ANALYSIS OF UNCERTAINTY PROPAGATION IN ACOUSTICS THROUGH A SPARSE GRID COLLOCATION SCHEME

Erb Ferreira Lins, erb@ufpa.br
Universidade Federal do Pará - Faculdade de Engenharia Mecânica, Av. Augusto Correa, 01, Belém, PA, 66075-110, Brasil

Fernando Alves Rochinha, faro@serv.com.ufrj.br
Universidade Federal do Rio de Janeiro, Programa de Engenharia Mecânica, sala G204, Ilha do Fundão, Rio de Janeiro, RJ, 21945-970, Brasil

Abstract. *In this work, the Finite Element Method is applied in the solution of acoustic problems when the boundary conditions present uncertainty. These boundary condition could represent sound absorbing materials which impedance has spatial random variation. The problem considered in this paper consists of a n-dimensional domain, representing a closed cavity, with boundary divided in three non overlapping parts. In the first two parts, pressure and normal velocities are prescribed. In the third one, the acoustic impedance, a relation between the velocity and pressure, is considered not completely known and thus should be modeled using a statistical description and random variables. These variables compose an additional dimension which is treated by an efficient, sparse collocation scheme, requiring simple modifications of the deterministic Finite Element code available to obtain the pressure field. This technique also reduces the amount of computational effort, when compared with the Monte Carlo technique and others stochastic approaches available in the literature. We apply this method to some numerical examples and the results obtained shows that uncertainty levels in the input data could result in large variability in the calculated pressure field in domain.*
Keywords: *sparse collocation, finite element method, uncertainty quantification*

1. INTRODUCTION

The sound propagation in enclosures is an important field of acoustics. This importance arises as consequence of the need in reducing or increasing the sound intensity in some regions of enclosures. A direct application of this knowledge can be seen in the design of vehicles cabins, acoustic rooms etc. Some techniques could be used to reduce the sound pressure field in enclosures. By far, the most applied method is damping the sound energy by using acoustic absorbers, generally located at the walls of the enclosure. As the sound waves hit the absorber, part of energy is converted to heat, another part is reflected back to the medium and a small fraction is transmitted to the absorbers' supporting structure (Cox and D'Antonio, 2004). The Helmholtz equation is normally used to model the propagation of acoustic waves. For a large set of problems, there is no analytical closed solution to this equation, so, a numerical procedure should be used to calculate the sound pressure field. The Finite Element Method (FEM) was successfully applied to the solution of this equation in a large variety of boundary conditions. The application of FEM to the Helmholtz equation has also been an object of study concerning the error estimation and propagation (see Ihlenburg and Babuska (1995a), Ihlenburg and Babuska (1995b) and the references therein). Recently, the probabilistic modeling of mechanical problems has attracted the attention of many researchers. In this type of simulation, the stochastic nature of some physical event is included in the description of mathematical model. This results in probabilistic information in model response, allowing a better design and increasing the reliability of mechanical system. There are two ways to include stochastical behavior in FEM: A statistical approach, like the Monte Carlo technique. In this method, a large number of samplings of the input variables is computed, the problem is then solved for each realization and the statistics are computed from solution population. The other way is a non-statistic approach, which results in the analytical treatment of stochastic process. In this case, a mathematical representation of the stochastic input variables should be included in mathematical formulation of problem. The first technique is widely used since it is easier to implement and very robust. However, a great number of samples to be solved could lead to a prohibitive computational cost. The second one allows a wiser use of numerical resources and more accurate statistical solutions, resulting in a more reliable design. In the non-statistical approaches, the main concern is how to achieve a mathematical description of stochastic process. The most widely technique used is the perturbation method, where the stochastic quantities are expanded around their mean by a Taylor series and higher order terms are neglected. The mathematical formulation could be solved with one or two terms, since more terms increase significantly the mathematical complexity of the analysis. This limits the method to the problems with small randomness. Some other techniques are based in a spectral representation of stochastic quantities like the Karhunen-LoËve expansion and Polynomial Chaos (Xiu and Karniadakis (2002), Ghanem and Spanos (1991)). The Polynomial Chaos was originally developed by Wiener (1938) and the technique was expanded by Xiu and Karniadakis (2002). Based in work of Wiener, Ghanem and Spanos (1991) use a spectral representation to represent the output stochastic variables in terms of stochastic input variables. When the FEM is the solution scheme used for spatial and temporal problem, this technique is named Spectral Stochastic Finite Element Method. Recently, new methodologies to handle the stochastic nature of some problems were developed. The Non-Intrusive Stochastic Galerkin (NISG, Acharjee and Zabaras (2007)) analysis is based in a finite element representation of stochastic quantities in a support space defined by the domain of input random variables. Based in previous work on stochastic collocation method (Babuška et al. (2007) and Babuška et al. (2005)), this technique decouples the random and spatial degrees of freedom, allowing great computational efficiency to stochastic modeling. Later, further improvement was achieved by the use of a sparse interpolation in the random space dimension. In the Stochastic Sparse Collocation Method (SSCM), an efficient, sparse interpolation technique is used to treat the random variables in the solution process. This technique significantly reduces the number of deterministic evaluations necessary and can be easily implemented, since it is also a non intrusive approach (Lins (2007), Asokan and Zabaras (2005)).

