APPLICATIONS OF THE HYBRID FE-SEA METHOD TO VIBRO-ACOUSTIC ANALYSIS OF COMPLEX ENGINEERING STRUCTURES

Júlio A. Cordioli, cordioli@emc.ufsc.br
Mechanical Engineering Department
Federal University of Santa Catarina, Florianópolis – SC, 88040-970, BRAZIL

Vincent Cotoni, vincent.cotoni@esi-group-na.com
ESI US R&D Inc
12555 High Bluff Drive, Suite 250, San Diego, CA 92130, USA

Abstract. *The Hybrid FE-SEA method is a recently developed vibro-acoustic analysis technique which is based on the rigorous coupling of deterministic approaches (Finite Element Method – FEM, for example) and statistical methods (Statistical Energy Analysis – SEA, for example). The method initially targeted the vibro-acoustic analysis of complex built-up systems associated with the so-called "mid-frequency problem", where the structure displays, in the same frequency range, a combination of long wavelength behaviour (better described by a deterministic approach) and a short wavelength behaviour (more suited to a statistical approach). The last couple of years have seen the Hybrid FE-SEA method being applied to problems where, previously, neither purely deterministic methods nor purely statistical methods have proven adequate. The objective of this paper is to review the method and the recently applications, and to verify the degree of success obtained by the Hybrid FE-SEA method. The cases considered in the paper cover different areas, including applications in the automotive, aeronautic and aerospace industries. The paper presents a brief description of each case, highlights the difficulties faced by other methods to analyze the problem, demonstrates the applicability of the Hybrid FE-SEA method to describe the physics of the problem, and validates the Hybrid FE-SEA models by comparison with experimental and/or numerical data.*

Keywords*: Hybrid FE-SEA, vibro-acoustics, SEA, FEM, BEM*

1. INTRODUCTION

The Hybrid FE-SEA method is a recently developed method that provides the rigorous coupling between statistical and deterministic subsystems in a fully coupled analysis (Shorter and Langley, 2005a, Shorter and Langley, 2005b, Langley *et al*, 2005, Cotoni *et al*, 2007 and Langley, 2007). In a Hybrid FE-SEA model, statistical subsystems are described using the Statistical Energy Analysis (SEA), while deterministic subsystems can be represented using Finite Element (FE) Method or other deterministic approach. The method was initially developed to address the so-called "mid-frequency problem" where the system under analysis contains both modally dense elements (short wavelength in comparison to the element dimensions) and elements with few modes (long wavelength in comparison to the element dimensions) in the frequency range of interest. Whereas the stiff elements (few modes) can be described using deterministic methods, the description of the modally dense components faces two major difficulties. First, a large number of degrees-of-freedom is required to properly describe the subsystem response, making the problem computationally expensive to solve. Secondly, the sensitivity of the modes (for example to manufacturing imperfections) restricts the use of purely deterministic methods. Traditional methods used in isolation have therefore proved inadequate to solve such problems, and the Hybrid FE-SEA model was presented as an alternative approach.

Once the theory behind the Hybrid FE-SEA method has been introduced, it was followed by the presentation of validation cases using numerical and experimental results for simple structures (Shorter *et al*, 2005). Since then, the Hybrid method has been used to model more complex industrial structures. The aim of this paper is to review the results of the studies involving the application of the Hybrid FE-SEA method to model engineering structures and to verify the degree of success of the method in each case. A brief review of the Hybrid FE-SEA equations is presented, followed by a discussion on the applications of the method and its validation process. Five examples of the application of the Hybrid FE-SEA method to engineering problems are presented. In each case, the reasons that leaded to the choice of the Hybrid FE-SEA method over other conventional methods are discussed. A brief description of the models is given, and some typical results are presented.

2. HYBRID FE-SEA METHOD

2.1. Theory

The derivation of Hybrid FE-SEA equations is described in details in (Shorter and Langley, 2005a; Shorter and Langley, 2005b; Langley, 2007a), and only a brief review of the equation is presented below. In the Hybrid FE-SEA method, some components of a system are described with FE, while other components are described with SEA. The FE part is represented by a set of degrees of freedom (dofs) $\mathbf{q}$, and the SEA subsystems by their average vibrational energy level E. The coupling of these two types of descriptions is at the center of the method, and it relies on a "diffuse field reciprocity principle" (Shorter and Langley, 2005b; Langley, 2007a). The principle provides a relation between the forces applied to the FE dofs and the average energy level E of the reverberant field in the connected SEA subsystem. The relation is written in terms of the cross-spectral matrix of force $\mathbf{S}_{ff,rev}$:

$$\mathbf{S}_{ff,rev} = \mathrm{E}\left[\mathbf{f}_{rev}\mathbf{f}_{rev}^{H}\right] = \left(\frac{4E}{\omega\pi n}\right)\mathrm{Im}\{\mathbf{D}_{dir}\} \tag{1}$$

