that are adaptative with respect to the last successful or unsuccessful generation, and with a step length that satisfies the upper and lower bounds.

2.2 The Fitness Function

As formulated by Kumar (Kumar, 2007), robust design optimization can be modeled as a multiobjective minimization of the expectation and variance of the performance. In our example, low drag was chosen as the performance item of interest. Then, a two objective fitness function is proposed to test the method. The problem is defined as follows: First objective: Minimize the drag coefficient Cd Subjected to:

1. Lower velocity boundaries = specified
2. Upper velocity boundaries = specified
3. Airfoil pressure side velocities = specified
4. Lift coefficient $Cl =$ specified
5. Airfoil thickness $t =$ specified
6. Number of inflection points on velocity curves < 2
7. $$\frac{dC_{p,s}}{dx} \leq 2.5$$
   , where $C_{p,s}$ is the pressure coefficient in the suction side of the airfoil.

The approximate formula for thickness is taken from [4]:

$$t = \frac{\sqrt{1 - M_\infty^2}}{2} \int_0^1 \frac{C_{p,p} + C_{p,s}}{2} dx \tag{1}$$

where M_{infty} is the Mach number of the free stream, and $C_{p,p}$ is the pressure coefficient on the pressure side of the airfoil.

Drag is calculated by integral boundary layer methods, using the known velocities of the individual being analyzed. The laminar layer is computed using Thwaites' method, the turbulent section uses Heads' method, and transition is set by Michel's criterion (Moran, 1984). These methods are empirical, but these same procedures will be used in all the airfoils. This allows for a comparative analysis, which is the main goal of the present study. The Squire-Young equation is used to relate the momentum thickness provided by the boundary layer calculation, and the potential velocity in the trailing edge, estimating the viscous drag as:

$$Cd = 2\Theta_{te} \left(\frac{U_{te}}{U_\infty} \right)^{3.2} \tag{2}$$

where Θ_{te} is the momentum thickness at the trailing edge, U_{te} and U_{infty} are respectively the trailing edge and free stream velocities.

Penalization of the constraints: For the equality constraints, airfoil thickness was allowed to vary within ś5% of the baseline, and lift coefficient should not be more than 5% lower than the baseline. For the inequality constraints, a violation of the inequality brings to the individual an automatic penalization. A penalization factor Fp of 10 was used for all the normalized constraints, shown below:

If Calc. .GT. (.LT.) Spec. then

$$Objct = Objct + Fp \left| 1 - \frac{Calc.}{Spec.} \right| \tag{3}$$

The second objective: Minimize σ , defined as

$$\sigma = \sqrt{\frac{\sum_{k=1}^{n} \left(Cd_k - \overline{Cd} \right)^2}{n}} \tag{4}$$

which is the standard deviation of the values of drag coefficient calculated over the perturbed velocity distribution of an individual. The perturbed velocity is obtained adding or decrementing from each control point a percentage of its allowable range of variation shown on Fig. 2. So, the amount of the perturbation is dependent on the abscissa of the control point. Since the abscissas of the control points are the same for all the individuals, this provides a way to comparatively measure the deviation from the non-perturbed state between individuals.

The multi-objective genetic algorithm used was a variant of NSGA II (elitist with , available on MatLabő Genetic Optimization toolbox, release 2008a. For the sake of processing speed, the fitness function was programmed in fixed format FORTRAN 90, compiled via Intel FORTRAN 9.0 and interfaced with MatLab using a MEX interface subroutine. The population number was set to 15 times the number of variables, or 150 individuals. Due to the low computing cost, a 1.46 GHz Dual CPU Intel Pentium T2310 processor, with 1GB RAM personal computer was used, achieving convergence in about 4 minutes.

2.3 The inverse design

For the inverse design, a controlled random search (CRS) algorithm was used to find the geometry of the airfoil. This is also an evolutionary algorithm, based on a random initial population, and a single objective fitness function. The objective is to minimize the difference between the velocity distribution between the airfoil geometry and the pressure distribution provided. To test the geometry, a two-dimensional Martensen-derived vortex panel method was used. In this panel method, a "fairing-in" correction to the trailing edge flow, as described by Gostelow (Gostelow, 1984) and implemented by Manzanares (Manzanares, 1994). This correction reduces the errors due to the lack of a viscous flow and transpiration effect correction, in this simplified method.