The SSFEM and NISG have a similar structure and both suffer the "curse of dimentionality", that is, the higher the number of terms used to represent the stochastic input variables, the higher is the computational effort to solve the problem. The SSCM alleviates this problem keeping the number of points to be evaluated in random dimension as the minimum possible.

In this work, the Stochastic Sparse Collocation Method (SSCM) will be applied to the stochastic modeling of impedance of an absorbing material. A one-dimensional sound propagation problem will be solved by the Finite Element method in the spatial dimension. The mathematical problem is based in Helmholtz equation with Dirichlet and Robin boundary conditions. The results will be compared with the analytical solution, NISG and the Monte Carlo technique.

2. STOCHASTIC ENCLOSED ACOUSTIC MODELING

2.1 Problem Setting

The main goal this work is to analyze the propagation of input uncertainties through internal acoustics models. The problem considered here consists of steady state acoustic waves propagation in a closed cavity, geometrically corresponding to a d-dimensional domain $D \in R^d$ with a boundary divided in three non overlapping parts, e.g.: $\partial D = \partial D_p \cup \partial D_v \cup \partial D_z$. In the first two parts, pressure (Dirichlet) and normal velocities (Neumann) are prescribed. The input uncertainty relies on ∂D_z, where the impedance velocity and pressure in the boundary is considered not completely known and thus modeled as a random variable $Z(x, \theta; \omega)$, with x and ω standing, respectively, to the geometric dimension and the circular frequency. Moreover θ indicates the randomness and is, formally, associated to the complete probability space (Ω, F, P) where Ω is the event space, $F \subset 2^\Omega$ is the σ-algebra, and $P : F \to [0, 1]$ is the probability measure.

Thus, the problem considered here is an stochastic extension of the Helmholtz equation in the sense that a pressure random field $p(x, \theta; w)$ satisfying the following equation is sought

$$\Delta p + k^2 p = 0 \tag{1}$$

$$p(x) = p_0(x) \text{ on } \partial D_p \tag{2}$$

$$\nabla p = v(x) \cdot n \text{ on } \partial D_p \tag{3}$$

$$Zv(x).n = p(x) \text{ on } \partial D_Z \tag{4}$$

where v is the boundary particle velocity, n is the boundary normal vector, p_0, a given pressure distribution. The wave number is given by $k = \omega/c$, with c being the wave propagation velocity in the media and ω is the wave frequency. The randomness embedded in the data for solving this problem rests on the boundary impedance, a complex variable, which, here, is modeled as a stochastic field, with real and imaginary part decomposition, e.g: $Z = Z_R(x, \omega; \theta) + jZ_I(x, \omega; \theta)$. The formulation can be generalized as

$$\mathcal{L}\,(x; u) = f(x), \quad x \in D \tag{5}$$

and the boundary conditions $\mathcal{B}(x; u) = g(x), \quad x \in \partial D$. This is a deterministic problem. The solution to this problem can be obtained in a variety of ways, like numerical or analytical methods. If the impedance is considered a random variable, this formulation can be restated as

$$\mathcal{L}(\theta, x; u) = f(\theta, x), \quad x \in D \tag{6}$$

and the boundary conditions $\mathcal{B}(\theta, x; u) = g(\theta, x), \quad x \in \partial D$. This is a random differential equation. In this work, we seek the solution of this equation (or, at least, some statistical properties of the solution), assuming that a deterministic solver is available. A detailed formulation to the Finite Element equations for acoustic problems can be found at Lins (2007).

2.2 Description of the Random Input

Previously, the generic stochastic acoustic model was set and the randomness was attributed to the impedance (conversely admittance) assigned to a part of the boundary which provides the mechanisms of sound energy attenuation. The method that will be assessed later can be extended with no major modifications to different random inputs. It is important to highlight that by assuming Z as a random field a nonlinear input-output relation is naturally established. Indeed, in many industrial and engineering applications, the general impedance of the system is designed towards enhancing acoustic comfort in an enclosed ambience, as it is frequently considered in the Automobile Industry. Thus, developing a sound methodology to assess the uncertainty propagation through numerical modeling could be understood as a crucial issue along the design of complex systems involving acoustics.

Specific aspects of the physical modeling itself are outside the scope of this work and they are partially covered in the references, although some comments, serving as motivation for the analysis carried out here, are included in the section devoted to the numerical examples. Indeed, those comments include more general situation in which impedance considered as a random field can be useful to account the inherent variability, as is the case of scattering over rough surfaces. The present section is devoted to introduce the main aspects of the formal description of the random input, which are crucial for the development of the numerical formulation.

The variability of the input is taken into account by enlarging the functional space in which Z is defined including random dimensions. More intuitively, this can be understood as associating the variability with random variables $\xi_\alpha(\alpha = 1, \ldots, \infty)$ as illustrated in Ghanem and Spanos (1991). It is to be noticed that once the input is now considered random, the output, here the acoustic pressure or any other function of it (e.g. sound intensity often used as a measure of acoustic comfort), has also to be considered a random field. Therefore, the mathematical problem set before is to be understood in a non deterministic framework.

