A NUMERICAL STUDY OF THE DISPERSION OF A CLOUD OF PARTICLES

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Abstract. *The passage of a particle by a larger body modifies the velocity, energy and angular momentum of this particle. There are many missions that used this concept, like the Voyager I and II that used successive close encounters with the giant planets to make a long journey to the outer Solar System. In this research we study the passage of a cloud of particles near a celestial body. This is the situation that occurs when a fragmented comet crosses the orbit of a planet like Jupiter, Saturn, etc. It is assumed that the dynamical system is formed by two main bodies that are in circular orbits around their center of mass and a cloud of particles that is moving under the gravitational attraction of the two primaries. The motion is assumed to be planar for all the particles and the dynamics given by the "patched-conic" approximation is used, which means that a series of two-body problems are used to generate analytical equations that describe the problem.*

Keywords*: Astrodynamics, Orbital maneuvers, Swing-By, Gravity assisted maneuvers, Orbital motion.*

1. INTRODUCTION

In astronautics, the close approach between a spacecraft and a planet is a very popular technique used to decrease fuel expenditure in space missions. This maneuver modifies the velocity, energy and angular momentum of a spacecraft. There are many important applications very well known, like the Voyager I and II that used successive close encounters with the giant planets to make a long journey to the outer Solar System; the Ulysses mission that used a close approach with Jupiter to change its orbital plane to observe the poles of the Sun, etc. Some examples of applications can be found in D'Amario, Byrnes, and Stanford (1982), Swenson (1992), Weinstein (1992), Farquhar and Dunham (1981) and Farquhar, Muhonen, and Church (1985).

In the present paper we study the close approach between a planet and a cloud of particles. It is assumed that the dynamical system is formed by two main bodies (usually the Sun and one planet) that are in circular orbits around their center of mass and a cloud of particles that is moving under the gravitational attraction of the two primaries. The motion is assumed to be planar for all the particles and the dynamics given by the "patched-conic" approximation is used, which means that a series of two-body problems are used to generate analytical equations that describe the problem. The standard canonical system of units is used and it implies that the unit of distance is the distance between the two primaries and the unit of time is chosen such that the period of the orbit of the two primaries is 2π.

The goal is to study the change of the orbit of this cloud of particles after the close approach with the planet. It is assumed that all the particles that belong to the cloud have semi-major axis $a \pm \Delta a$ and eccentricity $e \pm \Delta e$ before the close approach with the planet. It is desired to known those values after the close approach.

Among the several sets of initial conditions that can be used to identify uniquely one swing-by trajectory, a modified version of the set used in the papers written by Broucke (1988), Broucke and Prado (1993) and Prado (1993) is used here. It is composed by the following three variables: 1) V_p, the velocity of the spacecraft at periapse of the orbit around the secondary body; 2) The angle ψ, that is defined as the angle between the line M_1-M_2 (the two primaries) and the direction of the periapse of the trajectory of the spacecraft around M_2; 3) r_p, the distance from the spacecraft to the center of M_2 in the moment of the closest approach to M_2 (periapse distance). The values of V_p and ψ are obtained from the initial orbit of the spacecraft around the Sun using the "patched-conics" approximation and r_p is a free parameter that is varied to obtain the results.

The new aspect of the present research, when compared to the references cited, is the global study of a cloud of particles instead of the single particle study. With those results it is possible to predict the behaviour of many particles and how the close approach changes the relative positions of the particles,

2. ORBITAL CHANGE OF A SINGLE PARTICLE

This section will briefly describe the orbital change of a single particle subjected to a close approach with the planet under the "patched-conics" model. It is assumed that the particle is in orbit around the Sun with given semi-major axis (a) and eccentricity (e). The swing-by is assumed to occur in the planet Jupiter for the numerical calculations shown below, but the analytical equations are valid for any system of primaries. The periapse distance (r_p) is assumed to be known. As an example for the numerical calculations, the following numerical values are used: a = 1.2 canonical

units, e = 0.3, μ_J = 0.00094736, r_p = 0.0001285347 (100000 km = 1.4 Jupiter's radius), where μ_J is the gravitational parameter of Jupiter in canonical units (total mass of the system equals to one).
The first step is to obtain the energy (EB) and angular momentum (CB) of the particle before the swing-by. They are given by

$$EB = -\frac{(1-\mu_J)}{2a} = -0.4162, \quad CB = \sqrt{(1-\mu_J)a(1-e^2)} = 1.0445 \tag{1}$$

