INFLUENCE OF THE ENTRANCE FLOW FIELD ON THE DEVELOPMENT OF THE VELOCITY PROFILE IN A STRAIGHT PIPE FLOW

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Abstract. The purpose of this paper is to evaluate the influence of the entrance flow field on the development of the turbulent velocity profile in a straight pipe flow, so that meaningful data can be obtained from the computer simulation of the performance of an ultrasonic flow rate meter under different flow conditions. The commercially available software CFX 11.0 was used with RNG k- ε and k- ε models of turbulence, for the flow of water in a pipe at a 150000 Reynolds number. The development of the velocity profile was simulated in the software by specifying different profiles at the entrance of the pipe, (a) Flat velocity profile, (b) Software simulated completely developed velocity profile and (c) Power law (1/n) velocity profile. The flow was considered to be developed when the maximum deviation of the parameter at a given test section was less than 1 % of the value at a distance of 80 diameters (80D) from the entrance, used as a reference value. The distortion was defined as the difference between maximum and minimum values of the parameter before attaining development. Output software values of velocity at centerline and turbulence parameters k and ε were registered for comparison. The average velocity along an inclined path with respect to the pipe axis was calculated using software output data at different sections, and used to simulate the performance of an ultrasonic flow meter. Using a flat velocity profile at the pipe entrance, an overshoot of the centerline velocity profile was observed, which is difficult to explain on a physical basis. The same thing happened when using the power law (1/n) profile. The distortion was significantly reduced when the software simulated completely developed velocity profile and turbulence parameters k and ε were fed back to the entrance of the pipe. However, if only the completely developed velocity profile is fed back, the distortion is still high. The distortion of the average velocity along an inclined line was also shown to depend on the specification of the profile at the pipe entrance, which is bad for simulating the performance of an ultrasonic meter. It was concluded that if meaningful data are to be obtained by computer simulations of the flow field, a very careful specification of the velocity profile and turbulence parameters must be made, so that the errors can be kept to within acceptable limits, which means that the computer simulation models must be validated before analyzing any numerical result. The paper discusses the influence of the computational mesh and boundary conditions on the results, including velocity profile and turbulence parameters. It analyzes the results physically, aiming to understand the reasons for the discrepancies and how to correct them.

Keywords: computer simulation, ultrasonic flow meter, pipe-flow modeling

1. INTRODUCTION

Ultrasonic flow meters are an option for the pipeline industry, because they are robust and, under laboratory conditions, able to attain the international legal metrology requirements. However, they are influenced by flow profile, installation effects are not always possible to be eliminated, and the costs of experimentation are high.

Thus, computer simulation is an interesting option, but it is a challenge, as custody transfer flow meters should have maximum permissible measurement error of 0.2% for liquids and 0.5% for natural gas, and the numerical errors are to the same order of magnitude.

The power law (1/n) profile at the inlet produced significant distortions in the velocity parameters in pipe flow modeling, and the results of a flow-meter simulation could be distorted too. Therefore, a more detailed analysis of the parameters that may influence the simulation results was carried out.

The ultrasonic meter simulation was accomplished with inclined lines passing through the pipe axis, and the average velocity along these lines was used to simulate the velocity measured by the ultrasonic meter.

2. COMPUTER SIMULATION

2.1. Modeling Description

The domain was a 0.250m diameter and 25m long pipe, equivalent to 100 pipe diameters (100D). The software ICEM was used for the mesh, with 197904 elements, 532 in the inlet and 20832 in the wall. The inlet mesh was extruded along the cylinder.

The commercially available software CFX 11.0 (ANSYS CFX, 2007) was used for the simulations. The basic modeling configuration was:

- Geometry
 - 0.25m diameter and 25m long pipe.
- Domain Default
 - Fluid: Water (density = ρ = 997 kg m⁻³; dynamic viscosity = μ = 8.899×10⁻⁴ kg m⁻¹ s⁻¹).
 - \circ Reference Pressure = 1atm.
 - \circ Reynolds Number = 150000.
 - o Advection Scheme: High Resolution (default).
 - Timescale Control: Auto Timescale.
 - Convergence Criteria: RMS 10⁻⁶.
- Output
 - Average Static Pressure = 0Pa.
- Wall
 - No Slip, Smooth Wall.