A completely probabilistic description of Z would, then, require the knowledge of the marginal system of probabilities of the ξ_α variables. That seems to be non feasible in general terms, although a similar task has been carried out with success in Faverjon and Soize (2004a) and Faverjon and Soize (2004b) to obtain the probability density function (PDF) of a specific multi layer system. Even if it was possible having that description, the numerical formulation to be introduced later is built upon a finite number of random dimensions. The model to the random impedance field operator will be assumed to allow the following decomposition

$$\mathbb{Z}(\theta, x; u) = \mathbb{Z}(\xi_1(\theta), \xi_2(\theta), \ldots, \xi_N(\theta), x; u) \tag{7}$$

3. NUMERICAL FORMULATION: THE STOCHASTIC COLLOCATION METHOD

3.1 Random differential equations

Once the input variables are represented with by the use of a Karhunen-Lòeve series or any other mathematical representation using a set of random variables, the Sparse Stochastic Collocation Method can approximate the random differential equation using polynomial functions. The solution is build up by the tensor product of the functions used in each dimension of support space of random dimension. Let $y = (\xi_1, \ldots, \xi_N)$ be a point in the N-dimensional random space $\Gamma \subset \mathbb{R}^N$, Π_N is the space of all polynomials of dimension N with real coefficients and Π_N^p is this same space, limited to the polynomials with degree less than p.

Given the set $\{y_i\}_{i=1}^M \in \Gamma$ and the set of constants $\{b_i\}_{i=1}^M \in \mathbb{R}$ we want to find the polynomial function $l \in V_I$, where V_I is a subset of Π_N^p, such as

$$l(y_i) = b_i, \quad i = 1, \ldots, M \tag{8}$$

The points $\{y_i\}_{i=1}^M$ are the interpolation nodes and V_I is the interpolant space. In the case of a smooth function $f : \mathbb{R}^N \to \mathbb{R}$ this interpolation can be seen as find the approximation polynomial $\mathcal{I}(f) \in V_I$ such as $\mathcal{I}(f)(y_i) = f(y_i), \ i = 1, \ldots, M$. Using the Lagrangian polynomials it is possible to obtain an approximation to any point in space Γ with

$$\mathcal{I}(f)(y) = \sum_{i=1}^{M} f(y_i) L_i(y) \tag{9}$$

The Lagrangian polynomials are such that $L_i(y_j) = \delta_{ij}$, where δ_{ij} represents the Kronecker delta. So, if a problem has a set of values $u(y_i, x)$ of the function $u(y, x)$ it is possible to obtain a approximation $\hat{u}(y, x)$ in the form

$$\hat{u}(y, x) \equiv (u)(y, x) = \sum_{i=1}^{M} u(y_i, x) L_i(y) \tag{10}$$

To use this interpolation in a problem with random variables, we can set, from eq.(6)

$$\int_\Gamma \rho(y) \mathcal{L}(y, x; \hat{u})\, v(y) dy = \int_\Gamma \rho(y) f(y, x) v(y) dy, \quad \forall v(y) \in V_I, x \in D \tag{11}$$

Using the interpolation scheme given by eq.(10) and choosing $v(y) = L_j(y)$ we obtain

$$\int_\Gamma \rho(y)\mathcal{L}\left(y, x; \sum_{i=1}^{M} u(y_i, x)L_i(y)\right) L_j(y)dy = \int_\Gamma \rho(y)f(y, x)L_j(y)dy, \quad j = 1, \ldots, M \tag{12}$$

Since we use a Lagrangian interpolation, this equation results

$$\mathcal{L}\left(y_i, x; u(y_i, x)\right) = f(y_i, x), \quad i = 1, \ldots, M, \quad x \in D \tag{13}$$

This a deterministic problem, which can be solved at the point y_i. The boundary condition results

$$\mathcal{B}\left(y_i, x; u\left(y_i, x\right)\right) = g(y_i, x) \quad i = 1, \ldots, M \quad x \in \partial D \tag{14}$$

The operator $\mathcal{L}(y_i, .)$, restrained by the conditions given $\mathcal{B}(y_i, .)$, reduces to a spatial operator and this mathematical formulation can be approximated using any numerical method as the Finite Element or Finite Volume Method (which will result in a problem with N_{dof} degrees of freedom in spatial dimension), or even analytical. So, the solution at the M collocation points, $u(y_i, x)$ can be obtained and the interpolating function given by eq.(10) can be used. This can be great advantage compared with the solution given by Stochastic Spectral Finite Element Method, which each degree of freedom in random space is coupled with the *dofs* of the spatial dimension and results in system of linear equations of dimension $M \times N_{dof}$. Comparing with the Monte Carlo technique, it can be noted that SSCM has similar implementation with the evaluation points obtained not by using a random generator, but using the values of random variables at the collocation points of support space.