The velocity distribution resulting from the panel method is divided from the stagnation point, and the suction and pressure side lengths are normalized to be compared with the desired distribution. The sum of the velocity error at each point of both suction and pressure sides is the fitness value. The number of panels used was 120, with a finer discretization on the leading and trailing edges. The attack angle was forced to be close to the angle of the baseline airfoil within a hundredth of a degree.

It is worth to notice that the inversion method is independent from the genetic optimization step. Therefore, more efficient methods, and of higher fidelity could be used if available. In the present work, the results obtained were within 0.1% average error between the desired aerodynamic loading and the one produced by the airfoil shape found. For the objective of showing the applicability of the robust inverse methodology, this approximate shape reconstruction algorithm was considered acceptable.

3. RESULTS AND ANALYSIS

The methodology was tested using as baseline an airfoil of the NACA laminar series 65, the 65_1-412. The attack angle was set to 1.75^o, and the Reynolds number to $6x10^6$. The flow was considered incompressible, thus M_∞ was set to 0. The boundary layer transition points were allowed to vary according to the Michel's criterion. As said, the pressure side of the velocity distribution was fixed, and only the suction side was allowed to vary. The calculated values for the baseline airfoil, and some selected results, are given on Table 1:

Table 1. Baseline and results characteristics

Airfoil tested	Thickness, %chord	Attack angle, deg.	Lift coefficient	Drag coefficient	Std. Dev. σ
NACA 651-412	12.0	1.75	0.54	0.0049	-
A1	11.4	1.76	0.56	0.0055	2.41×10^{-4}
B1	12.1	1.76	0.54	0.0058	1.81×10^{-4}
C1	12.1	1.76	0.54	0.0059	1.35×10^{-4}
D1	12.7	1.76	0.54	0.0061	1.28×10^{-4}
A2	13.3	1.74	0.55	0.0051	1.24×10^{-4}
B2	11.4	1.76	0.54	0.0060	1.03×10^{-4}

The Pareto plot with the optimization results is shown on Fig. 4. We chose two distinct groups in this plot to be analyzed, that we called families 1 and 2. In Fig. 5, the velocity e geometric plots of family 1 (airfoils A1, B1, C1 and D1) are plotted, after the inversion of the velocity profiles.

We can see a progressive trend towards a more blunt peak in airfoil D1 compared to airfoil A1, which shows a sharp velocity increase followed by a plateau, resembling a supercritical airfoil. This plateau, though, causes the speed to decrease more steeply towards the trailing edge, than on the remaining airfoils B1, C1 and D1. It will control the boundary layer growth for a big portion of the airfoil, but the following steeper negative gradient will be more sensitive

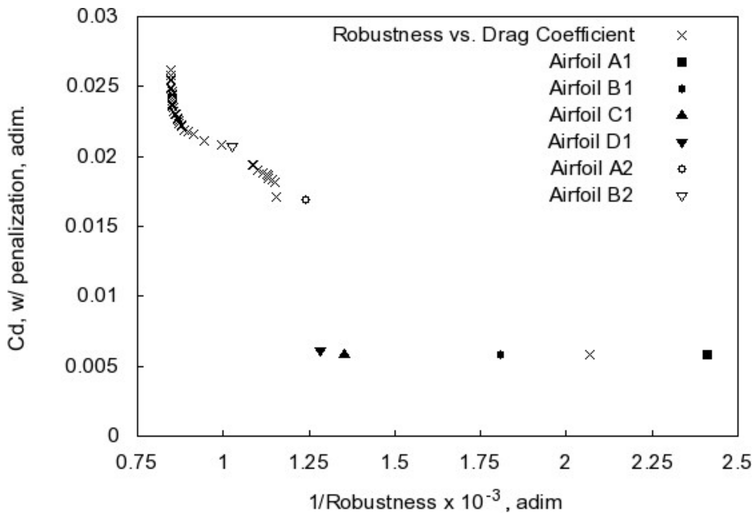

Figure 4. Optimization results.