The modelings were configured with different inlet boundary conditions and turbulence models (k- ε (Launder and Spalding, 1974) and RNG-k- ε (Yakhot and Orszag, 1986)). The modelings nomenclature is:

- PLX: Inlet = 1/7 power law profile, medium turbulence intensity; Turbulence Model = k- ϵ .
- PL: Inlet = 1/7 power law profile, medium turbulence intensity; Turbulence Model = RNG k- ϵ .
- UVWKEX: Inlet = u, v, w, k and ε imported profiles; Turbulence Model = k- ε .
- UVWKE: Inlet = u, v, w, k and ε imported profiles; Turbulence Model = RNG k- ε .
- UVW: Inlet = u, v and w imported profiles, medium turbulence intensity; Turbulence Model = RNG k- ε .
- PLKE: Inlet = 1/7 power law, k and ε imported profiles; Turbulence Model = RNG k- ε .
- FLAT: Inlet = flat velocity profile, medium turbulence intensity; Turbulence Model = RNG k-ε.
- FLATKE: Inlet = flat velocity profile; k and ε imported profiles; Turbulence Model = RNG k- ε .

Charts of dimensionless velocity ($W = w/\overline{w}$) by dimensionless distance (Y = y/D) were made to check the profile development along 100D. The cross-section average velocity (\overline{w}) is calculated from Re, and it was 0.5355 m/s for all pipe flow simulations. The chart in Fig. 1 shows the axisymmetric velocity profiles for the PL modeling. The profile with higher centerline velocity (more than 1.3) is 23D away from the inlet. The inlet profile shows a centerline velocity significantly smaller, close to 1.2.



Figure 1. Velocity profiles along 100D for PL.

Figure 2 presents the centerline velocity along 100D for the pipe-flow modelings described here. The chart shows the centerline dimensionless velocity (W) by the number of diameters (Z = z/D).

PLX and PL were configured with power law velocity profile at the entrance, and showed a development with a maximum value between 17D and 25D, while UVWKEX and UVWKE used imported profiles and obtained a more

stable development along 100D. UVW imported velocity distribution, but k and ε were not imported and UVW showed a significant variation at the centerline velocity along the 100D length.



Figure 2. Centerline dimensionless velocity along a 100D length.

PLKE used the power law profile at the entrance and k and ε imported profiles, and its centerline velocity development showed less distortion than UVW that imported velocity profiles and used default medium turbulence intensity at the entrance.

The difference between PLX and PL is their turbulence model configuration. Their patterns of velocity development are similar, but with two visible differences: the maximum values and the distances from the inlet. Other turbulence models available in CFX followed the same pattern, differing in the maximum value and in the distance from the inlet.

The centerline velocity near the end of the pipe can be considered constant but it is clearly a function of the turbulence model.

PL and UVW were configured with the same turbulence model, but with different entrance profiles, and their maximum values are different, even though they are located at almost the same distance from the inlet.

The pattern of the development of the FLAT pipe- flow modeling, with uniform profile at the inlet is relatively close to the pattern of PLX, PL and UVW.

Therefore, the profile development can be described by few values of centerline dimensionless velocity: W_0 (velocity at 0D); W_{max} (the maximum centerline velocity); W_{min} (the minimum centerline velocity); and W_{80} (velocity at 80D). Also, the following parameters were defined to quantify the deviations: E_{max} (maximum value deviation); E_{min} (minimum value deviation); and E_{total} (total deviation or distortion).

The distortion is here defined as the sum of the module of E_{min} and E_{max} . All the deviations were calculated referring to the centerline velocity at 80D. Table 1 shows the principle values of velocity and the deviations for the flow modelings illustrated in Fig. 2.