Then, it is possible to calculate the magnitude of the velocity of the particle with respect to the Sun in the moment of the crossing with Jupiter's orbit (V_i), as well as the true anomaly of that point (θ). They come from

$$V_i = \sqrt{(1-\mu_J)\left(\frac{2}{r_{SJ}} - \frac{1}{a}\right)} = 1.0796 \text{ and } \theta = \cos^{-1}\left[\frac{1}{e}\left(\frac{a(1-e^2)}{r_{SJ}} - 1\right)\right] = 1.2591 \tag{2}$$

using the fact that the distance between the Sun and Jupiter (r_{SJ}) is one and taking only the positive value of the true anomaly.

Next, it is calculated the angle between the inertial velocity of the particle and the velocity of Jupiter (the flight path angle γ), as well as the magnitude of the velocity of the particle with respect to Jupiter in the moment of the approach (V_∞). They are given by (assuming a counter-clock-wise orbit for the particle)

$$\gamma = \tan^{-1}\left[\frac{e\sin\theta}{1+e\cos\theta}\right] = 0.2558 \text{ and } V_\infty = \sqrt{V_i^2 + V_2^2 - 2V_i V_2 \cos\gamma} = 0.2767 \tag{3}$$

using the fact that the velocity of Jupiter around the Sun (V_2) is one. Fig. 1 shows the vector addition used to derive the equations.

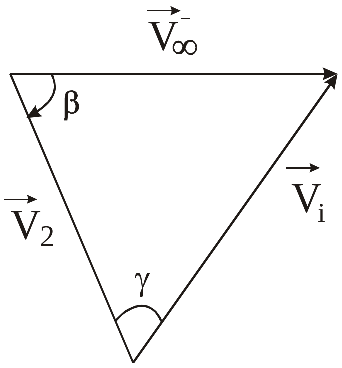


Fig. 1 – Vector addition during the close-approach.

The angle β shown is given by $\beta = \cos^{-1}\left[-\frac{V_i^2 - V_2^2 - V_\infty^{-2}}{2V_2 V_\infty^-}\right] = 1.7322$.

Those information allow us to obtain the turning angle (2δ) of the particle around Jupiter, from
$\delta = \sin^{-1}\left(1 \Big/ 1 + \frac{r_p V_\infty^2}{\mu_J}\right) = 1.4272$.

The angle of approach (ψ) has two values, depending if the particle is passing in front or behind Jupiter. These two values will be called ψ_1 and ψ_2. They are obtained from $\psi_1 = \pi + \beta + \delta = 6.3011$ and $\psi_2 = 2\pi + \beta - \delta = 6.5882$.

The correspondent variations in energy and angular momentum are obtained from the equation
$\Delta C = \Delta E = -2\, V_2\, V_\infty \operatorname{sen}\delta \operatorname{sen}\psi$ (since ω = 1). The results are: $\Delta C_1 = \Delta E_1 = -0.009811$ and $\Delta C_2 = \Delta E_2 = -0.1644$.

By adding those quantities to the initial values we get the values after the swing-by. They are: E_1 = -0.4260, C_1 = 1.0346, E_2 = -0.5806, C_2 = 0.8801.

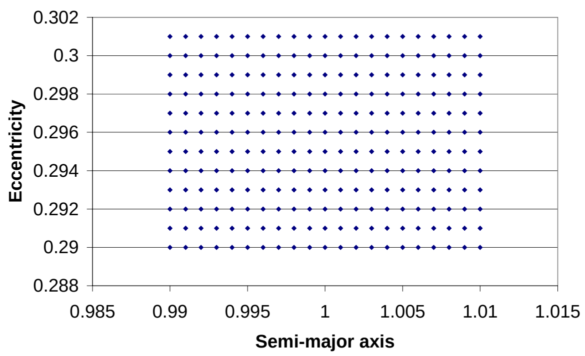
Finally, to obtain the semi-major axis and the eccentricity after the swing-by it is possible to use the equations $a = -\frac{\mu}{2E}$ and $e = \sqrt{1 - \frac{C^2}{\mu a}}$. The results are: a_1 = 1.1723, e_1 = 0.2937, a_2 = 0.8603, e_2 = 0.3144.