Given the solution $u(y_i, x)$ at M points, the solution statistics can be obtained using, for example to the mean value of $\hat{u}(y_i, x)$

$$\mathbb{E}\left[\hat{u}(y, x)\right] = \sum_{i=1}^{M} u(y_i, x) \int_\Gamma L_i(y)\rho(y)dy \tag{15}$$

The integration is determined by the choices of interpolation polynomials. Numerical quadrature techniques as Gauss-Kronrod can be used.

The Doob-Dinkin theorem (Øksendal, 1998) states that the stochastic variables as $u(x, \omega, \theta)$ could be represented as functions of the variables defining the support space or $u(x, \omega, \theta) \approx u(x, \omega, \xi_1(\theta), \xi_2(\theta))$. With this in mind, it is possible to compute the mean value for this variable with integration in this two dimensional space. In this work, it was used a Gauss quadrature to compute the k-th statistical moment in the form:

$$\begin{aligned} \mathbb{E}(u^k) &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u^k(x, \omega, \xi_1, \xi_2) f_1(\xi_1) f_2(\xi_2) d\xi_1 d\xi_2 \\ &= \sum_{n=1}^{N_{gp}} \sum_{m=1}^{N_{gp}} w_n w_m I(u^k(\xi_{1_n}, \xi_{2_m})) f_1(\xi_{1_n}) f_2(\xi_{2_m}) \end{aligned} \tag{16}$$

Where w_n is the weight in the integration point n. f_1 and f_2 represent the probabilistic density function of ξ_1 and ξ_2. To compute this numerical integrations it is only necessary evaluate the sparse interpolant for the pressure variable in the integration point (ξ_{1_n}, ξ_{2_m}). This could be done in the post processing stage, so any integration technique (even Monte Carlo Integration) could be used.

3.2 Sparse Grids

The Collocation Technique developed at section 3.1 can find an approximation to a random differential equation using the solution of M deterministic problems. To reduce even more the computational effort these points should be chosen in a way to achieve the maximum accuracy using the smaller set of points. These points can be determined by the used of tensor product of complete polynomial functions in each dimension or by the use of sparse grids. In this work, we use the sparse grid technique. The Smoljak algorithm can be used to compute the interpolant function for a multidimensional sparse grid set, by extending the one dimensional interpolating functions to multidimensional space using a particular form of tensor product. This combination can be chosen in order that some interpolation property for $N = 1$ is maintened for $N > 1$. The number of nodes in the set is much less than the one obtained by the usual tensor product. This algorithm was developed by Smoljak (1963) and information can be found in Barthelmann et al. (2000), Ritter and Novak (1996) and Lins (2007).

The point set chosen should have some important properties. The Smoljak algorithm requires that the point set for some interpolation level should contains all the points from levels below $\left(X^i \subset X^{i+1}\right)$ then one of the best choice is the Chebyshev-Gauss-Lobatto points. These nodes distribution, in the i-th interpolation level, is given by $X^i = \{x_1^i, \ldots, x_{m_i}^i\}, i \in \mathbb{N}$ so

$$\begin{array}{l} m_i = 2^{i-1} + 1, \quad i > 1 \\ x_j^i = -\cos \frac{\pi(j-1)}{m_i - 1}, \quad j = 1, \ldots, m_i \end{array} \tag{17}$$

and $x_1^i = 0$ if $m_i = 1$. The number of points required grows considerably with the interpolation level when complete meshes are applied.

Figure (1) shows the point distribution for two and three dimensional spaces using a sparse interpolant and meshes generated using the tensor products of one dimensional set of Chebyshev-Gauss-Lobatto points.

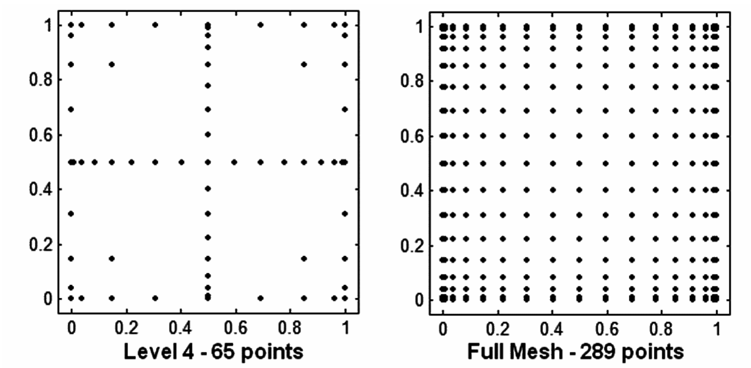

Figure 1. – Collocation points for two dimensional space. Left: Nodes used in the Smoljak algorithm. Right: Tensor product of one dimensional nodes

4. NUMERICAL EXAMPLES

4.1 1-D - The Kundt Tube

The Kundt tube is an experimental apparatus commonly used to measure the acoustical properties of a sample of an absorbing material. The experimental setup is quite simple: a loudspeaker generates plane waves in the interior of tube, the waves are reflected by the sample at the other end of tube and a standing wave is set up within the tube (figure 1). As the sample changes the amplitude and phase of the sound wave, it is possible to calculate the impedance of the sample by the measure of the pressure in some points of this standing wave (Cox and D'Antonio (2004), Schultz et al. (2007)).