2.2. Velocity Values

The dimensionless centerline velocity for Reynolds number 150000 was calculated in this paper for comparison purposes for different models resulting in 1.174 for the Nikuradse's, 1.187 for Bogue-Metzner's, 1.174 for Reichard's and 1.225 for the 1/7 power law model. (Kays, 1993) (Bogue and Metzner, 1963) (Schlichting, 1968) Therefore, these values can be compared with the values in Tab. 1.

The values of W_{80} in Tab. 1, in conformity with the chart in Fig. 2, indicate that the centerline velocity of profiles developed for the k- ε turbulence model is 1.161, whereas for the RNG k- ε model it is around 1.177. Comparing these values to those obtained from Nikuradse, Bogue-Metzner and Reichard, the values of the RNG k- ε model are more coherent, with deviations of -0.84% and 0.26%.

Mattingly and Yeh (1999) tested ultrasonic flow meters and they modeled fully developed pipe flows. Pipe flows were measured using laser Doppler velocimetry (LDV) along horizontal and vertical diameters. The Bogue-Metzner distribution attained an approximation to experimental results better than other distributions.

Here, the results of Bogue-Metzner distribution were closer to RNG k-ε distribution than to k-ε distribution.

The value of the centerline velocity of the pipe-flow modelings configured with the power law profile at the inlet differs from the value of centerline velocity of the power law model by -0.57%. This difference is basically due to the mesh and it could be reduced if necessary.

For PLX and PL, the difference between values of W_{80} and the centerline velocity of the 1/7 power law profile is -3.8% for the RNG k- ε model and -4.5% for the k- ε model and, therefore, following the criterion that the flow can be considered developed if the analyzed parameter varies less then 1%, the power 1/7 law profile can not be considered a developed profile.

Modeling	W_0	W_{max}	W_{min}	W_{80}	E_{max} (%)	E_{min} (%)	E_{total} (%)
PLX	1.218	1.316	1.148	1.161	13.39	-1.16	14.55
PL	1.218	1.331	1.157	1.178	13.18	-1.57	14.76
UVWKEX	1.162	1.164	1.160	1.161	0.30	-0.08	0.38
UVWKE	1.177	1.180	1.176	1.177	0.23	-0.08	0.31
UVW	1.178	1.295	1.160	1.179	9.84	-1.61	11.64
PLKE	1.218	1.218	1.173	1.177	3.48	-0.34	3.82
FLAT	1.000	1.232	1.168	1.176	4.76	-0.68	5.44
FLATKE	1.000	1.215	1.170	1.175	3.40	-0.42	3.82

Table 1. Centerline dimensionless velocities and distortions.

Table 1 indicates that the distortion of PLX and PL are over 14%, and even the distortion of UVW is over 11%. These values are high for different simulations and not only for flow-meter related simulations.

Profile importation is considered the best solution to obtain a fully developed profile along the pipe, but the imported profile itself shows some numerical errors. The profile at 80D of PLX was exported to UVWKEX, and the profile at 80D of PL was exported to UVWKE and UVW, and differences of 0.08% may appear if the velocity values at 80D of PLX (or PL) are compared with the inlet velocity values of UVWKEX (or UVWKE and UVW).

Maximum and minimum deviations occurred in the following positions: 18D and 36D for PLX, and 23D and 42D for PL and UVW. In the case of development of the uniform profile with the RNG k- ε turbulence model, these maximums were at 34D and 58D. The values of W_0 are different for each modeling, but the slopes during the first 20D, are very similar.

The distortion for the FLAT modeling was 5.44%, and the FLATKE distortion was 3.82%. However, this is not a solution, because the difference is already very high. The k and ε parameters were imported from the profile at 80D of the PL.

In the case of the FLAT pipe flow modeling, the aim was not to obtain a developed profile along the pipe, but to obtain a profile in development with the objective of evaluating the performance of the meter in this situation. Therefore, to calculate the deviation of the minimum value and the distortion, only the velocities from 25D to 100D were considered.