3. ORBITAL CHANGE OF A CLOUD OF PARTICLES

The algorithm just described can now be applied to a cloud of particles passing close to Jupiter and Saturn. The idea is to simulate a cloud of particles that have orbital elements given by: a ± Δa and e ± Δe. The goal is to map this cloud of particles to obtain the new distribution of semi-major axis and eccentricities after the swing-by. Fig. 2(a-f) show some results for Jupiter, for the case Δa = Δe = 0.001, r_p = 1.1 R_J and 5.0 R_J.

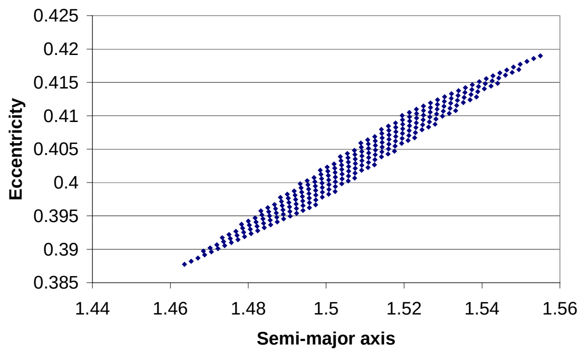
There are two solutions, depending if the particle passes in front of the larger mass or behind it. It causes the gain or loss of energy and generates different possible applications. Those figures allow us to get some conclusions. The solution called "Solution 1" has larger amplitude than the Solution 2 in both orbital elements, but it concentrates the orbital elements in a line, while the so-called "Solution 2" generates a distribution close to a square. The area occupied by the points is smaller for "Solution 1". Both vertical and horizontal lines are rotated and become diagonal lines with different inclinations. The effect of increasing the periapsis distance is to generate plots with larger amplitudes, but with the points more concentrated, close to a straight line.

Those plots and next ones has the same type of information. Every dot corespond to one trajectory and they are mapped from the initial to the final orbit after the gravitational effects of the close approach. So, the reader can observe the orbital elements of the particle before and after the close approach.



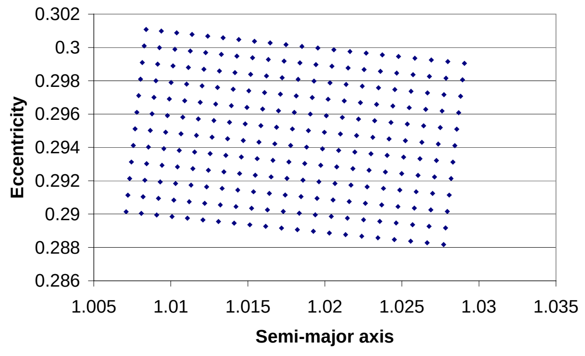
Eccentricity vs. Semi-major axis before the Swing-By

Fig. 2(a) – Swing-by for a cloud of particles with Jupiter for rp = 1.1 R_J



Eccentricity vs. Semi-major axis after Swing-By for "Solution 1"

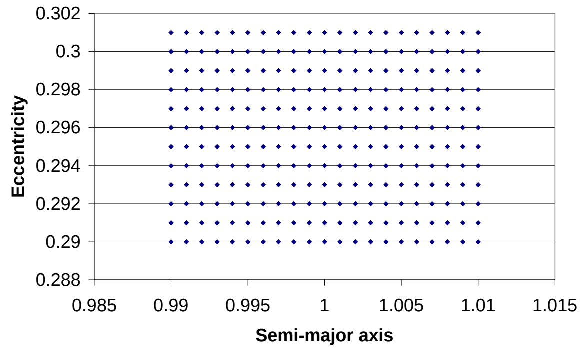
Fig. 2(b) – Swing-by for a cloud of particles with Jupiter for rp = 1.1 R_J



Eccentricity vs. Semi-major axis after Swing-By for "Solution 2"

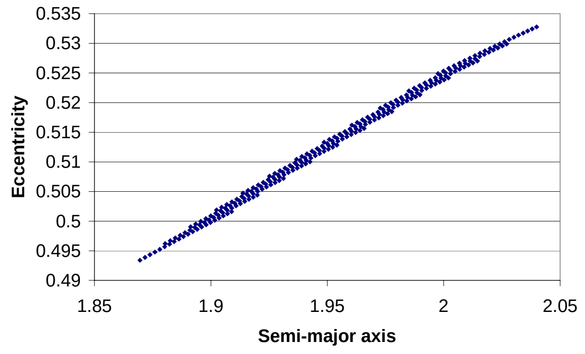
Fig. 2(c) – Swing-by for a cloud of particles with Jupiter for rp = 1.1 R_J

To understand better the importance of the paeriapsis distance, we also made simulations using the value of r_p = 5.0 R_J. The results are shown below.