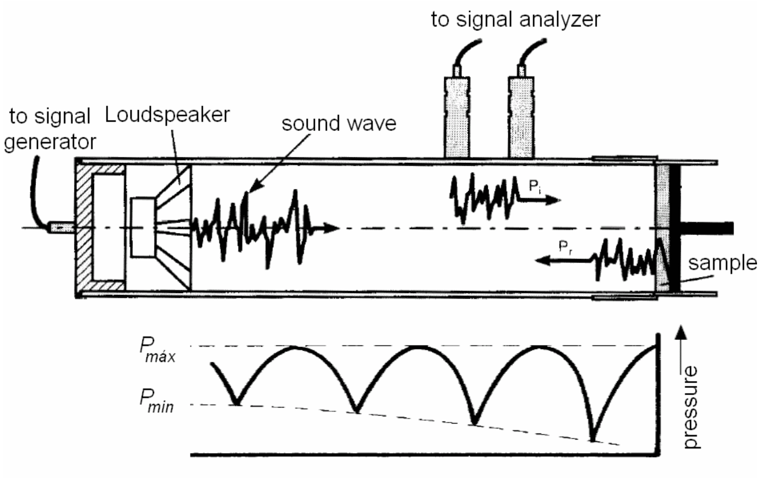

Figure 2. – Impedance tube

A simplification of this impedance tube is showed in fig.(2). We approximate this setup as an one dimensional domain, with a Dirichlet and Robin boundary conditions. The Robin condition represents the impedance of the sample at the end of the tube. With this in mind, the Helmholtz eq. (1) and the boundary conditions given by equations (2) and (4) will result in the following mathematical formulation

$$\frac{d^2p(x)}{dx^2} + k^2 p(x) = 0 \tag{18}$$

$$\begin{array}{ll} p(x) = p_0 \quad \text{at} \quad x = 0 \\ \frac{dp}{dx} - \frac{j\rho\omega}{Z} p(x) = 0 \quad \text{at} \quad x = L \end{array} \tag{19}$$

Given the idealized domain show in fig.(2), these equations represent the acoustic source by a Dirichlet boundary condition and the material impedance could be represented by a Robin boundary condition. Z is the impedance of absorbing material. The analytical solution of eq. (18) and (19) is

$$p(x) = p_0 \frac{Z \cos k(L-x) - j\rho c \sin k(L-x)}{Z \cos kL - j\rho c \sin kL} \tag{20}$$

It should be noted that the pressure field is a nonlinear function of Z. In this numerical experiment, given a probabilistic distribution for the absorbing material impedance, we compute the pressure distribution in this domain. As long as the analytical solution is available, the statistics of pressure field can be obtained using Monte Carlo method, in a fast and reliable way.

Schultz et al. (2007) presents a detailed analysis of uncertainty propagation in this experimental setup. The uncertainty levels obtained by these authors were considered in numerical experiments presented in this work.

The finite element solution will be obtained by a one-dimensional mesh with $n = 100$ elements. The node coordinates, using a uniform mesh are

$$x_k = \frac{k-1}{n}, \quad k = 1, \ldots, n+1 \tag{21}$$

Figure (3) shows the elements distribution along the tube length.

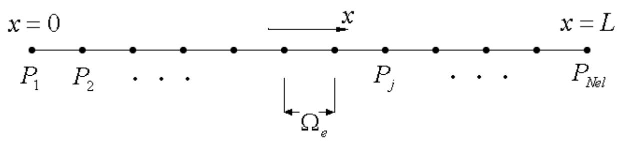

Figure 3. – Finite Element Mesh

The impedance Z is represented by a complex function with the following form

$$Z(\omega) = i\frac{\tilde{\alpha}(\omega)}{\omega} + \beta(\omega) \tag{22}$$

$\tilde{\alpha}(\omega)$ and $\beta(\omega)$ are two frequency dependent parameters. For the sake of clarity, we can set $\tilde{\alpha}(\omega)/\omega \equiv \alpha(\omega)$. This impedance model is widely applied since data for real and imaginary part of impedance is commonly obtained in experimental tests. Despite of non-causality of this model, when transposed from frequency to time domain (Berthelot, 2001), this formulation will be applied only for uncertainty analysis in calculated pressure field. The real and imaginary parts impedance values considered in this numerical test correspond to experimental data available for glass wool *Manville* (Bermúdez and Rodríguez, 1999).

