PASSIVITY BASED CONTROL OF A 3D OVERHEAD CRANE

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Abstract. *One of the main challenges in positional control of an overhead crane is its tendency to oscillate, especially when stopping. This paper presents a passivity based solution for this problem by inducing an artificial damping. Overhead cranes are similar to a gantry cranes and consist of three major parts: the hoist, providing up/down motion to lift items; the trolley, providing left/right motion for the hoist and payload; and the bridge, providing back/forward motion for trolley, hoist, and payload. Usually, they are permanently installed in factories, shops, or warehouses to move items not moveable by humans or forklifts. Since the payload is suspended by a somewhat flexible cable, an unactuated joint or coupling is formed. So, the mechanical system consisted by the crane and the payload are sub-actuated because it has more degrees of freedom that actuators. The goal is to control the payload position reducing the swing. In order to achieve this goal is proposed a nonlinear control based on passivity. The dynamic model is derived using the Lagrangian formalism using a distinct set of coordinates from the literature. This set of coordinates is singularity free which is the main difference from other author choice. A Lyapunov function based control is applied. The results are verified by means of simulation.*

Keywords: *nonlinear control; passivity based control; Lagrangian formalism; anti-swing control; crane control*

1. INTRODUCTION

An overhead crane is a type of crane very common in factories, warehouses, harbors and other environments where there is the need for moving heavy payloads. What makes the overhead crane so interesting for the study of non-linear control techniques is:

1. it's a underactuated system in which the number of degrees of freedom is greater than the number of actuator;
2. it's non-linear in nature having a tridimensional (non-planar) pendulum characteristic; and
3. it has a huge practical interest since it's widely used in the industry.

Many researches have dedicated some work to the improving of this equipment, in particular aiming the elimination or reduction of swing that the payload is submitted when stopping the translational movement. Strip (1989) showed that, for an object hanging in a wire, there is a family of control strategies that results in movements free of swing. Nevertheless, he employed an approximation for little angles to simplify the mathematical treatment of the system under study.

Toxqui et al. (2006) applied a PD (proportional-derivative) controller tuned in real-time by a neural network. This attempt led to a satisfactory performance only when the angle between the cable and the vertical line is small. Mahfouf et al. (2000) used a fuzzy controller applied to overhead crane that could move in only one direction to evaluate the employment of fuzzy control techniques in swing attenuation. More recently, Hayajneh et al. (2006) has worked to reduce the number of rules and extend fuzzy control to the problem of two directions movement with swing reduction.

Banavar et al. (2006) used an approach called IDA-PBC – Interconnection and Damping Assignment-Passivity Based Control also to address the two directions movement problem. The results discourage the use of this technique in two direction movement problem due the difficulties in solving the partial differential equations (PDE) that comes out in the controller design. Gmes-Estern et al. (2001) and Viola et al. (2005) state that it is only practical to apply this technique when the system has only one sub-actuated variable. In these particular cases, the PDE's are transformed in total differential equations and a solution can be found. When there are more than one sub-actuated variable, as it is so with overhead crane, the PDE usually became exceptionally complicated to be solved.

Chen et al. (2005) took in to account the cable length variations and developed a partial feedback linearization[1]. Despite of controlling the displacement in three dimensions, the control has little efficiency because the obtained trajectory is quite different from the planed one. Furthermore, the authors' goal does not seem be in syntony with the main goal of the overhead cranes: displacing the payload to a final 2D position. There is no need for completely controlling the displacement in 3D; only the final 2D position is important and not the intermediary one. The tacks performed by an overhead crane are far different from the task preformed by a robot.

[1] A partial feedback linearization splits the variable into two groups: actuated and non-actuated.

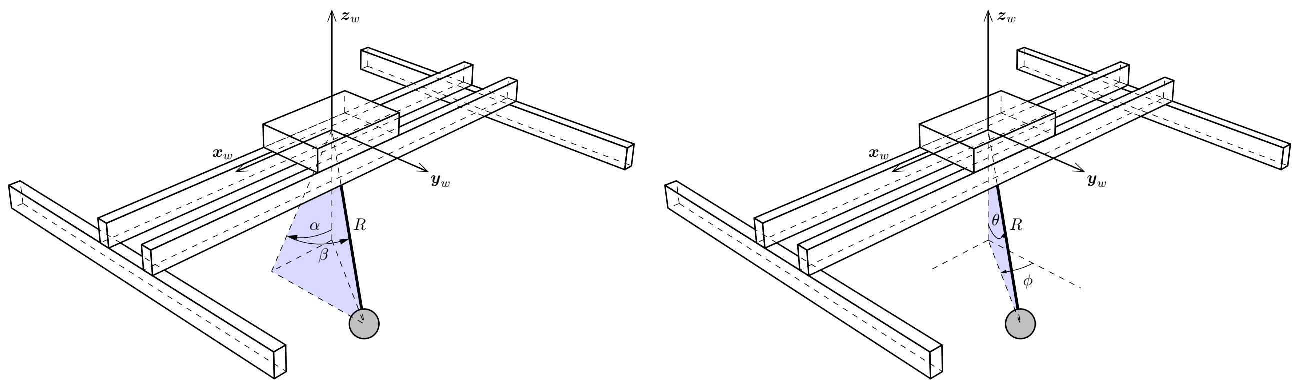
Fang et al. (2001, 2003) used a control law based on passivity adapting the ideas from Lozano et al. (2000) originally formulated to control an inverted pendulum. Despite of presenting very interesting results, they used a kinematically singular model in which the value of one generalized variable is undetermined when the cable becomes vertical. The very same definition are employed by Al-Garni et al. (1995).

In this paper, it is used a controller based in the same control law employed by Fang et al. (2001, 2003), but build upon a non-singular kinematic model. A secondary goal of this work is to determinate the impact of this change over the controller performance. In Section 2the constrain hypothesis are stated, the dynamic model of the system is derived using Lagrangian formulation and passivity of underactuated Euler-Lagrange system is proved. Note that, usually, only fully actuated or completely unactuated Euler-Lagrangee systems are proved to be passive (Ortega et al., 1998). In Section 3the passivity based controller is designed and its stability is established. In Section 4the system is simulated and the obtained results are compared with Fang et al. (2001). In Section 5some conclusions are drawn and some goal for futures works are stated.