where $\mathbf{D}_{dir}$ is the "direct field" dynamic stiffness matrix (i.e. the dynamic stiffness matrix of the equivalent semi-infinite SEA subsystem expressed on the FE dofs), ω is the angular frequency and n is the modal density of the SEA subsystem. Based on Eq. (1), the Hybrid FE-SEA equations were derived for the FE and SEA subsystems fully coupled response (Shorter and Langley, 2005a). The response of the FE dofs is given in cross-spectral form by:

$$\mathbf{S}_{qq} = \mathbf{D}_{tot}^{-1}\left[\mathbf{S}_{ff,d} + \sum_k \left(\frac{4E_k}{\omega\pi n_k}\right)\mathrm{Im}\left\{\mathbf{D}_{dir}^{(k)}\right\}\right]\mathbf{D}_{tot}^{-H} \tag{2}$$

where $\mathbf{D}_{dir}^{(k)}$, E_k, and n_k are the "direct field" dynamic stiffness matrix, the average vibrational or sound energy level, and the modal density of the k^{th} statistical subsystem, while $\mathbf{S}_{ff,d}$ is the cross-spectral force matrix for excitations directly applied to the FE part, and

$$\mathbf{D}_{tot} = \mathbf{D}_d + \sum_k \mathbf{D}_{dir}^{(k)} \tag{3}$$

with $\mathbf{D}_d$ being the dynamic stiffness matrix of the FE part of the system.

In order to solve Eq. (2), it is first necessary to know the average energy response of the SEA subsystems. This is obtained by solving the SEA power balance equations, written as

$$P_j + P_{in,j}^{ext} = \omega\left(\eta_j + \eta_{d,j}\right)E_j + \sum_k \omega\eta_{jk}\eta_j\left(\frac{E_j}{n_j} - \frac{E_k}{n_k}\right) \tag{4}$$

where the coupling loss factors η_{jk} and the damping loss factor $\eta_{d,j}$ due to the FE part, and the power input from direct excitations on the FE subsystems $P_{in,j}^{ext}$ are given by

$$n_{d,j} = \left(\frac{2}{\omega\pi n_j}\right)\sum_{r,s}\mathrm{Im}\{\mathbf{D}_{d,rs}\}\left(\mathbf{D}_{tot}^{-1}\,\mathrm{Im}\left\{\mathbf{D}_{dir}^{(j)}\right\}\mathbf{D}_{tot}^{-H}\right)_{rs} \tag{5}$$

$$n_{jk} = \left(\frac{2}{\omega\pi n_j}\right)\sum_{r,s}\mathrm{Im}\left\{\mathbf{D}_{dir,rs}^{(j)}\right\}\left(\mathbf{D}_{tot}^{-1}\,\mathrm{Im}\left\{\mathbf{D}_{dir}^{(k)}\right\}\mathbf{D}_{tot}^{-H}\right)_{rs} \tag{6}$$

$$P_{in,j}^{ext} = \left(\frac{2}{\omega}\right)\sum_{r,s}\mathrm{Im}\left\{\mathbf{D}_{dir,rs}^{(j)}\right\}\left(\mathbf{D}_{tot}^{-1}\mathbf{S}_{ff,d}\mathbf{D}_{tot}^{-H}\right)_{rs} \tag{7}$$

Once the energy levels of the SEA subsystems are found through Eq. (4), the response of the FE part of the system can be calculated using Eq. (2). Extensions of the theory were recently proposed to compute the variance of the response (Langley and Cotoni, 2007) and to extend the equations to couplings through the domain of FE subsystems (Langley and Cordioli, 2009).

2.2. Applications: System level vs. Component Level

The Hybrid FE-SEA method was firstly though as an alternative approach to the so-called mid-frequency problem, where a system has both short wavelength and long wavelength components. Typically, it can be seen as a way to introduce details in existing SEA models, or as a way to introduce statistics in existing FE models (therefore, reducing computational expenses and predicting mean values). In the case of a system level SEA model where more details are needed, one can use the Hybrid method to either (i) introduce an FE description of some complex parts (like connections or loading areas) into the SEA model; and (ii) create local component level models of transmission paths to compute more accurate CLFs and power input (according to Eq. (6-7)) to be used in the SEA model. In (i), the analysis is performed using a single model, but it becomes more intensive than with the original SEA model. In (ii), the analysis becomes a two-step process, and the second step is as or more efficient than the original SEA model.

As an example, consider the prediction of the interior noise of an aircraft. The energy flow between the fuselage and floor panels through the floor framework may be an important transmission path. Although the floor panel and the fuselage skin are modally dense, the floor framework usually has few modes, and the problem is well suited for the Hybrid FE-SEA method. Applying the Hybrid FE-SEA method to the entire aircraft can be computationally expensive since it means that the entire framework structure of the aircraft needs to be modelled using FE. Alternatively, a Hybrid FE-SEA component level model can be used to calculate the coupling loss factor between the fuselage and the floor panels. The information can then be included in a system level SEA model. In addition, the Hybrid FE-SEA component level model can be directly used to minimize the vibrational energy transmission through the floor beams.

In what follows, both system and component level models using the Hybrid FE-SEA model are presented and discussed.