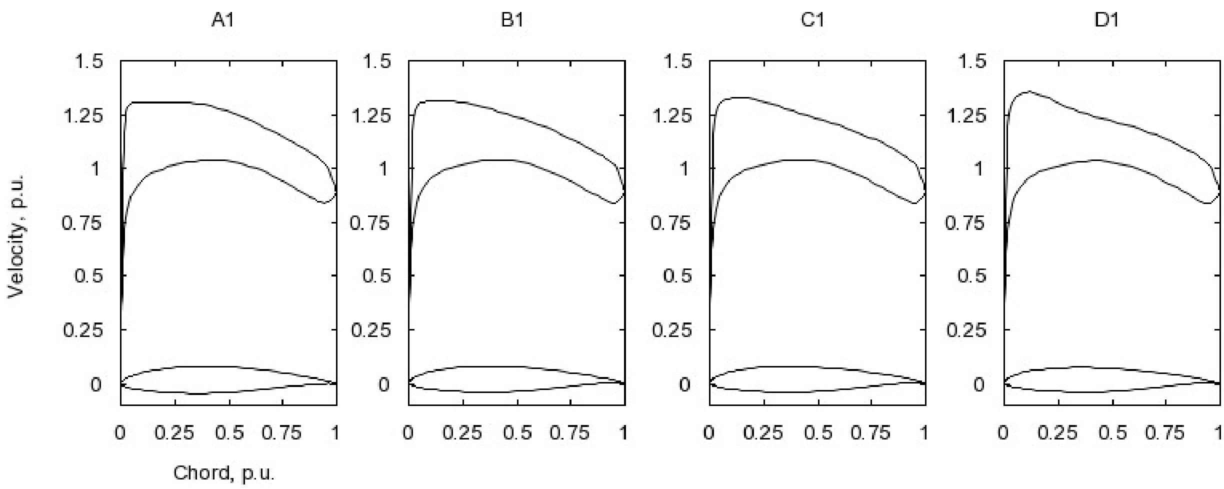

Figure 5. Family 1 airfoils.

to variations. This agrees with the relative position of these airfoils in the Pareto distribution, showing that A1 would be less robust than the other airfoils in that family.

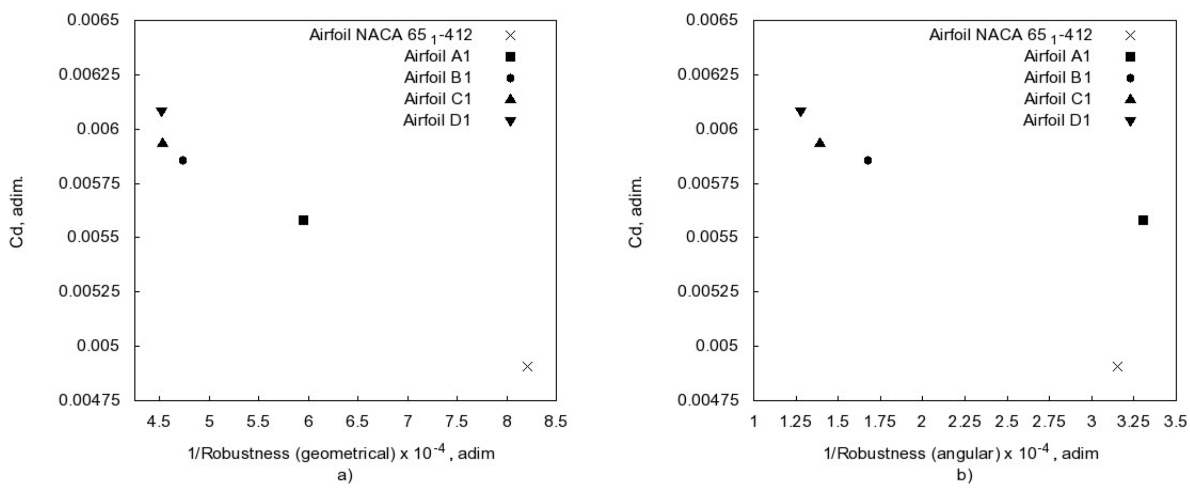

Figure 6. Robustness to geometrical and angular variations - family 1.