To proceed the stochastic analysis it is necessary to setup statistical information in the input data. We assume that the real and imaginary parts of impedance could be represented by two stochastic variables with the following form

$$\begin{array}{l} \alpha(\omega,\theta) = \bar{\alpha}(\omega)\,(1 + \sigma\,\xi_1(\theta)) \\ \beta(\omega,\theta) = \bar{\beta}(\omega)(1 + \sigma\,\xi_2(\theta)) \end{array} \tag{23}$$

Where ξ_1 and ξ_2 are two independent random variables with range $\xi_{1,2} \in [-1, 1]$ and zero mean. σ represents how much the values of α and β fluctuates around the mean values $\bar{\alpha}$ and $\bar{\beta}$, and it can be directly related with the standard deviation as described below. The maximum uncertainty principle can be used to define the statistical distribution of ξ (Soize, 2001). As long as only the mean values of $\alpha(\omega,\theta)$ and the standard deviation are known, this principle states that the uniform distribution should be selected to represent the random variables. If ξ_1 and ξ_2 are two uniform random variables and σ_α and σ_β are the standard deviation of $\alpha(\omega,\theta)$ and $\beta(\omega,\theta)$ respectively, we can calculate

$$\begin{array}{l} \sigma_\alpha = std(\alpha(\omega,\theta)) = \frac{1}{3}\bar{\alpha}(\omega)\sigma \\ \sigma_\beta = std(\beta(\omega,\theta)) = \frac{1}{3}\bar{\beta}(\omega)\sigma \end{array} \tag{24}$$

Later the following ratio will be used to measure the input uncertainty level:

$$r_{in} = \frac{\sigma_\alpha}{\bar{\alpha}(\omega)} = \frac{\sigma_\beta}{\bar{\beta}(\omega)} = \frac{\sigma}{3} \quad (25)$$

For the input variables, this ratio is constant in all frequency range. Similarly, for output data the rate

$$r_{out} = \frac{\sigma_p}{\mathbb{E}[p]} \quad (26)$$

Where $\mathbb{E}[p]$ is the mean value of pressure field and σ_p is the pressure standard deviation. It should be note that both values varies spatially and with frequency.

The one dimensional problem will be solved in the frequency range from 1Hz to 2kHz in steps of 0,5Hz. To each frequency, the Monte Carlo method, the Non Intrusive Stochastic Galerkin (NISG) as proposed by Acharjee and Zabaras (2007) and Stochastic Sparse Collocation Method (SSCM, Lins (2007)) were applied to calculate the statistical data from output pressure field. The Table 1 shows the main features of each technique.

Table 1. Numerical data of the Methods applied

Method	Features	Number of evaluations (per frequency)
Monte Carlo	Number of samples: 15,000	15,000
NISG	20×20 finite elements in support space (4 integration points per elements)	1,600
SSCM	Maximum approximation order: 4 Statistical moments calculation: Gaussian integration in support space	Maximum: 114

All the techniques above used the Finite Element Method described in this section except the Monte Carlo Technique which used the analytical solution (equation (20)). The SSCM has converged, in vast number of frequencies, with only approximation of order 3, requiring only 49 evaluations of Finite Element Code.

The results obtained are shown in figs.(4)-(7).

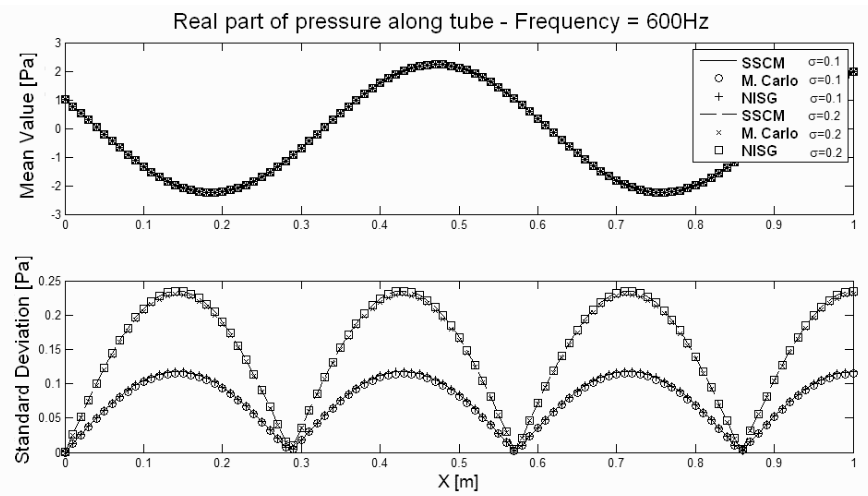

Figure 4. – Pressure along spatial domain: mean and standard deviation of real part of pressure. Frequency=600Hz.

As can be seen the results obtained agree very well with data from NISG and Monte Carlo for the mean and standard deviation both from real and imaginary part of stationary wave. To analyze the output uncertainty level, it is necessary to choose one point in the domain and proceed to the computation of rate r_{out}. In the figs.(8)-(9) it is shown how the output ratio varies according to the excitation frequency for two different positions in the domain ($X = 0.1$ and $X = 0.5$).

It should be noted that the agreement between the statistical information from these three formulations is quite good in all frequency range, for each of input standard deviation. A gradual discrepancy increase can be observed in higher frequency range: the Monte Carlo solution is slightly different from NISG and SSCM. This could be expected, as long as some difference is expected from different solutions used by each of the methods (FEM for NISG and SSCM, and analytical solution applied by Monte Carlo).