2.3. Validation process

Due to its deterministic-statistical nature, the prediction of the Hybrid FE-SEA method corresponds to the system response averaged over an ensemble of similar systems. The ensemble is also associated with the system partitioning adopted in the model. Components (usually with few modes) which are robust to variations, and therefore do not change from one ensemble member to another, are modeled deterministically with FE. On the other hand, sensitive components (usually modally dense) whose dynamic properties vary from one ensemble member to another are modeled statistically.

Ideally, test results for experimental validation of a Hybrid FE-SEA model are available in the form of those given by Hill *et al* (2009) and Kompela and Bernard (1993), where measurements were performed for a real ensemble of structures (like cars leaving the production line). For numerical validation, Monte Carlo techniques can be used to compute the ensemble average from multiple models run individually (Cotoni *et al*, 2007 and Shorter *et al*, 2005).

In practice, the costs involved with testing an ensemble of articles is prohibitive, and the prediction of the Hybrid FE-SEA model of engineering structures is usually compared with experimental results for a single structure. In this case, it is common to average the results over frequency bands assuming ergodicity of the frequency and ensemble statistics. This practical approach is similar to what is commonly done with standard SEA validation process. In order to gain insight into the validity of a Hybrid model, it is recommended that the deterministic and the statistical parts be validated separately (see for example the approach used by Cotoni *et al* (2008) for an aircraft overhead bin).

3. SYSTEM LEVEL APPLICATIONS

3.1 Automotive interior noise due to structure-borne excitation

In this work, the Hybrid FE-SEA method is used to predict the interior noise levels in a trimmed full vehicle due to structure-borne excitation in the frequency range of 200 to 1000 Hz (Charpentier *et al*, 2008a and Charpentier *et al*, 2008b). The case is a classical mid-frequency problem: the A-pillar is an example of a stiff component with long wavelength behaviour, and the car roof an example of a modally dense component. Attempts of using pure FE models have resulted in models involving around 6 million dofs up to 250 Hz, and solution times already impractical for this frequency. SEA has being widely applied for the design of sound packages for cars under airborne excitation at frequencies higher than 300 Hz. However, the low modal density and complexity of some components involved in structure-borne excitations and structure-borne transmission make it much more difficult to model with SEA. The Hybrid FE-SEA method was therefore chosen to add the required level of detail to the transmission paths.

The Hybrid FE-SEA model of the vehicle is shown in Fig. 1. Stiff elements of the car, like the rails, engine supports, and shock towers are modeled as FE subsystems, whereas flexible components, like the roof, windows, and doors are represented by SEA subsystems. A FE model used for structural analysis was the main source of information for the Hybrid FE-SEA model. The damping loss factors of the SEA subsystems were obtained through experimental tests with similar structures, while the CLF through complex junctions was calculated using local FE models (Energy Flow Method) or local Hybrid FE-SEA models (component level models). Local Hybrid FE-SEA models were also used to efficiently calculate the radiation efficiency of panels with complex geometry. A detailed description of the model building process is given by Charpentier *et al* (2008a) and Charpentier *et al* (2008b).

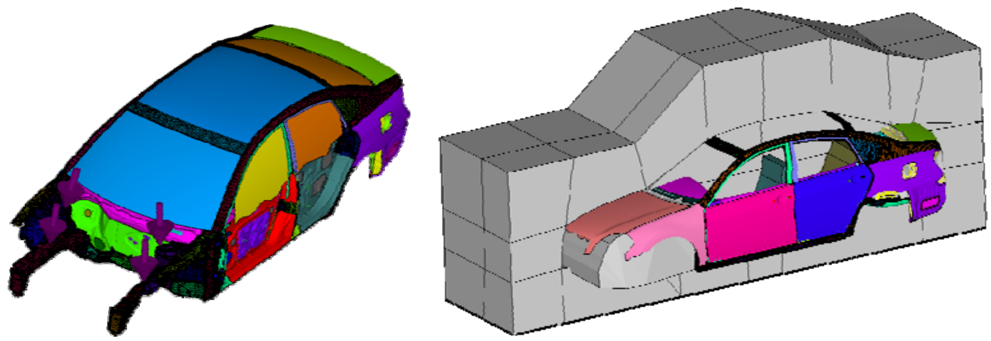

Figure 1. Hybrid FE-SEA model of a vehicle for structure-borne excitations (Charpentier *et al* 2008a).

The partition of the structure into FE and SEA components was initially defined from simple engineering judgement, and was later confirmed by some numerical experiments (Charpentier *et al* 2008a) involving: (i) global modal analysis of the structure (localized modes indicate potential SEA subsystems), (ii) local modal analysis of potential SEA subsystems (checking modes in band, sensitivity to boundary conditions, etc), and (iii) a forced response analysis (checking the spatial uniformity of the response). In order to provide an accurate Hybrid FE-SEA model, the partitioning process must ensure that global and local transmission paths are correctly accounted for in the model. Indeed, it was shown by Charpentier *et al* (2008a) that a version of the Hybrid model with the rails as SEA subsystems failed to properly describe the direct transmission path through these elements from the front of the car to the rear, underestimating the vibration levels at the rear part of the vehicle. By describing the rails as FE subsystems, the critical "long range" transmission paths are properly accounted for. The partitioning defined by Charpentier *et al* (2008a) is expected to be valid for any similar vehicles (and the partitioning process does not have to be repeated unless the analysed vehicle is very different).