Similar reasoning cannot be used in the analysis of PLX and PL, because the maximums are greater than the initial value, but could be used with PLKE. If PLKE were to be considered a profile in development, the deviation of the maximum value and the distortion would change to 0.08% and 0.42%, respectively.



Figure 3. Velocity profile along a 100D length for UVW.

The UVWKE modeling only differs from the PL by the import of the u, v, w, k and ε profiles. In other words, these imported profiles reduce the centerline velocity distortions from 14.76% to 0.31%.

Based on the criterion of variation of $\pm 1\%$ at the centerline velocity to consider the flow as fully developed, the pipe-flow modelings can be considered fully developed only from certain pipe positions far from the inlet: 42D for PLX; 52D for PL; 51D for UVW; 6D for PLKE; 45D for FLAT; and 43D for FLATKE.

The pipe flow of UVWKEX and UVWKE modelings may be considered fully developed along the 100D if the criterion of variation of $\pm 1\%$ in the centerline velocity were used. They would be considered fully developed even within the interval of $\pm 0.5\%$. Figure 3 shows the profile development for the UVW.

Based on the values of centerline velocity along the pipe, the flow at 80D is considered as fully developed for all modeling flows.

2.3. Small Mesh Troubles

Results of different meshes were compared, varying the number of elements, and the cross-section or wall mesh. A small difference in the results from one mesh to another may occur, but the same pattern in the profile development was maintained.

Searching for the minor errors, two items were observed that generated deviations easy to avoid. One is the appearance of little noises, as shown in Fig. 4. They occur at the borders of primitive meshes. These peaks were generated in a domain, formed by an assembly with four primitive meshes. The primitive mesh had only 25D and had to be reflected by reaching 100D. The inlet and the outlet meshes of the primitive meshes were exactly the same. The velocity peaks, generated at these borders, are approximately 0.04% of the expected velocity.



Figure 4. Centerline velocity along a 100D length for UVWKE.

A mesh influence on the turbulent kinetic energy contour was verified, too. The mesh of the cross-section was built of 5 blocks. The velocity contour did not indicate mesh influence, but the mesh influence on the turbulent kinetic energy was visible. Fig. 5 shows the mesh of the cross-section and its influence on the κ contour.



Figure 5. Mesh and the contour of turbulent kinetic energy (κ) at 80D for PL.

2.4. Pressure and Turbulence Parameters

Velocity profile does not change alone; when there are changes in velocity, others parameters change too. Total pressure (p_{tot}) , defined here as the pressure that would exist in one point, if the fluid dynamic energy due to flow were converted without losses, is a function of the velocity (U): $p_{tot} = p_{stat} + \rho(U \cdot U)/2$. Therefore, total pressure will shows distortions too.

The chart in Fig. 6 presents the total pressure (blue) and the static pressure (red) for UVWKE (left) and PL (right) along the pipe. The static and total pressures decline continually in UVWKE, but the total pressure remains practically constant in PL until 23D, that is, where the centerline velocity is ascendant.



Figure 6. Total pressure (blue) and static pressure (red) for UVWKE (left) and PL (right).

Likewise, the turbulent kinetic energy shows a different pattern of development if the flow is not fully developed along the entire pipe. Turbulent kinetic energy remains almost constant, varying from $6.04 \times 10^{-4} \text{ m}^2/\text{s}^2$ to $6.21 \times 10^{-4} \text{ m}^2/\text{s}^2$, for the UVWKE. However, when velocity distortions occur, *k* varies from $1.25 \times 10^{-4} \text{ m}^2/\text{s}^2$ to zero, returning to $6.0 \times 10^{-4} \text{ m}^2/\text{s}^2$, as seen in Fig. 7. The development of turbulent viscosity (μ_t) presents the same pattern of the *k* development.



Figure 7. Turbulent kinetic energy (k) for UVWKE (left) and PL (right).

Therefore, in the first 23 diameters of PL pipe-flow modeling, the centerline velocity will increase, while the total pressure remains constant and the turbulent viscosity and the kinetic energy drop to zero. In other words, there would be no turbulence far from the wall. Yet, the flow for UVWKE will be turbulent all over the pipe.