2. DYNAMIC MODEL

Figure 1a shows the schematic representation of an overhead crane with a coordinate reference system, or frame, $O_w\boldsymbol{x}_w\boldsymbol{y}_w\boldsymbol{z}_w$, where w stands for *world*. It is also pointed out how the angles α and β are measured. These angles establish how the payload position is affected by the cable inclination. The reference system origin is conveniently located where the cable touches the wheel when the positioning system is at null displacement. Fang et al. (2001, 2003) and Al-Garni et al. (1995) use another angles to define the cable orientation, as shown in Fig. 1b, but this representation is singular because angle ϕ is undefined where $\theta = 0$. They also claim that this fact seems to be unimportant, but we suspect that this choice leads to less smooth or even noisy control forces (see Section 4). Still, the angle θ is appealing because it measures the cable inclination in respect to the vertical line and its value is the ultimate quantity to be minimized in anti-swing control.

In normal operation, an overhead crane lifts the payload until a safety height above the floor, usually determined by the operator; moves the payload to a point $(x,\, y)$ in a plan parallel to the plan $x_w y_w$ of the coordinate reference system; and lows the payload or keep it steadily in this position, it depends on task to be performed. The challenge is to move the payload until the final position and stop at the final position without swinging.



(a) Non-singular representation

(b) Singular representation

Figure 1: Schematic representation of an overhead crane

The dynamic model of the system was elaborated using Lagrangian formalism (Sciavicco and Siciliano, 2000) and its correctness was verified using the Recursive Newton Euler algorithm presented by Craig (1989). The constants employed in the model development are: cable length, R (considered constant in what concerns the control[2]); the trolley mass, m_t; the bridge mass, m_b; the payload mass, m_p; the moment of inertia of the payload (assumed uniform and symmetric), I_p; and acceleration due gravity, g.

It was assumed that:

1. the friction is negligible;

2. the cable mass is negligible;

3. the cable is always stretched out what is equivalent to a spherical joint connected to the trolley;

[2]Since the payload movement is divided in steps, the controller designed in Section 3is used only to control the movement of the bridge and trolley. The hoist, responsible the lift and low the payload, is moved only with the bridge and trolley stopped.

4. the payload does not spin around its own axis;
5. the axes x and y are absolutely horizontal and mutually orthogonal;
6. all constant are perfectly known;
7. all positions and velocities, linear and angular, are measured;
8. all external perturbations like wind forces, vibration produced by other machines, etc. are negligible;
9. the angles α and β are restricted[3] to the open range $(-\pi,\, \pi)$ rad.

The payload position in respect to the reference coordinated is:

$$x_p = x + R\,\cos\beta\,\sin\alpha; \qquad y_p = y + R\,\sin\beta; \qquad z_p = -R\,\cos\alpha\,\cos\beta \tag{1}$$

where the angles α and β are defined in Fig. 1a and x and y are coordinates of the trolley (cable connection point). The kinetic energy $\mathcal{T}$ can be split in four terms: 1) the translational kinetic energy of the bridge; 2) the translational kinetic energy of the trolley; 3) the translational kinetic energy of the payload; and 4) the rotational kinetic energy of the payload. So:

$$\mathcal{T} = \mathcal{T}_1 + \mathcal{T}_2 + \mathcal{T}_3 + \mathcal{T}_4 \tag{2}$$

$$\mathcal{T}_1 = \frac{m_b}{2}\,\dot{y}^2 \tag{3}$$

$$\mathcal{T}_2 = \frac{m_t}{2}\,\left(\dot{x}^2 + \dot{y}^2\right) \tag{4}$$

$$\mathcal{T}_3 = \frac{m_p}{2}\left(\dot{\alpha}^2\,R^2\,\cos^2\beta + 2\,\cos\alpha\,\dot{\alpha}\,R\,\dot{x}\,\cos\beta + \dot{\beta}^2\,R^2 - 2\,\sin\alpha\,\sin\beta\,\dot{\beta}\,R\,\dot{x} + 2\,\dot{\beta}\,R\,\dot{y}\,\cos\beta + \dot{x}^2 + \dot{y}^2\right) \tag{5}$$

$$\mathcal{T}_4 = \frac{I_p}{2}\,\dot{\alpha}^2 + \frac{I_p}{2}\,\dot{\beta}^2\,. \tag{6}$$

The systems potential energy, $\mathcal{U}$, is equal to the payloads potential energy alone given that the remaining components moves only horizontally. This potential energy is given by:

$$\mathcal{U} = R\,g\,m_p\,\left(1 - \cos\alpha\,\cos\beta\right)\,. \tag{7}$$

The Lagrangian is given by (Sciavicco and Siciliano, 2000):

$$\mathcal{L} = \mathcal{T} - \mathcal{U}\,. \tag{8}$$

The vector of external forces applied by actuator, $\boldsymbol{\tau}$, and the vector of system generalized variable, $\boldsymbol{q}$, are:

$$\boldsymbol{\tau} = \begin{bmatrix} F_x \\ F_y \\ 0 \\ 0 \end{bmatrix} \qquad \text{and} \qquad \boldsymbol{q} = \begin{bmatrix} x \\ y \\ \alpha \\ \beta \end{bmatrix} \tag{9}$$

where F_x is the force that must be applied by the actuator in the trolley, F_y is the force that must be applied by other actuator in the bridge, x é the x-axis coordinate of the trolley (not the payload), y is the y-axis coordinate of the trolley, α and β are the angle that define the cable orientation and are measured according to Fig. 1a.