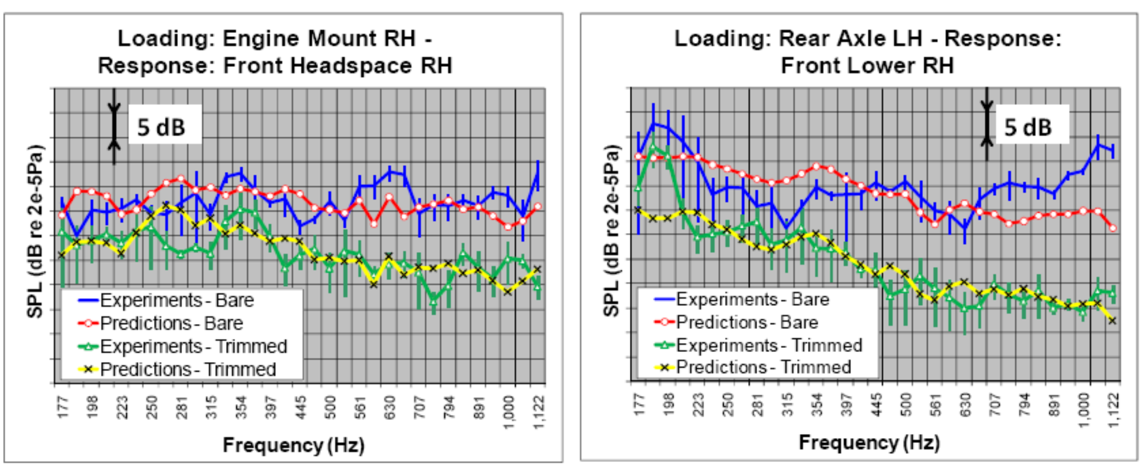

Figure 2. Sound pressure inside the vehicle for excitation at the engine mount (left) and at the rear axle (right) without the sound package (bare) and after its installations (trimmed) - Hybrid vs. experimental (Charpentier *et al* 2008b).

Typical results for the interior noise level are shown in Fig. 2. The results for trimmed and untrimmed versions of the vehicle are included in the graphs. The numerical results are, in general, within 3 - 5 dB, and correctly account for the effect of the noise control treatments applied in the trimmed version. The level of agreement is similar to the values reported in the literature for the differences in measurements within identical vehicles (Hill *et al*, 2009 and Kompela and Bernard, 1993). The surprisingly better agreement obtained with the trimmed vehicle is due to the better characterization of the interior cavity damping (mainly governed by the applied noise control treatments).

3.2 Satellite dynamic response due to diffuse acoustic field

Knockaert *et al* (2007) describes the Hybrid FE-SEA modeling of a satellite. The aim of the work is to predict the dynamic response of the structure due to the diffuse acoustic field environment during the acoustic qualification test. In the frequency range of interest (0 - 1200 Hz), the structure comprises both stiff elements and elements with a large number of modes. In addition to the need to reduce computational cost due to the description of the structure, there is an interest in reducing the costs associated with describing the diffuse acoustic excitation (traditionally modeled with deterministic methods such as BEM or Infinite Elements).

An overall view of the Hybrid FE-SEA model of the satellite is shown in Fig. 3. The definition of the FE and SEA subsystems followed the general rules discussed in the previous section, with rigid components modeled as FE subsystems, and modally dense components as SEA subsystems. However, in the present case, the partitioning process also took in account the type of result required from the analysis. For example, the top part of the satellite includes a telescope with internal mirrors. The noise level imposed to the mirrors is a critical output of the analysis. Although some comparisons showed that a SEA model of the component provides accurate prediction of the spatial average vibration, it was decided to model the component with FE and the internal cavity with BEM to allow the prediction of vibration and pressure results at specific locations.

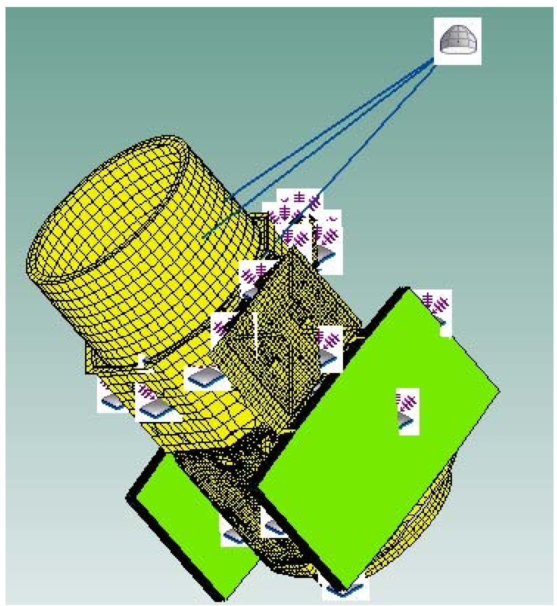

Figure 3. Hybrid FE-SEA model of a satellite (Knockaert *et al*, 2007).