PLX and PL show distortion in k and ε above 100%, while k and ε variations for UVWKEX and UVWKE do not go beyond 5%. Fig. 8 shows the ε development for UVWKE and PL.

The k and ε patterns are similar for PLX, PL and UVW modelings. On the other hand, UVWKEX, UVWKE and PLKE modelings show that k and ε have similar patterns, too.

In those cases where the values of k and ε were not imported, entrance default with an average intensity of 5% was used, with the consequence that the inlet profile becomes proportional to the velocity, with high turbulent kinetic energy

in the centre and zero at the walls, but after 1D the situation has already been reverted. Using in the inlet average intensity of 1% or 10%, the results are close to the results with average intensity of 5%.



Figure 8. Turbulent eddy dissipation (ɛ) for UVWKE (left) and PL (right).



Figure 9. Velocity profiles (left) and total pressure profiles (right) for FLAT.

The behavior of k and ε along the pipe axis for the FLAT pipe-flow modeling shows the same standard as PLX and PL. However, the pipe-flow modelings configured with a uniform distribution at the inlet do not have the highest velocity of the profiles at the centerline. As shown in Fig. 9, the sides of the profiles are slightly higher at 0D, 1D, 5D and 10D.

It can be observed in Fig. 9 the total pressure profiles development along the pipe. Their sides are slightly higher at 0D, 1D, 5D and 10D, too.

2.5. Average Velocity in Inclined Lines

The transit-time ultrasonic meter comprises one or more pairs of transducers. Each transducer sends and receives acoustic pulses in high frequency, transversally to the pipe. When the pulse travels in the flow direction (in Fig. 9, from T1 to T2) it spends less time than when it travels in the opposite direction (from T2 to T1). The difference between these propagation times will be proportional to the average velocity of the particles on the acoustic way, in the time interval in which the pulses cross the pipe. Ultrasonic meters are influenced by the flow profile and researches have been accomplished to quantify this influence. (Yeh and Mattingly, 1997) (Moore *et al.* 2000)

The simulation of these meters was made here by tracing along the pipe 100 straight lines with the inclination of 45°. The average velocity was calculated for each of the lines. The average velocity in one inclined line (\overline{w}_T) is not equal to the average velocity in the cross-section (\overline{w}), but there exists a relationship between them. This relationship has been theoretically analyzed by Orlando and Val (2006). The correction factors may be determined in calibration.

The parameter used to evaluate the meters will be the average dimensionless velocity in the inclined lines $(W_{T=} \overline{w}_T / \overline{w})$. Distortions in the line velocities were evaluated using as a reference the value of the line velocity at 80D from their own modeling.



Figure 10. Scheme of a transit-time ultrasonic meter.

The deviations in the line average velocities for PLX, PL, UVWKEX, UVWKE, UVW and PLKE are shown in the chart in Fig. 11. From 60D to 100D, deviations can be considered zero for all the pipe-flow modelings. Significant correlation between the development of line velocity and centerline velocity is seen comparing Fig. 11 with Fig. 2. Line velocity deviations are smaller than the centerline velocity deviations, but they are significant for custody transfer meter simulation for the modelings that did not import the *k* and ε developed profiles.



Figure 11. Line velocities along a 100D length for fully developed pipe-flow modelings.

Table 2. Distortions in line velocities along	100D for fully developed	pipe-flow modelings.
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Modeling	$E_{maxT}(\%)$	E_{minT} (%)	W_T distortion (%)	W distortion (%)
PLX	3.45	-0.41	3.86	14.55
PL	3.25	-0.60	3.85	14.76
UVWKEX	0.14	-0.03	0.17	0.38
UVWKE	0.12	-0.03	0.15	0.31
UVW	2.62	-0.51	3.13	11.64
PLKE	0.67	-0.13	0.80	3.82

Table 2 shows the deviation values for the line velocities: E_{maxT} (deviation of maximum value of W_T), E_{minT} (deviation of minimum value of W_T), distortion in W_T and distortion in W.