The Euler-Lagrange equation is (Sciavicco and Siciliano, 2000):

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\boldsymbol{q}}} - \frac{\partial\mathcal{L}}{\partial\boldsymbol{q}} = \boldsymbol{\tau} \tag{10}$$

or, in scalar form:

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{x}} - \frac{\partial\mathcal{L}}{\partial x} = F_x \tag{11}$$

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{y}} - \frac{\partial\mathcal{L}}{\partial y} = F_y \tag{12}$$

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\alpha}} - \frac{\partial\mathcal{L}}{\partial \alpha} = 0 \tag{13}$$

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\beta}} - \frac{\partial\mathcal{L}}{\partial \beta} = 0 \tag{14}$$

[3] In practice, since the real cable is somewhat flexible, the model is valid only in some neighborhood of zero.

which, when rewritten in matrix form, gives:

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{q})\,\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\,\dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) \tag{15}$$

where

$$\boldsymbol{M}(\boldsymbol{q}) = \begin{bmatrix} m_t + m_p & 0 & R\,m_p\,\cos\alpha\,\cos\beta & -R\,m_p\,\sin\alpha\,\sin\beta \\ 0 & m_t + m_p + m_b & 0 & R\,m_p\,\cos\beta \\ R\,m_p\,\cos\alpha\,\cos\beta & 0 & m_p\,R^2\,\cos^2\beta + I_p & 0 \\ -R\,m_p\,\sin\alpha\,\sin\beta & R\,m_p\,\cos\beta & 0 & m_p\,R^2 + I_p \end{bmatrix}, \tag{16}$$

$$\boldsymbol{C}(\boldsymbol{q},\,\dot{\boldsymbol{q}}) = \begin{bmatrix} 0 & 0 & -R\,m_p\,\cos\beta\,\sin\alpha\,\dot{\alpha} - R\,m_p\,\cos\alpha\,\sin\beta\,\dot{\beta} & -R\,m_p\,\cos\alpha\,\sin\beta\,\dot{\alpha} - R\,m_p\,\cos\beta\,\sin\alpha\,\dot{\beta} \\ 0 & 0 & 0 & -R\,m_p\,\sin\beta\,\dot{\beta} \\ 0 & 0 & -R^2\,m_p\,\cos\beta\,\sin\beta\,\dot{\beta} & -R^2\,m_p\,\cos\beta\,\sin\beta\,\dot{\alpha} \\ 0 & 0 & R^2\,m_p\,\cos\beta\,\sin\beta\,\dot{\alpha} & 0 \end{bmatrix} \tag{17}$$

and

$$\boldsymbol{G}(\boldsymbol{q}) = \begin{bmatrix} 0 \\ 0 \\ R\,g\,m_p\,\sin\alpha\,\cos\beta \\ R\,g\,m_p\,\cos\alpha\,\sin\beta \end{bmatrix} \tag{18}$$

Notice that the model given by Eq. (15) aims the design of a controller and no friction was included. It is possible to modify Eq. (15) to take into account one or more friction sources. This can be important, for example, for actuators specification, but is not important for controller design since friction injects even more passivity into the system, exactly the characteristic which is needed to damp the swinging. So, if the model in open loop is excited by an impulsive force it will oscillate forever because there is nothing to damp the swinging.

Isolating $\ddot{\boldsymbol{q}}$ in Eq. (15) one can obtain:

$$\ddot{\boldsymbol{q}} = \boldsymbol{M}(\boldsymbol{q})^{-1}\,\left(\boldsymbol{\tau} - \boldsymbol{C}(\boldsymbol{q},\,\dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} - \boldsymbol{G}(\boldsymbol{q})\right)\,. \tag{19}$$

Then, the system can be model in the form:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\,\boldsymbol{u} \tag{20}$$

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}) \tag{21}$$

where[4]

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{bmatrix}; \qquad \dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{q}} \\ \ddot{\boldsymbol{q}} \end{bmatrix}; \qquad \boldsymbol{u} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{\tau} \end{bmatrix}; \qquad \boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{0} \\ \dot{\boldsymbol{q}} \end{bmatrix}; \tag{22}$$

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} \dot{\boldsymbol{q}} \\ -\boldsymbol{M}(\boldsymbol{q})^{-1}\,\left(\boldsymbol{C}(\boldsymbol{q},\,\dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q})\right) \end{bmatrix} \quad \text{and} \quad \boldsymbol{g}(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M}(\boldsymbol{q})^{-1} \end{bmatrix}. \tag{23}$$

The matrix $\boldsymbol{M}(\boldsymbol{q})$ in Eq. (16) is symmetric and positive defined.

A very important property of this system is that:

$$\frac{1}{2}\,\dot{\boldsymbol{M}}(\boldsymbol{q}) - \boldsymbol{C}(\boldsymbol{q},\,\dot{\boldsymbol{q}})$$

is skew-symmetric. This propriety is usually referred as *the passivity property of a system* and it is used to prove that an Euler-Lagrange system is passive according to Byrnes et al. (1991). The skew-symmetry property implies that:

$$\boldsymbol{\xi}^T\,\left(\frac{1}{2}\,\dot{\boldsymbol{M}}(\boldsymbol{q}) - \boldsymbol{C}(\boldsymbol{q},\,\dot{\boldsymbol{q}})\right)\,\boldsymbol{\xi} = 0 \tag{24}$$

[4]The vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ shall not be confused with the coordinates x and y is the trolley.

for any vector $\boldsymbol{\xi}$. A second important property is that the time derivative of the potential energy, given by Eq. (7), is:

$$\frac{d\mathcal{U}(\boldsymbol{q})}{dt} = \dot{\mathcal{U}}(\boldsymbol{q},\, \dot{\boldsymbol{q}}) = \dot{\boldsymbol{q}}^T \boldsymbol{G}(\boldsymbol{q}) \tag{25}$$

$$= \begin{bmatrix} \dot{x} & \dot{y} & \dot{\alpha} & \dot{\beta} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ R\,g\,m_p\, \sin\alpha\, \cos\beta \\ R\,g\,m_p\, \cos\alpha\, \sin\beta \end{bmatrix} = \dot{\alpha}\, R\,g\,m_p\, \sin\alpha\, \cos\beta + \dot{\beta}\, R\,g\,m_p\, \cos\alpha\, \sin\beta\,. \tag{26}$$