Figure 6 and 8 show the comparative results for the airfoils (respectively families 1 and 2), but now with their drag and robustness measure calculated over the obtained geometrical shapes. These results are for a perturbation of 2% of the chord in the ordinate of the control points that define the suction side geometry (Fig. 6a and 8a). A perturbation in the angle of incidence was made separately, in order to observe its relation to the performance variation, and the value of the attack angle is changed by $\pm$ 0.5 ^{o}. The same boundary layer subroutine was used in both inverse and direct calculations, for a comparative analysis. In both a) and b) plots, the perturbed NACA 65_1-412 is also shown for comparative purposes.

In Figs. 6a and 8a we can see that for the same amount of geometrical variation, the airfoils follow the same trend seen on Fig. 4, meaning that the relative robustness is maintained after the inversion process. The non-dominance of the solutions was also maintained, meaning that for some loss in one of the objectives (the body drag), the second objective (the drag standard deviation) diminishes against the geometrical perturbations applied. In the decision-making process, the analyst has now to evaluate which airfoil suits better the environment he will be inserted. If manufacture, erosion, impacts, or other agents that could cause a deformed shape can be precisely controlled, lower drag airfoils are a better choice. If the environmental agents are too difficult or costly to be dealt with, maybe a less sensitive airfoil of those listed could be used instead, with a price to be paid in performance.

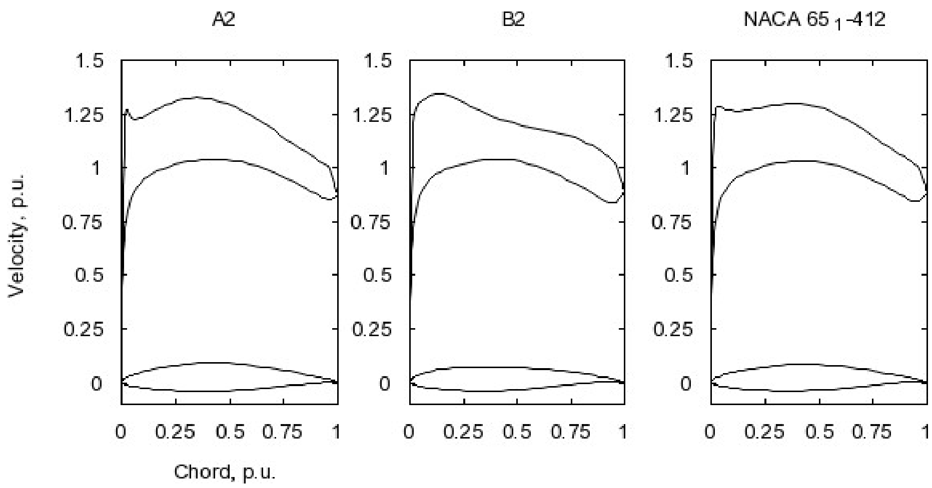

Figure 7. Family 2 airfoils.