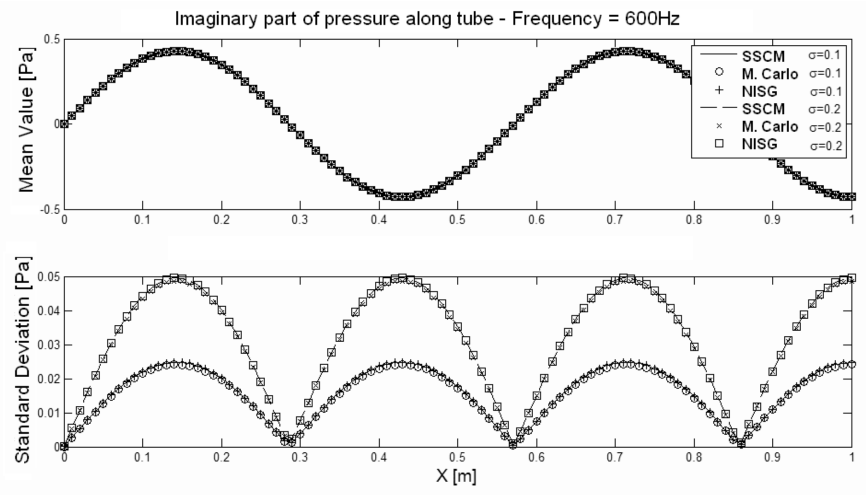

Figure 5. – Pressure along spatial domain: mean and standard deviation of imaginary part of pressure. Frequency=600Hz.

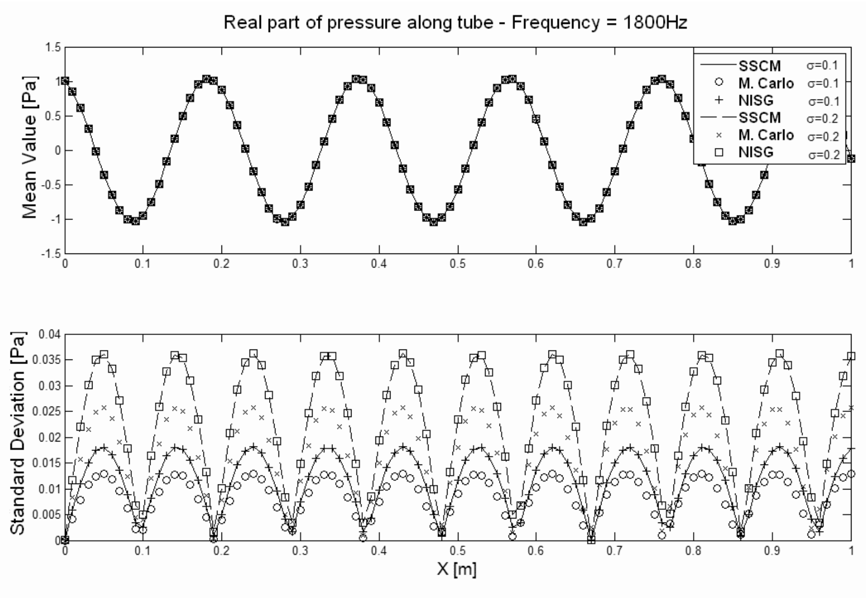

Figure 6. – Pressure along spatial domain: mean and standard deviation of real part of pressure. Frequency=1800Hz.

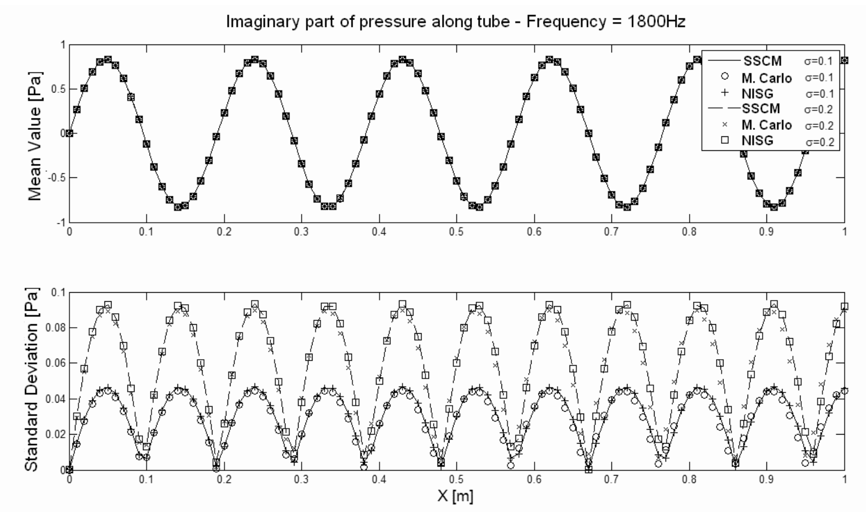

Figure 7. – Pressure along spatial domain: mean and standard deviation of imag. part of pressure. Frequency=1800Hz.