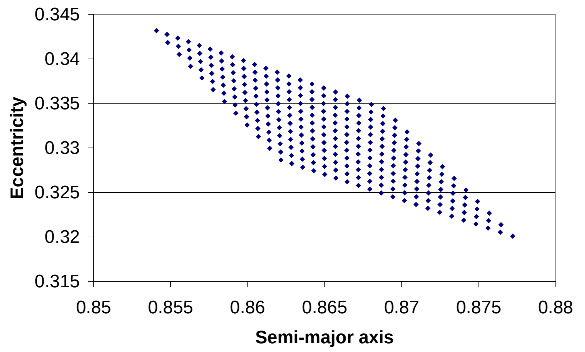
Eccentricity vs. Semi-major axis before the Swing-By

Fig. 2(d) – Swing-by for a cloud of particles with Jupiter for rp = 5.0 R_J



Eccentricity vs. Semi-major axis after Swing-By for "Solution 1"

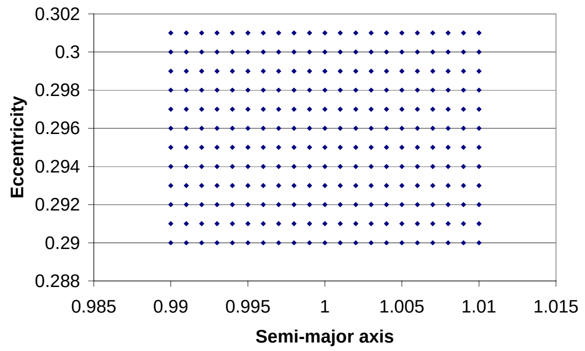
Fig. 2(e) – Swing-by for a cloud of particles with Jupiter for rp = 5.0 R_J



Eccentricity vs. Semi-major axis after Swing-By for “Solution 2”

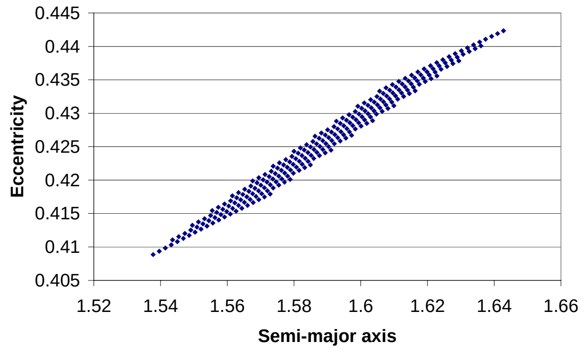
Fig. 2(f) – Swing-by for a cloud of particles with Jupiter for rp = 5.0 R_J

The following figures 3(a-f) show the equivalent results for Saturn. The idea is to verify the differences that are obtained when the main planet is changed, which means that we see the effects of a different mass for the planet.



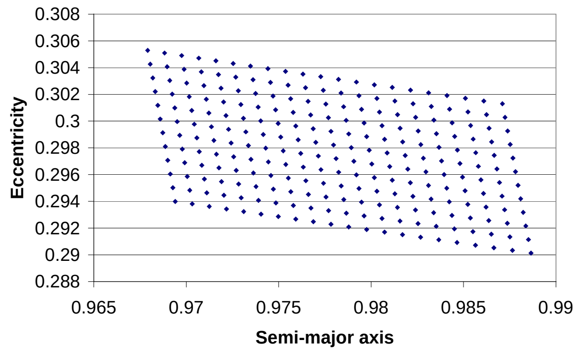
Eccentricity vs. Semi-major axis before the Swing-By

Fig. 3(a) – Swing-by for a cloud of particles with Saturn for rp = 1.1 R_S



Eccentricity vs. Semi-major axis after Swing-By for “Solution 1”

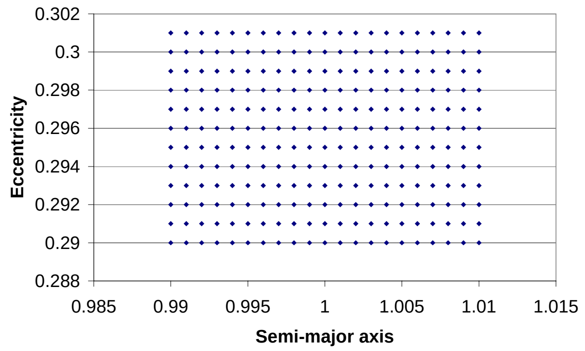
Fig. 3(b) – Swing-by for a cloud of particles with Saturn for rp = 1.1 R_S