In the modeling process of the satellite using only deterministic methods, a significant part of the computational expenses is due to the description of the diffuse acoustic field. Within the context of the Hybrid FE-SEA model, it is possible to efficiently model a diffuse acoustic field exciting an FE structure, and the calculation of forces makes use of the same equations used to couple a FE structural subsystem to a SEA acoustic subsystem (Eq. (1)). Here, the cross-spectral matrix of forces is defined from the “direct field” dynamic stiffness matrix associated with the acoustic medium (i.e. the radiation impedance matrix). To quickly compute the radiation matrix, an algorithm based on the computation of the Green’s function of a fluid half space by means of a wave number transform is used. The approach assumes that a planar baffle defines the fluid half space, and the results are therefore approximate for highly non-planar structures. More details on the algorithm used can be found in references by Langley and Cordioli (2009), Shorter (2005) and Langley (2007b). For the satellite model, all surfaces, except the telescope (top part excited using BEM) were excited through Hybrid descriptions of the diffuse acoustic field.

Typical results for the vibrational level of some components of the satellite are shown in Fig. 4. Although the numerical results slightly over predict the vibrational levels at some frequencies, the main peaks of the response are correctly predicted (usually the most important point in the qualification test).

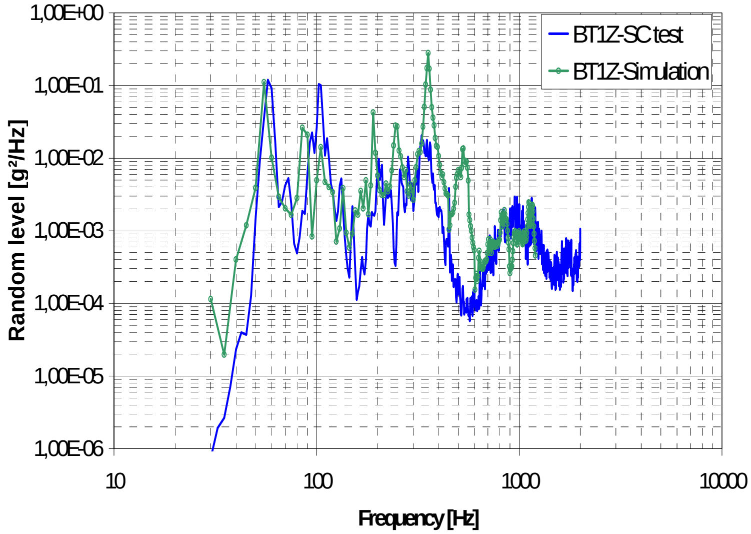

Figure 4. Vibrational response of a component of the satellite due to a diffuse acoustic field environment– Experimental vs. simulated results (Knockaert *et al*, 2007).

4. COMPONENT LEVEL APPLICATIONS

4.1 Automotive dash panel transmission loss

A Hybrid FE-SEA model is used to predict the transmission loss of a front-of-dash component of a vehicle (Shorter *et al*, 2007). The model can be used to perform the optimization of the acoustic performance of the component and the noise control treatments applied to it as well as to update a SEA system level model of the vehicle.

Figure 5 shows two versions of the Hybrid FE-SEA model of the dash panel and the source and receiving chambers associated with a transmission loss test set-up. In Fig. 5(a), the dash panel is modeled using FE, and the chambers are represented by SEA cavities. Hybrid area junctions are used to couple FE and SEA subsystems, and the same algorithm mentioned in section 3.2 is used to compute the direct field stiffness matrix.

Figure 5(b) shows the Hybrid FE-SEA model for an alternative approach (Shorter *et al*, 2007) to represent the chambers in the hybrid model. In the alternative approach, the chambers are represented by a series of energy sinks (representing the impedance of a semi-infinite fluid) and the excitation is applied directly to the FE subsystems. The representation of the diffuse acoustic field follows the same approach described in the previous section. Such approach allows the subdivision of the dash panel in several regions, (each one connected to an energy sink and excited by a SEA diffuse acoustic field), and the component TL can be calculated for each region. As a consequence, it is possible to calculate the contribution of each region to the total transmission loss and identify the dominant transmission path in a computationally efficient way.