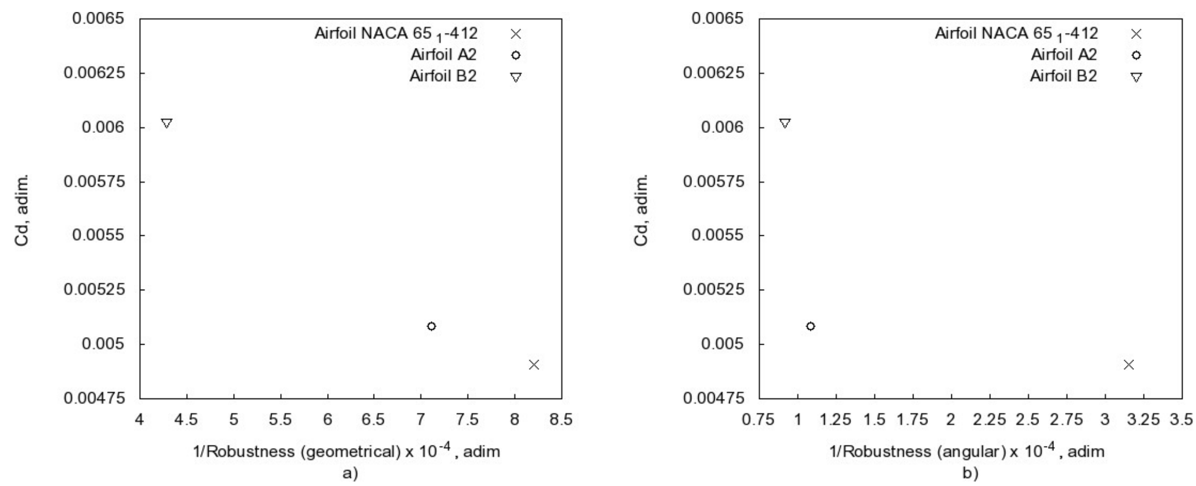

Figure 8. Robustness to geometrical and angular variations - family 2.

In family 2 of airfoils, the penalization shifted the values of the minimum drag objective during the opimization. This is seen on table 1, where the thickness of airfoil A2 is above the 5% allowance. In airfoil B2, the factors that activate the penalization are the thickness and number of inflections of the velocity curve. Even though these shapes violate the constraints, they still show good values of drag and robustness. Airfoil B2 has good performance characteriscs, although slightly thinner than allowed. Airfoil A2 shows a very low drag coefficient, but is thicker than the specification alows for. This is a weakness of the method presented. One has to limit the number of individuals in the Pareto front, and to limit the number of individuals in the population (for the sake of computing efficiency). But eliminating non-conforming individuals reduces overall diversity, and may exclude an individual that could evolve to a better design. The penalty function is a compromise solution, and has to be carefully chosen to avoid reducing too much the diversity of the population.

In Figs. 6b and 8b, it is shown the robustness against the oscilation in the angle of attack. For both families, they agree with qualitatively with the results shown in Fig. 4. Despite this agreement, it is important to note that the study presented cannot fully differentiate between geometrical and angular robustness of an airfoil in advance (during the optimization of the velocity curves), since the incidence of the free stream is unknown. An analysis after the inversion of the airfoils has to be made, to show which influence of these separate effects is more pronounced, and if the airfoil will fit the design requirements. One way to consider *a priori* these effects is to include the thin airfoil theory into the evaluation of the fittness function. Again, only estimated values of airfoil camber and angle of incidence could be inferred from the analysis the load distribution, as for the estimated thickness and lift coefficient.

4. CONCLUSION

In the work presented, an attempt was made to verify the applicability of the robust design techniques to the inverse analysis of airfoils, using multiobjective genetic algorithms. A simple and computationally inexpensive integral boundary layer method was used to assess the performance of the individuals, while a perturbation was applied to their velocity curves on the suction side, to estimate their sensitivity. A Pareto distribution containing the fittest airfoils was achieved, and selected individuals were studied to verify if the comparative robustness would be maintained now analysing their geometries.

It was noted that, for the selected individuals, their sensitivity to both angular and geometrical perturbation was maintained after the inversion process. This indicates, at a first look, that the method can be applied, saving computational time and reducing costs. The weaknesses observed were that, for a given velocity distribution, it is not possible to know in advance whether the resulting airfoil will be more affected by geometric or angular errors. Also, due to the fact that the performance constraints like thickness and lift are only approximately evaluated during the optimization, it is difficult to assess whether the airfoils generated will possess the desired characteristics.

A further step in this line of reseach now is to use a higher fidelity inversion method, and to evaluate the resulting airfoils using a Navier-Stokes algorithm. This will require a higher computational effort, but still comparatively affordable when employing inverse methods during the genetic optimization step.

5. ACKNOWLEDGEMENTS

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