The UVWKEX and UVWKE pipe-flow modelings presented the best results. Their centerline velocity distortions were 0.38% and 0.31% and their line velocity distortions were 0.17% and 0.15%. Values to the order of 0.17% and 0.15% are already very close to the maximum permissible measurement error of 0.2%, but they may be accepted.

A distortion in W_T above 3%, as happens with PLX, PL and PLKE, is excessive once the use of the 1/7 profile would provide a fully developed flow over the entire domain. But even for flows in development, the average flow velocity does not vary, so the velocity in the lines that simulate the flow meters should not vary so much.

The line velocity distortion for the modelings with uniform profile at the inlet is shown in Fig. 12. It can be observed that the distortions in the development of the uniform distributions are smaller than the distortions generated in the development of the 1/7 power law distributions.



Figure 12. Line velocities along a 100D length for pipe-flow modelings with uniform profile at the entrance.

Table 3. Distortions in line velocities along a 100D length for pipe-flow modelings with uniform profile at the entrance.

Modeling	$E_{maxT}(\%)$	E_{minT} (%)	W_T distortion (%)	W distortion (%)
FLAT	1.16	-0.21	1.37	5.44
FLATKE	0.91	-0.17	1.08	3.82

Table 3 shows that the distortion in W_T for FLAT pipe-flow modeling is 1.37%, and so, it is smaller than the distortion for PL (Tab. 2). However, the measurement of flow velocity through calculation of the line velocity for FLAT flow modeling at the beginning of the pipe is approximately –5.5%, as shown in Fig. 12.



Figure 13. Meter factor along the pipe.

The profile of a uniform distribution is different from the profile of a fully developed flow, but if the cross-section average velocity were the same, it would be interesting if the meter could give the same result. But this does not happen.

It is not possible to assure that a flat profile would be developed along a 100D pipe in exactly the same way they did in the FLAT or FLATKE pipe-flow modeling, but it is possible to say that if inclined lines could be "calibrated" in a flow with a fully developed profile to evaluate the average flow velocity, and then they were "installed" along the 100D pipe with the profiles of FLAT or FLATKE modelings they would give the errors shown in Fig. 12.

Figure 13 presents meter factors for PLX, PL, UVWKEX, UVWKE, UVW and PLKE. They are the opposite of the W_T velocity. This factor would be found if the "lines" could be "calibrated". It has been used to evaluate the influence, for example, of the number of Reynolds in measurement of values.

3. CONCLUSIONS

The distortions of the profile along the pipe using the k-ɛ model or the RNG-k-ɛ model are similar.

The best result for pipe flow modeling along the 100D length was achieved by using u, v, w, k and ε imported fully developed profiles.

For pipe flow modeling of a fully developed flow along the 100D length, it is more efficient to configure the inlet with power law profiles and import the fully developed k and ε profiles, instead of configuring the inlet with imported fully developed u, v and w profiles and using the medium turbulence intensity.

The distortion in inclined line velocity is lower than the distortion in the centerline velocity, but it is already high and the correlation between them is significant.

The distortions in centerline velocity or in line velocity for the modelings with power law profile at the entrance are higher than these distortions for the modelings with uniform profile at the entrance.

In the modelings with significant distortions along the first pipe diameters, the total pressure in the center of the pipe remains constant, the turbulent kinetic energy and turbulent viscosity drop to zero, and the line velocities and the centerline velocity rise continuously. This condition is interrupted with the abrupt increase of turbulent kinetic energy, a significant fall in dynamic pressure and a fall in line velocity and centerline velocity.

In simulations of ultrasonic measurement along a 100D long pipe, only UVWKEX and UVWKE pipe-flow modelings reach distortions lower than the maximum permissible measurement error for meters of custody transfer.

When the analysis concerns the error produced by using a line velocity to measure the average flow profile, it refers to limitations of the measurement method, but when the analysis is about the error produced by the distortions in a fully developed flow, it refers to the modeling (configuration or limitations of the modeling method).

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