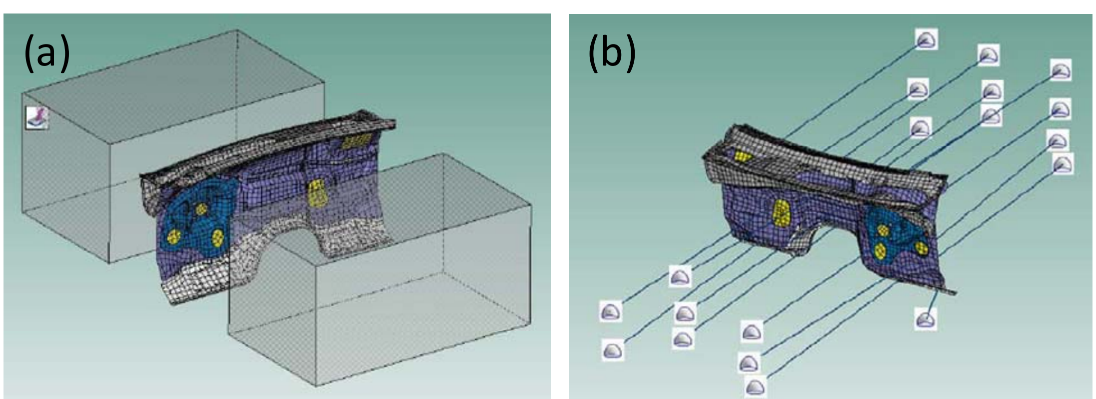

Figure 5: Hybrid FE-SEA of a vehicle dash panel. (a) Chambers as SEA cavities; (b) – Chambers as energy sinks (SEA semi-infinite fluids) (Shorter *et al*, 2007).

The prediction of the transmission loss of a region of the dash panel computed using the hybrid model is compared in Fig. 6 with experimental results and the mass-law curve. The Hybrid model correctly predicts the peaks and valleys in the TL at low frequency (behavior associated with local resonances and anti-resonances).

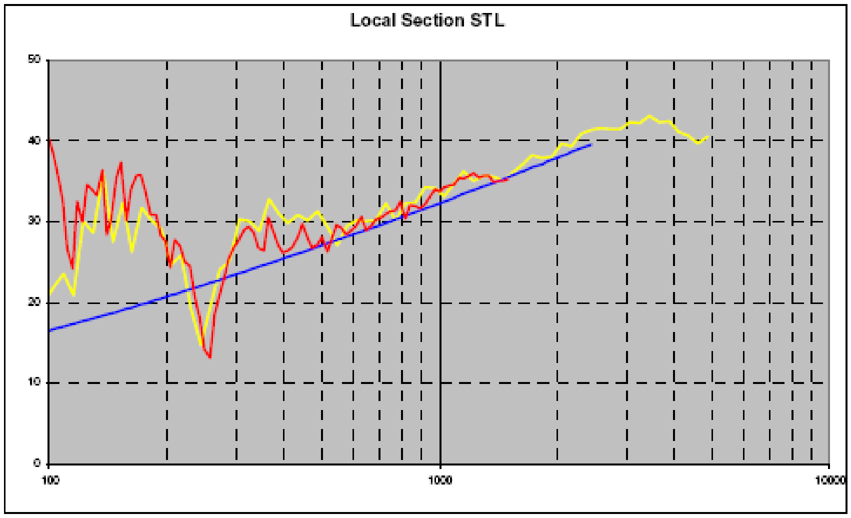

Figure 6. Transmission loss of a region of the dash panel – Numerical results (red), experimental results (yellow) and mass-law curve (blue) (Shorter *et al*, 2007).

4.2 Structure-borne transmission to aircraft overhead bin

In this section, a Hybrid FE-SEA model of the overhead bin and the frame used to connect to the fuselage is discussed. The aim of the model is to analyze the structure-borne noise transmission from the aircraft fuselage to the overhead bin through the frame (Cotoni *et al*, 2008). As a component level model, the results may be used to directly optimize the component or to calculate input parameters to a system level SEA model.

The structure considered in the analysis and the corresponding Hybrid FE-SEA model are shown in Fig. 7. The structure is composed of the overhead bin, the frame supporting the bin, and the tie rod connecting the structure to the fuselage. The low modal density of the frame and the high number of modes of the bin panels make the system an ideal candidate for the Hybrid FE-SEA method. The modeling process is described in details by Cotoni *et al*, 2008. The FE model of the frame and the SEA model of the overhead bin panels were developed and validated separately. The models were later coupled through Hybrid point junctions.

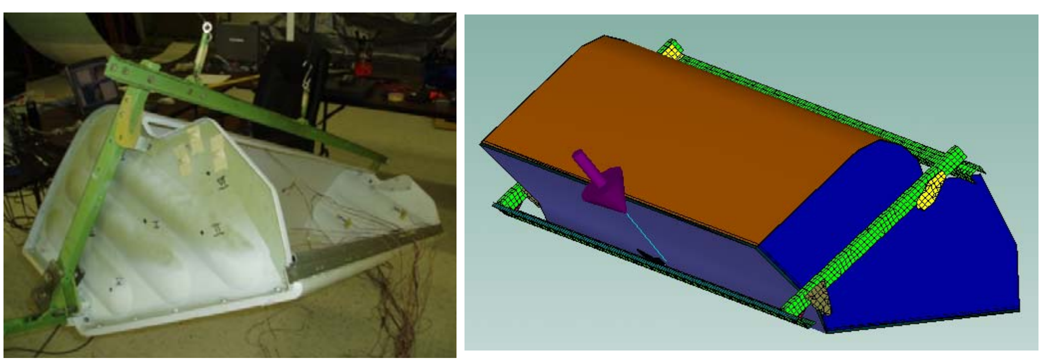

Figure 7. Overhead bin and corresponding Hybrid FE-SEA model (Cotoni *et al*, 2008).