The total energy of the system is given by:

$$\begin{aligned} E(\boldsymbol{u}) = E(\boldsymbol{q},\, \dot{\boldsymbol{q}}) &= \mathcal{T}(\boldsymbol{q},\, \dot{\boldsymbol{q}}) + \mathcal{U}(\boldsymbol{q}) \\ &= \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q})\, \dot{\boldsymbol{q}} + \mathcal{U}(\boldsymbol{q}) \end{aligned} \tag{27}$$

where $\boldsymbol{M}(\boldsymbol{q})$ is given by Eq. (16). It can proved that $E(\boldsymbol{0}) = 0$ and $E(\boldsymbol{x}) \geq 0$ using Eq. (27). Furthermore, $E(\boldsymbol{x}) > 0$ for $\dot{\boldsymbol{q}} \neq \boldsymbol{0}$ or $\alpha,\, \beta \neq 0$. Computing the time derivative of Eq. (27) and using Eq. (25), (19), (24), and (22), it is obtained:

$$\begin{aligned} \dot{E}(\boldsymbol{q},\, \dot{\boldsymbol{q}}) &= \frac{1}{2}\ddot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q})\, \dot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T \dot{\boldsymbol{M}}(\boldsymbol{q})\, \dot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q})\, \ddot{\boldsymbol{q}} + \dot{\mathcal{U}}(\boldsymbol{q},\, \dot{\boldsymbol{q}}) \\ &= \dot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q})\, \ddot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T \dot{\boldsymbol{M}}(\boldsymbol{q})\, \dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T \boldsymbol{G}(\boldsymbol{q}) \\ &= \dot{\boldsymbol{q}}^T \left(\boldsymbol{M}(\boldsymbol{q})\, \ddot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q})\, \dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) \right) \\ &= \dot{\boldsymbol{q}}^T \left(\boldsymbol{M}(\boldsymbol{q})\, \boldsymbol{M}(\boldsymbol{q})^{-1} \left(\boldsymbol{\tau} - \boldsymbol{C}(\boldsymbol{q},\, \dot{\boldsymbol{q}})\, \dot{\boldsymbol{q}} - \boldsymbol{G}(\boldsymbol{q})\right) + \frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q})\, \dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) \right) \\ &= \dot{\boldsymbol{q}}^T \left(\boldsymbol{\tau} - \boldsymbol{C}(\boldsymbol{q},\, \dot{\boldsymbol{q}})\, \dot{\boldsymbol{q}} - \boldsymbol{G}(\boldsymbol{q}) + \frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q})\, \dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) \right) \\ &= \dot{\boldsymbol{q}}^T \left(\boldsymbol{\tau} + \frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q})\, \dot{\boldsymbol{q}} - \boldsymbol{C}(\boldsymbol{q},\, \dot{\boldsymbol{q}})\, \dot{\boldsymbol{q}} \right) \\ &= \dot{\boldsymbol{q}}^T \boldsymbol{\tau} + \dot{\boldsymbol{q}}^T \left(\frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) - \boldsymbol{C}(\boldsymbol{q},\, \dot{\boldsymbol{q}}) \right) \dot{\boldsymbol{q}} \\ &= \dot{\boldsymbol{q}}^T \boldsymbol{\tau} = \boldsymbol{y}^T \boldsymbol{u} \end{aligned} \tag{28}$$

which is the mechanical power supplied to the system by the actuators. Integrating both sides of Eq. (28) in respect of time, one gets:

$$\int_0^t \dot{E}(\boldsymbol{q},\, \dot{\boldsymbol{q}})\, dt = \int_0^t \boldsymbol{y}^T \boldsymbol{u}\, dt \tag{29}$$

$$E(t) - E(0) = \int_0^t \boldsymbol{y}^T \boldsymbol{u}\, dt \tag{30}$$

which implies that the system is lossless passive according to Definition 2.6 of Byrnes et al. (1991) with storage function $E(\boldsymbol{q},\, \dot{\boldsymbol{q}}) = E(\boldsymbol{x})$. Note that, is some friction terms are included in the model, Eq. (30) will have an additional term in the right side and the system will be strictly passive according to Definition 2.7 of Byrnes et al. (1991). It is general propriety of the Euler-Lagrange systems.

3. PASSIVITY BASED CONTROLLER DESIGN

The controller goal is to compute the forces necessary to be applied by the actuators in the bridge and trolley in order to move the payload to a specified location given by the desired coordinates x_d and y_d. So, the actual and desired position vectors are defined as:

$$\boldsymbol{r} = \begin{bmatrix} x \\ y \end{bmatrix} \qquad \text{and} \qquad \boldsymbol{r}_d = \begin{bmatrix} x_d \\ y_d \end{bmatrix} \tag{31}$$

and the position error and its time derivative are defined as

$$\boldsymbol{e} = \boldsymbol{r}_d - \boldsymbol{r} \qquad \text{and} \qquad \dot{\boldsymbol{e}} = -\dot{\boldsymbol{r}}\,. \tag{32}$$

The system passivity characteristic indicates that the total energy, given by:

$$E(t) - E(0) = \int_0^t \boldsymbol{y}^T \boldsymbol{u}\, dt \tag{33}$$

may be employed in the controller design (Lozano et al., 2000). When the payload completely stops the quantities $\boldsymbol{e}$, $\dot{\boldsymbol{r}}$ and E are null. Hence, the following functions is adopted as a Lyapunov function candidate:

$$V\left(q\,,\dot{q}\right) = \frac{k_E}{2}\,E\left(\boldsymbol{q},\,\dot{\boldsymbol{q}}\right)^2 + \dot{\boldsymbol{r}}^T\,\frac{k_v}{2}\,\dot{\boldsymbol{r}} + \boldsymbol{e}^T\,\frac{k_p}{2}\,\boldsymbol{e}\,. \tag{34}$$

where k_v and k_p are positive defined square matrices[5] and k_E is a positive scalar. Equaition (34) is a vector form for the scalar function proposed by Byrnes et al. (1991) in which the lasts two terms are biquadratics. The output of the controller are the forces applied by the actuator on the mechanical subsystem and they are given by:

$$\boldsymbol{F} = \begin{bmatrix} F_x \\ F_y \end{bmatrix}. \tag{35}$$

Using Eq. (9), (31) and (35) it can shown that:

$$\dot{\boldsymbol{q}}^T\,\boldsymbol{\tau} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{\alpha} & \dot{\beta} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{x} & \dot{y} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} = \dot{\boldsymbol{r}}^T\,\boldsymbol{F}\,. \tag{36}$$