Eccentricity vs. Semi-major axis after Swing-By for "Solution 2"

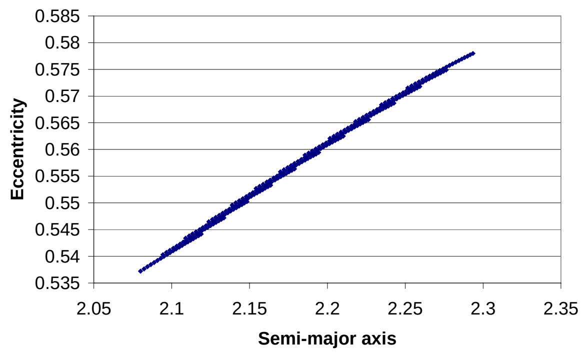
Fig. 3(c) – Swing-by for a cloud of particles with Saturn for rp = 1.1 R_S

To understand better the importance of the paeriapsis distance, we also made simulations using the value of r_p = 5.0 R_S for Saturn. The results are shown below.



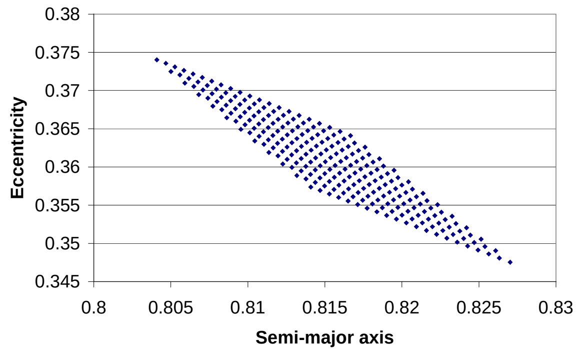
Eccentricity vs. Semi-major axis before the Swing-By

Fig. 3(d) – Swing-by for a cloud of particles with Saturn for rp = 5.0 R_S



Eccentricity vs. Semi-major axis after Swing-By for "Solution 1"

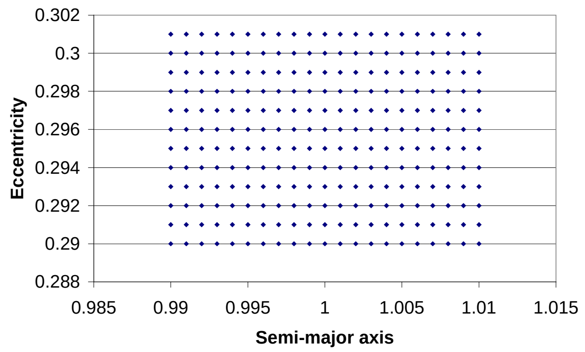
Fig. 3(e) – Swing-by for a cloud of particles with Saturn for rp = 5.0 R_S



Eccentricity vs. Semi-major axis after Swing-By for "Solution 2"

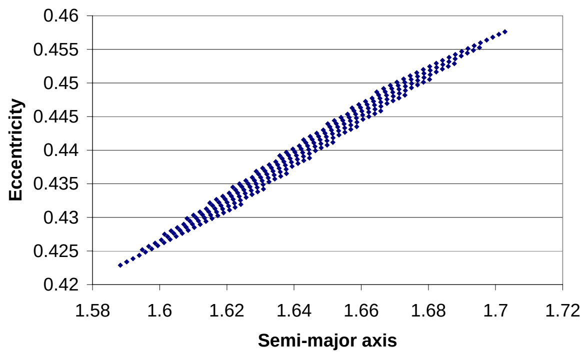
Fig. 3(f) – Swing-by for a cloud of particles with Saturn for rp = 5.0 R_S

In order to make this study more general, we also made some simulations for the planet Uranus. The following figures 4(a-f) show the equivalent results for this planet. The idea is also to verify the differences that are obtained when the main planet is changed, which means that we see the effects of a different mass for the planet.