Typical results obtained with the model are shown in Fig. 8 for excitation at the tie rod end and response of one of the bin panels (top panel). The numerical results are in general within 5 dB of the experimental results. Figure 8 also shows how the Hybrid model was used to diagnose the very high transmission peak around 300 Hz. By analyzing the modes of the frame (described with FE), it was found that a local resonance of the bracket of the supporting frame at 316 Hz allows high levels of energy to be injected into the structure. This type of information could be used to modify the design of the bracket and reduce the vibration of overhead bin.

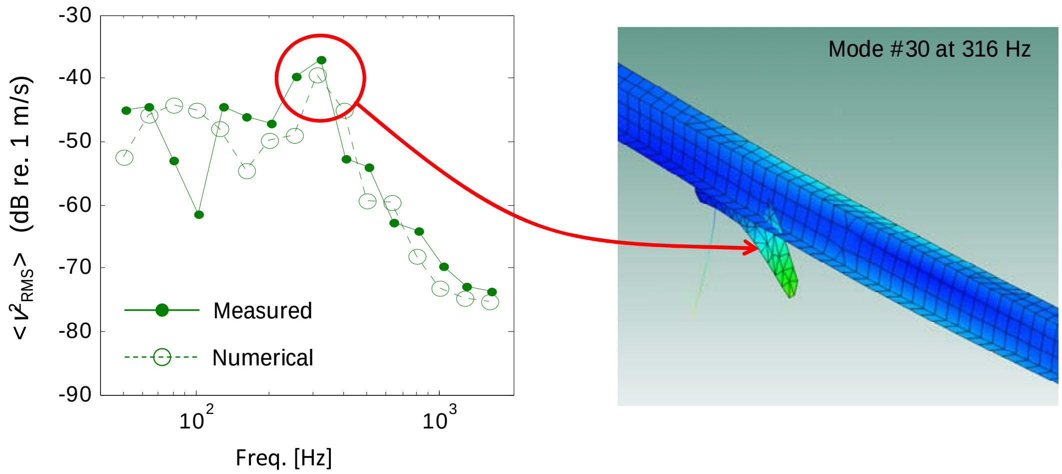

Figure 8. RMS velocity of the top panel of the bin due to point force excitation at the end of the tie rod (Cotoni *et al*, 2008).

4.3 Structure-borne transmission to aircraft interior panel

Consider the double wall structure of a commercial aircraft showed in Fig. 9(a). The system includes elements with high modal density (fuselage and interior panel) connected through a complex junction. The interior panel is connected to the fuselage through vibration isolators, which are mounted on the fuselage frames. In this study, the Hybrid FE-SEA method was used to efficiently calculate the CLF between fuselage and interior panel (Cotoni *et al*, 2008). A component level Hybrid FE-SEA model of the transmission through the rubber mount is shown in Fig. 9(b). The frame and the vibration isolator were modeled as FE subsystems and were connected through Hybrid junctions to SEA subsystems (representing the fuselage and the interior panel). The CLF is directly calculated using Eq. (6). The SEA subsystems connected at the ends of the frame are here to define "average" boundary conditions for the FE subsystem. Indeed, the addition of the direct-field dynamic stiffness of the SEA subsystem in the Hybrid equations is equivalent to adding a semi-infinite subsystem whose remote boundaries are random.

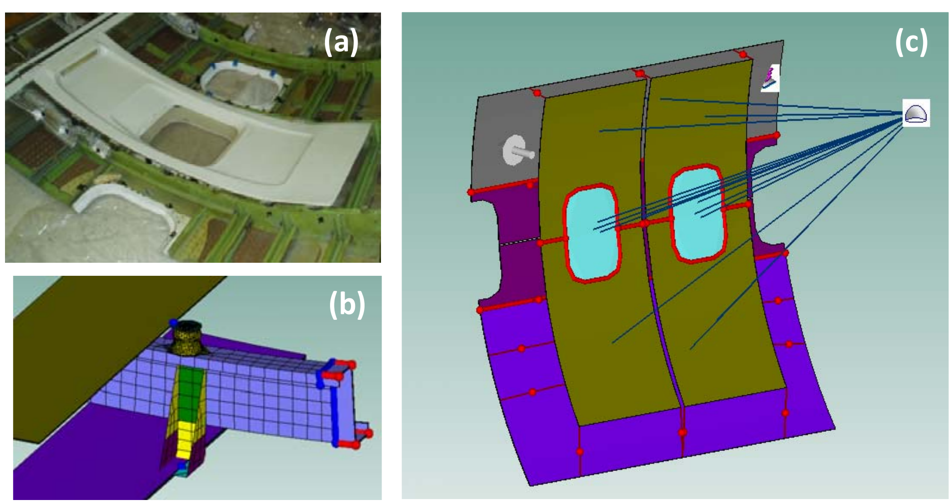

Figure 9. (a) Fuselage and interior panel considered in the analysis; (b) Hybrid FE-SEA component level model; and (c) SEA system level models.