After replace the inverse of $\boldsymbol{M}\left(\boldsymbol{q}\right)$, the Eq. (9), (17), (18) and first and second derivatives in respect to times of Eq. (36) in Eq. (19), it becomes:

$$\ddot{r} = \frac{1}{\det\left(\boldsymbol{M}\left(\boldsymbol{q}\right)\right)}\,\left(\boldsymbol{P}\,\boldsymbol{F} + \boldsymbol{W}\right) \tag{37}$$

where

$$\begin{aligned}
\boldsymbol{P} &= \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}, \qquad \text{and} \qquad \boldsymbol{W} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \qquad \text{and} \\
p_{11} &= \left(m_p\,R^2\,\cos^2\beta + I_p\right)\,\left(m_p\,\left(m_p\,\sin^2\beta + m_t + m_b\right)\,R^2 + I_p\,\left(m_t + m_p + m_b\right)\right) \\
p_{12} &= -R^2\,m_p^2\,\cos\beta\,\sin\alpha\,\sin\beta\,\left(m_p\,R^2\,\cos^2\beta + I_p\right) \\
p_{21} &= p_{12} \\
p_{22} &= I_p^2\,\left(m_t + m_p\right) + R^4\,m_p^3\,\cos^4\beta\,\sin^2\alpha + I_p\,R^2\,m_p^2\,\cos^2\alpha + I_p\,R^2\,m_t\,m_p + \\
&\quad + R^2\,m_p\,\cos^2\beta\,\left(m_t\,m_p\,R^2 + I_p\,\left(2\,m_p\,\sin^2\alpha + m_t\right)\right) \\
w_1 &= R^5\,m_p^3\,\left(m_t + m_b\right)\,\sin\alpha\,\cos^3\beta\,\sin^2\beta\,\dot{\alpha}^2 + I_p\,R^3\,m_p^2\,\left(m_t + m_b\right)\,\sin\alpha\,\cos\beta\,\sin^2\beta\,\dot{\alpha}^2 - \\
&\quad - R^3\,m_p^2\,\left(m_p\,\left(m_t + m_b\right)\,R^2 + I_p\,\left(m_t + m_p + m_b\right)\right)\,\sin\alpha\,\cos^3\beta\,\left(\dot{\alpha}^2 + \dot{\beta}^2\right) - \\
&\quad - I_p\,R\,m_p\,\left(m_p\,\left(m_t + m_b\right)\,R^2 + I_p\,\left(m_t + m_p + m_b\right)\right)\,\left(\sin\alpha\,\cos\beta\,\left(\dot{\alpha}^2 + \dot{\beta}^2\right) + 2\,\cos\alpha\,\sin\beta\,\dot{\alpha}\,\dot{\beta}\right) + \\
&\quad + I_p\,R^2\,m_p^2\,g\,\left(m_t + m_p + m_b\right)\,\sin\alpha\,\cos\alpha\,\left(\sin^2\beta - \cos^2\beta\right) - \\
&\quad - R^4\,m_p^3\,g\,\left(m_t + m_b\right)\,\sin\alpha\,\cos\alpha\,\cos^4\beta - 2\,I_p\,R^3\,m_p^3\,\cos\alpha\,\sin^3\beta\,\dot{\alpha}\,\dot{\beta} \\
w_2 &= \sin\beta\,\Big[I_p\,R^3\,m_p^3\,\left(\left(\cos\alpha\,\cos\beta\,\dot{\alpha} + \sin\alpha\,\sin\beta\,\dot{\beta}\right)^2 - \dot{\beta}^2 - 2\,\left(\cos\alpha\,\cos\beta\,\dot{\alpha}\right)^2\right) - \\
&\quad - I_p\,R^3\,m_p^2\,m_t\,\cos^2\beta\,\left(\dot{\alpha}^2 + \dot{\beta}^2\right) - R^5\,m_t\,m_p^3\,\cos^2\beta\,\left(\cos^2\beta\,\dot{\alpha}^2 + \dot{\beta}^2\right) - \\
&\quad - I_p\,R^2\,m_p^2\,g\,\left(m_t + m_p\right)\,\cos\alpha\,\cos\beta - R^4\,m_t\,m_p^3\,g\,\cos\alpha\,\cos^3\beta - \\
&\quad - I_p\,R\,m_p\,\left(m_t\,m_p\,R^2 + I_p\,\left(m_t + m_p\right)\right)\,\dot{\beta}^2\Big]
\end{aligned} \tag{38}$$

The derivative in respect to times of Eq. (34) is:

$$\begin{aligned}
\dot{V}\left(\boldsymbol{q},\,\dot{\boldsymbol{q}}\right) &= \frac{k_E}{2}\,2\,E\left(\boldsymbol{q},\,\dot{\boldsymbol{q}}\right)\,\dot{E}\left(\boldsymbol{q},\,\dot{\boldsymbol{q}}\right) + \ddot{\boldsymbol{r}}^T\,\frac{k_v}{2}\,\dot{\boldsymbol{r}} + \dot{\boldsymbol{r}}^T\,\frac{k_v}{2}\,\ddot{\boldsymbol{r}} + \dot{\boldsymbol{e}}^T\,\frac{k_p}{2}\,\boldsymbol{e} + \boldsymbol{e}^T\,\frac{k_p}{2}\,\dot{\boldsymbol{e}} && (39) \\
&= k_E\,E\left(\boldsymbol{q},\,\dot{\boldsymbol{q}}\right)\,\dot{\boldsymbol{q}}^T\,\boldsymbol{\tau} + \dot{\boldsymbol{r}}^T\,k_v\,\ddot{\boldsymbol{r}} + \dot{\boldsymbol{e}}^T\,k_p\,\boldsymbol{e} && (40) \\
&= \dot{\boldsymbol{r}}^T\,\left(k_E\,E\left(\boldsymbol{q},\,\dot{\boldsymbol{q}}\right)\,\boldsymbol{F} + k_v\,\ddot{r} - k_p\,e\right) && (41) \\
&= \dot{\boldsymbol{r}}^T\,\left(k_E\,E\left(\boldsymbol{q},\,\dot{\boldsymbol{q}}\right)\,\boldsymbol{I}_2\,\boldsymbol{F} + k_v\,\frac{1}{\det\left(\boldsymbol{M}\left(\boldsymbol{q}\right)\right)}\,\left(\boldsymbol{P}\,\boldsymbol{F} + \boldsymbol{W}\right) - k_p\,e\right) && (42) \\
&= \dot{\boldsymbol{r}}^T\,\left(\left(k_E\,E\left(\boldsymbol{q},\,\dot{\boldsymbol{q}}\right)\,\boldsymbol{I}_2 + \frac{k_v}{\det\left(\boldsymbol{M}\left(\boldsymbol{q}\right)\right)}\,\boldsymbol{P}\right)\,\boldsymbol{F} + \frac{k_v}{\det\left(\boldsymbol{M}\left(\boldsymbol{q}\right)\right)}\,\boldsymbol{W} - k_p\,\boldsymbol{e}\right) = -\dot{\boldsymbol{r}}^T\,k_d\,\dot{\boldsymbol{r}} && (43)
\end{aligned}$$

[5]They can be or not a diagonal matrices or even positive scalars, as adopted in the simulations (see Section 4).

where the Eq. (28), (36), (32) and (37) were applied, $\boldsymbol{I}_2$ is the identity matrix of order 2, and k_d is a positive defined square matrix[6] in order to make the system stable. Note that the passivity propriety implies that $\dot{E}\left(\boldsymbol{q},\,\dot{\boldsymbol{q}}\right) = \dot{\boldsymbol{q}}^T\,\boldsymbol{\tau}$ and it is quite important to derive Eq. (34).

Isolating $\boldsymbol{F}$ in Eq. (43), we have:

$$\boldsymbol{F} \;=\; \boldsymbol{\Omega}^{-1}\left(k_p\,\boldsymbol{e} - k_d\,\dot{\boldsymbol{r}} - \frac{k_v}{\det\left(\boldsymbol{M}\left(\boldsymbol{q}\right)\right)}\,\boldsymbol{W}\right) \tag{44}$$

where

$$\boldsymbol{\Omega} = k_E\,E\left(\boldsymbol{q},\,\dot{\boldsymbol{q}}\right)\,\boldsymbol{I}_2 + \frac{k_v}{\det\left(\boldsymbol{M}\left(\boldsymbol{q}\right)\right)}\,\boldsymbol{P}\,. \tag{45}$$

Equation (44) is the control law employed.

It can be shown that matrix $\boldsymbol{P}$ is positive defined and, as a consequence, also $\boldsymbol{\Omega}$ in Eq. (44) is positive defined because it results from the sum of two positive defined matrices. So, it is assured that the inverse of $\boldsymbol{\Omega}$ exists.

Observing the right hand side of Eq. (43), it can be confirmed that $\dot{V}$ can be null in others points apart from the origin[7]. It is only enough that $\dot{\boldsymbol{r}} = \boldsymbol{0}$ to make $\dot{V} = 0$. Consequently, asymptotic stability of origin cannot be assured by the Lyapunov's theorem (Khalil, 2002, p. 114). However, it can be proved, using LaSalle's invariance theorem (Khalil, 2002, p. 128), that $\boldsymbol{x}$ approaches to $\left[\begin{array}{cccccccc} x_d & y_d & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]^T$ when $t \to \infty$. The proof follows the steps of Lozano et al. (2000).

4. SIMULATION

The system was simulated using Matlab `symulink` (The MathWorks Inc., 2007). Figure 2 shows the block diagram that carries out the physical (mechanical) subsystem. The core of this subsystem is block `cranesys`, shown in magenta in Fig. 2. This block contains an implementation of Eq. (19), and also Eq. (16)[8], (17) and (18), and was written in Matlab metalanguage using a computer algebra system (MuPAD, 2008). The initial conditions can be specified in the cyan blocks, but for the purpose of this work the system was assumed to be at rest in the beginning of the simulations. In the first simulation, the open loop model of the mechanical subsystem shown in Fig. 2 was excited by a time finite duration force to confirm the accuracy of the model. As a result, the output oscillated without damping, as expected. Also, the trolley and bridge moved with an almost constant speed (there are some speed ripple caused by the interaction of the trolley/bridge translation with payload swing).

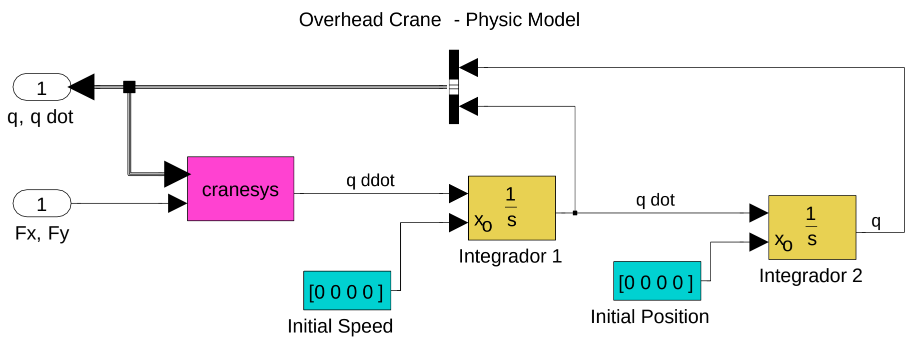


Figure 2: Physic model simulation diagram

The control law, given by Eq. (44), is implemented by four blocks shown in Fig. 3: `Omega`, `Fcc`, `Geneal Inverse (LU)`, and `Matrix Multiply`. The two latter ones are standard Matlab blocks. The two former ones were written in Matlab metalanguage with the help of MuPAD (2008) and implement Eq. (45) and the right term of Eq. (44), respectively. Note that Ω, given by Eq. (45), is a quite complicated 2×2 matrix, but the computational effort the invert it is not too big. It doesn't worth to invert Ω analytically because the result demands much more computation effort to be carried out than the actual scheme that consists in computing Ω, then inverting it. The blocks `x_d` and `y_d` are used to specify the desired final position on plane xy and the block `abqp` converts angles α and β to θ according to Eq. (46).