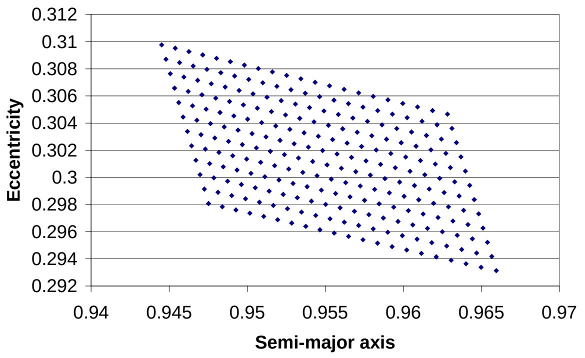
Eccentricity vs. Semi-major axis before the Swing-By

Fig. 4(a) – Swing-by for a cloud of particles with Uranus for rp = 1.1 R_U



Eccentricity vs. Semi-major axis after Swing-By for "Solution 1"

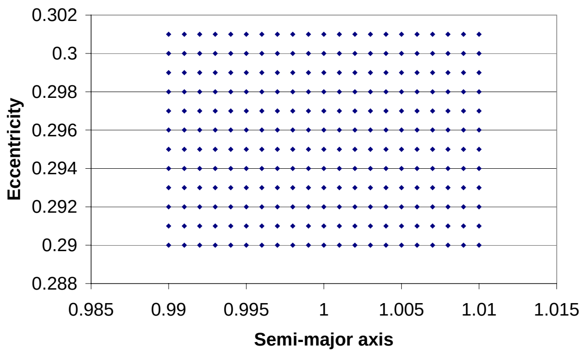
Fig. 4(b) – Swing-by for a cloud of particles with Uranus for rp = 1.1 R_U



Eccentricity vs. Semi-major axis after Swing-By for "Solution 2"

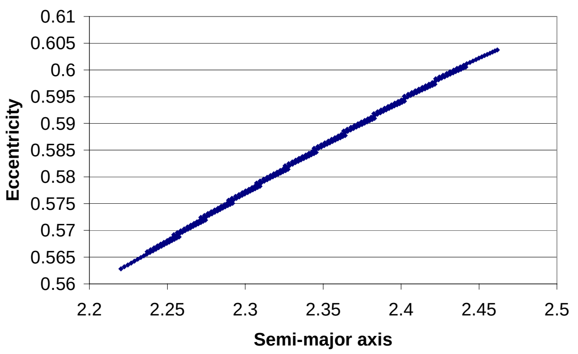
Fig. 4(c) – Swing-by for a cloud of particles with Uranus for rp = 1.1 R_U

Once more, to understand better the importance of the periapsis distance, we also made simulations using the value of r_p = 5.0 R_U for Uranus. The results are shown below.



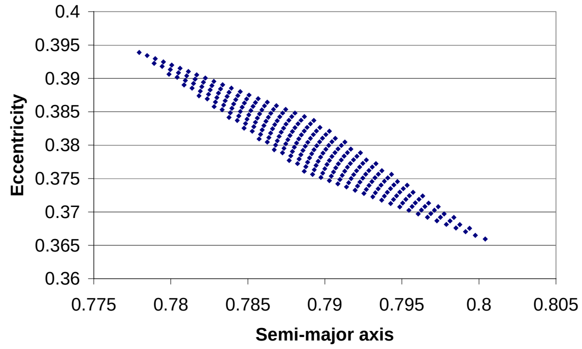
Eccentricity vs. Semi-major axis before the Swing-By

Fig. 4(d) – Swing-by for a cloud of particles with Uranus for rp = 5.0 R_U



Eccentricity vs. Semi-major axis after Swing-By for "Solution 1"

Fig. 4(e) – Swing-by for a cloud of particles with Uranus for rp = 5.0 R_U



Eccentricity vs. Semi-major axis after Swing-By for "Solution 2"

Fig. 4(f) – Swing-by for a cloud of particles with Uranus for rp = 5.0 R_U

4. CONCLUSIONS

The figures above allow us to get some conclusions. The solution called "Solution 1" has larger amplitude than the Solution 2 in both orbital elements, but it concentrates the orbital elements in a line, while the so-called "Solution 2" generates a distribution close to a square. The area occupied by the points is smaller for "Solution 1". Both vertical and horizontal lines are rotated and become diagonal lines with different inclinations. The effect of increasing the periapse distance is to generate plots with larger amplitudes, but with the points more concentrated, close to a straight line. In general, results like those ones shown here can be used to understand better the effects of the periapsis distance.

5. ACKNOWLEDGEMENTS

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