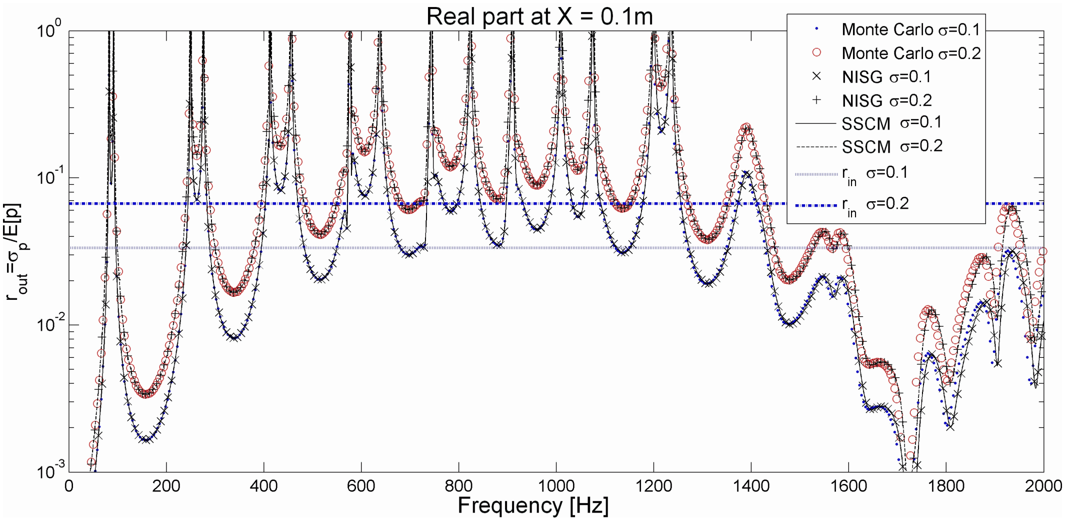

Figure 8. – Ratio $r_{out} = \sigma/\mathbb{E}[p]$ for real part of pressure at $X = 0, 1$.

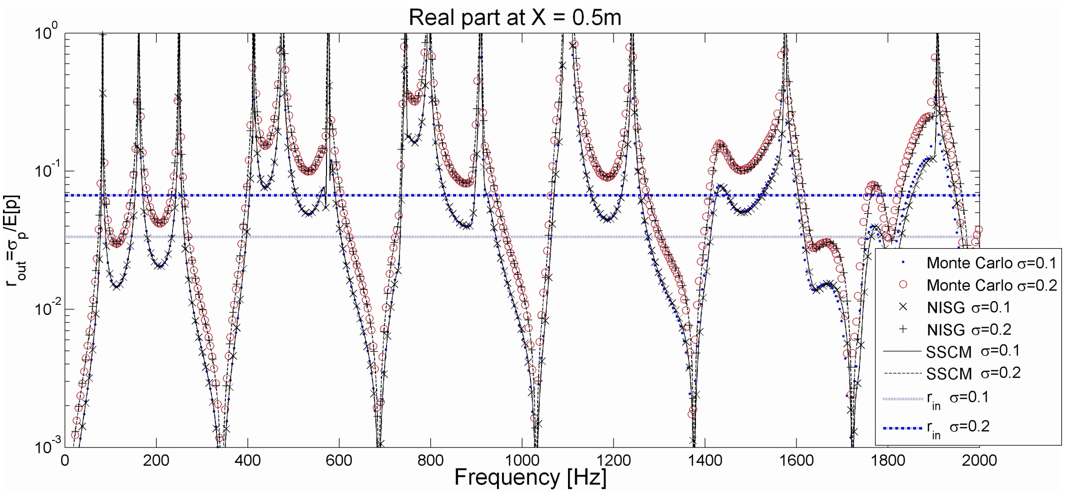

Figure 9. – Ratio $r_{out} = \sigma/\mathbb{E}[p]$ for real part of pressure at $X = 0, 5$.

It should be observed that the uncertainty levels in the output are quite large when compared with the input uncertainty levels (The straight dotted lines). In the vicinity of natural frequencies the uncertainty levels can be quite large.

The main information to experimental setup of Kundt's tube that can be extracted from figures is the following: There is a combination of sensor position and frequency ranges that can minimize the uncertainty in output data even if uncertaintyies in the input information are high (as can be observed in high frequency range of position $X = 0.1$). Another conclusion is that despite of similarity with these figures and the FRF for each of these points, it should be quite clear that the later can not provide some important information regarding the better positions and frequency range for measure points in this experimental apparatus.

5. CONCLUSIONS

In this work, a efficient, sparse collocation technique was applied to the approximation of random differential equations. The solution was developed in order to estimate the pressure field in an enclosure, given the randomness of acoustic absorbers panel. It is necessary to develop a mathematical model to represent this randomness using random variables. If this model can be obtained, it will be possible to apply a space decomposition and represent this information using a series of simple random variables. Each random variables defines a new dimension, composing a random space support. The use of Lagragian polynomial interpolation in each dimension of random space allows to uncouple the random and spatial dimension, and the solution could be obtained with usual FEM acoustic solver, with simple modifications. The computational effort was greatly reduced by the use of a sparse set of nodes in interpolation. The results obtained shows that the technique is faster and more efficient than the ones based in Wiener Chaos polynomial approximation or other non-intrusive approaches. The technique was applied in the solution of one dimensional problem, showing that the Kundts' tube device, originally planned to obtain the acoustic impedance of sample of material, can suffer great uncertainty in some combinations of input randomness, microphone position and frequency.

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