[6]It can be or not a diagonal matrices or even positive scalars, as adopted in the simulations (see Section 4).

[7]The origin must be dislocated to $\boldsymbol{x}_e = \left[\begin{array}{cccccccc} x_d & y_d & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]^T$ because this is the closed loop equilibrium point

[8]Actually, the inversion of EQ. (16) was implemented.

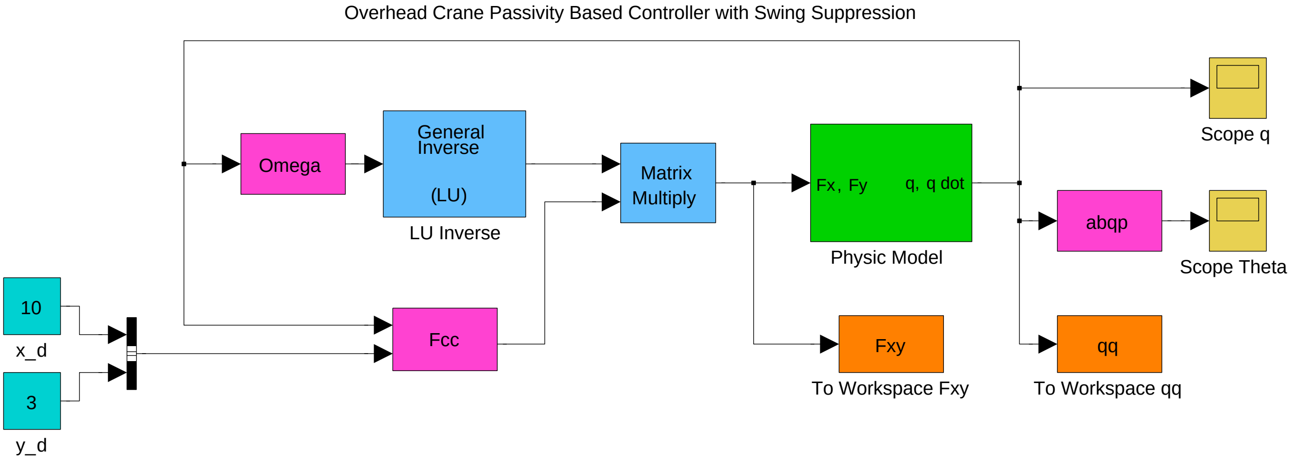


Figure 3: Control simulation diagram

Table 1 brings the simulation parameters and their values. The parameters associated with the plant have the same values employed by Fang et al. (2001).

The simulation scenario used to validate the controller is the execution of a moving command: departing from rest at position $(0,\ 0)$, the payload was sent to position $(x_d;\ y_d) = (10;\ 3)$ m. The same scenario was employed by Fang et al. (2001) and the use of this particular scenario here allows a more straight forward result comparing.

Table 1: Simulation Parameters

Parameters Associated with the Plant			Controller Gains		
Parameters	Symbol	Value	Gains	Symbol	Value
Cable Length	R	$2.5\,\mathrm{m}$	Derivative	k_d	600
Bridge Mass	m_b	$190\,\mathrm{kg}$	Proportional	k_p	120
Trolley Mass	m_t	$23\,\mathrm{kg}$	Energetic	k_E	0.05
Payload Mass	m_p	$160\,\mathrm{kg}$	Non-Linear Derivative	k_v	50
Payload Moment of Inertia	I_p	$1.5\,\mathrm{kg\,m^2}$			
Acceleration Due Gravity	g	$9.81\,\mathrm{m/s^2}$			

Figure 4 shows the position $(x,\ y)$ of the trolley as a time function and the angles α, β . The simultaneous analysis of these four graphs (Fig. 4a to d) allows to evaluate the controller performance in respect to the requirement of moving and swing suppression. Both position, x and y, behaves approximately like an overdamped second order linear system output. The angles shown in Fig. 4c and d display a desired damped behavior.

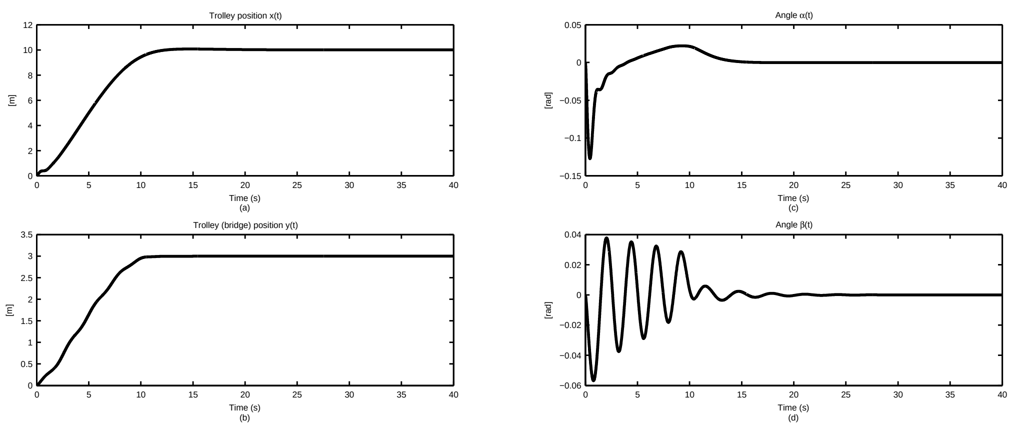


Figure 4: Simulation Results – positions x and y, and angles (α and β)

The force graphs (Fig. 5a and b) give an idea of how the forces delivered by actuators behave.

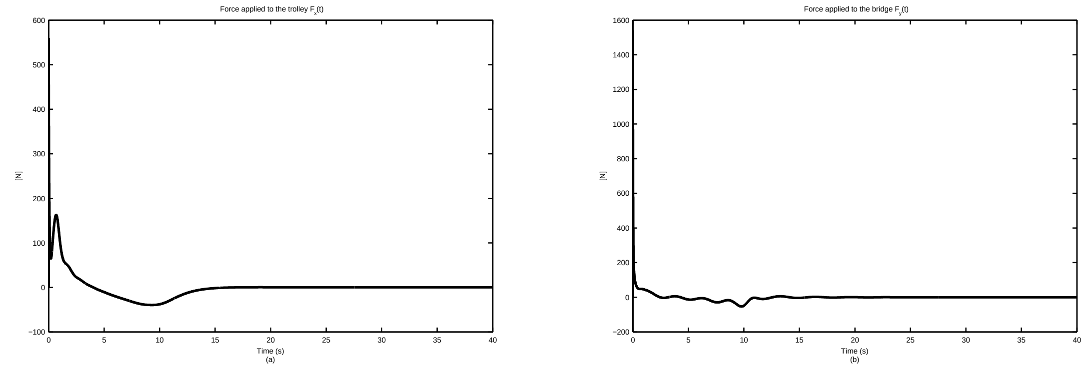


Figure 5: Simulation Results – forces applied by the actuator to (a) the trolley and (b) the bridge

The payload relative position (Fig. 6b) shows how an observer moving with the trolley whould see the payload which gives a good understanding of how the system behaves.

In order to compare the simulation results with the results reported by Fang et al. (2001), which were obtained using a different definition of angles, it is necessary to convert the angle α and β in ϕ and θ as follow:

$$\theta = \arccos\left(\cos\alpha\,\cos\beta\right) \tag{46}$$

$$\phi = \operatorname{atan2}\left(\sin\alpha\,\cos\beta,\,\sin\beta\right)\,. \tag{47}$$

The most important information is given by angle $\theta\,(t)$ which measures the inclination of cable in respect to the vertical line (Fig. 6a). The faster θ goes to zero the higher the swing attenuation. Figure 6a is much simple to analyze than the combined data of Fig. 4c and d.

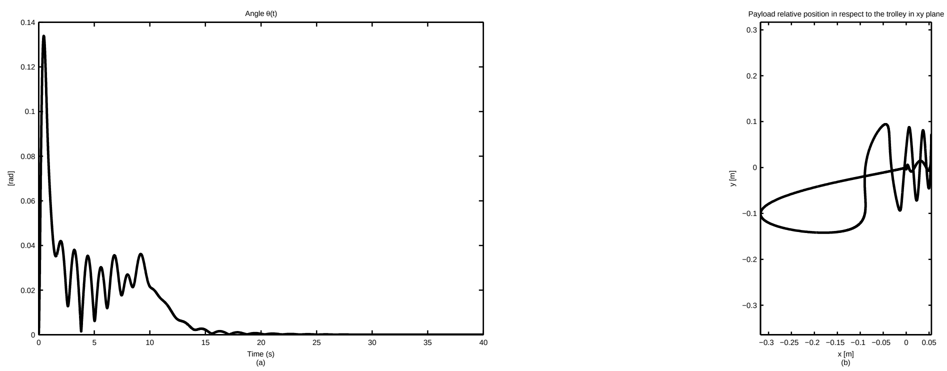


Figure 6: Simulation Results – (a) angles θ and (b) payload relative position

The results obtained are similar to the ones obtained by Fang et al. (2001). The smaller values of θ obtained here can be credited to a difference in controller tuning and not necessarily means that this approach is better than theirs. Neither the smaller setting time obtained by Fang et al. (2001) means that theirs approach is superior. Much need to be done in the tuning procedure before such a conclusion be reached.

5. CONCLUSIONS

The presented results are similar to the results presented by Fang et al. (2001), but the obtained forces (control actions) are smoother than theirs. It is not possible to say yet if the reference angles, as defined in Fig. 1a, have any practical advantages or disadvantages in respect to the angle definitions used by Fang et al. (2001, 2003) and Al-Garni et al. (1995) (see also Fig. 1b). Since the controller was tuned by trial and error, it is believed that it is possible to obtain a better performance than the presented in this work if more time would be spent in the tuning process, possibly by employing a more elaborated tuning method. It is also important to note that application of k_v, k_p, and k_d as scalars consists in a subutilization of the controller capabilities. If k_v, k_p, and k_d are taken as matrices they can be used to compensate the differences perceived by the different actuator and it is very likely that, in this case, the performance will be improved significantly. Figure 6b gives an insight of how the tendency of swing in x and y is not even. Clearly, the swing amplitude in y direction is smaller but also smaller is the damping. This tendency cannot be attributed to the difference in the desired travel distances since the same behavior can be observed if the travel distances are equal. The matrices k_v, k_p, and k_d are of dimension 2×2 what means that, together with the scalar k_E, thirteen parameters, at most, can be used to tune the controller. This can be used in the elaboration of a very sophisticated tuning procedure which shall satisfy very stringent requirements.

The intense application of computer algebra systems has minimized the mistake possibility in the equation derivation and simplified the construction of the simulation blocks written in Matlab metalanguage (The MathWorks Inc., 2007). Besides, the time needed for mathematical manipulation was largely reduced. The main goals of the work: to build up the dynamic model of an overhead crane; to propose a controller and to show by means of simulation and theoretically that this controller can effectively reduce the payload swing, were satisfactorily reached. In future works a more effective method for controller tuning must be developed and others energy or power based Lyapunov function shall be proposed.

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