The CLF of the component level model was later included in a SEA model of the aircraft wall shown in Fig. 9(c). Figure 10 shows the results of the system response to rain-on-the-roof loading of the top part of the fuselage panel. The SEA model correctly predicts the distribution of the vibrational energy in the system. The predicted results are within 5 dB of the experimental results for most frequency bands.

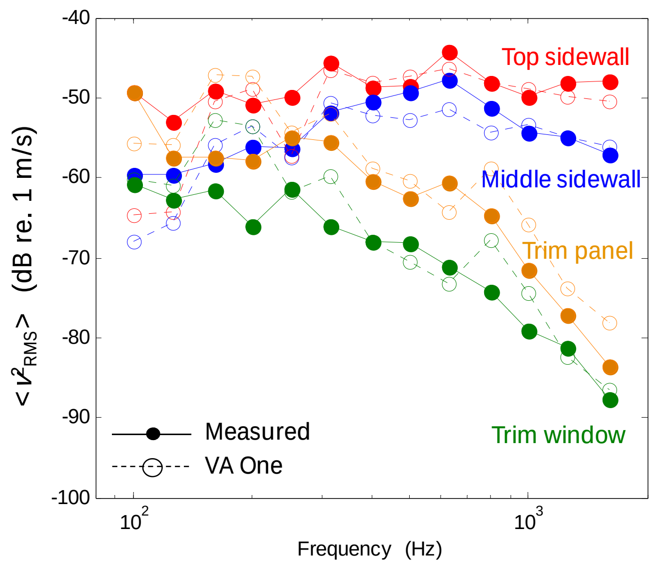

Figure 10. RMS velocity of subsystems of the SEA system level model.

5. CONCLUDING REMARKS

Application of the Hybrid FE-SEA method to the prediction of vibration and noise response of various engineering systems from different industries were reviewed. In general, the predictions are in very good agreement with experimental results. In some cases, it would have been difficult or impossible to obtain similar predictions using standard methods due to computational costs or lack of accuracy.

A number of different uses of the method were explored: (i) reduced number of dof and add statistics to an existing FE model (full car and satellite examples), (ii) add deterministic details to an existing SEA model (overhead bin and fuselage double wall examples), (iii) quickly add acoustics to an existing FE model (satellite and automotive dash examples). Both system level and component level models can be built and used for prediction and diagnosis. For the particular application where the Hybrid method is used to add details to an SEA model, a component level Hybrid model can be used to compute accurate CLFs through complex junctions, power input from loading on complex components, and radiation from complex components. The computed quantities can then be used to update a system level SEA model (the SEA model stays computationally very efficient).

The nature of the Hybrid FE-SEA method also allows the use of standard diagnosis tools from both the FE method (mode contribution analysis for example) and SEA method (noise path analysis for example).

Active areas of research include extending the number of Hybrid junction types (which means finding efficient way to compute the direct-field dynamic stiffness of components), making use of the recently developed variance prediction within the Hybrid method framework, and developing algorithms to guide the partitioning of a system into deterministic and statistical subsystems.

7. REFERENCES

Charpentier, A., Sreedhar, P. and Fukui, K., 2008a, “Efficient model of structure-borne noise in a fully trimmed vehicle from 200Hz to 1kHz”, Proceeding of Internoise 2008, Shanghai, China.

Charpentier, A., Sreedhar, P., Cordioli, J.A. and Fukui, K., 2008b, “Modelling process and validation of Hybrid FE-SEA method to structure-borne noise path in a trimmed automotive vehicle”, SAE technical paper 2008-36-0574.

Cotoni, V., Shorter, P.J. and Langley, R.S., 2007, “Numerical and experimental validation of a hybrid FE–SEA method for the harmonic analysis of structure-borne noise” Journal of the Acoustical Society of America, Vol.122, pp. 259-270.

Cotoni, V., Gardner, B., Cordioli, J.A., Carneal, J. and Fuller, C., 2008, “Advanced modelling of aircraft interior noise using the Hybrid FE-SEA method”, SAE technical paper 2008-36-0575.

Hills, E., Mace, B.R. and Fergunson, N.S., 2009 (in press), “Acoustic response variability in automotive vehicles”, Journal of Sound and Vibration.

Knockaert, R., Frikha, S. and Cotoni, V., 2007, " Simulation of a spacecraft acoustic test by Hybrid FE-SEA method: application to the CALIPSO spacecraft and comparison with experimental data", First CEAS European Air and Space Conference, Berlin – Germany.

Kompella, M.S. and Bernhard, R.J., 1993, "Measurement of the statistical variation of structural–acoustic characteristics of automotive vehicles", SAE technical paper 931272.

Langley, R.S., Shorter, P.J. and Cotoni, V., 2005, "A hybrid FE-SEA method for the analysis of complex. vibro-acoustic systems" Proceedings of the International Congress on Noise and Vibration Emerging Methods (NOVEM2005), Saint-Raphael, France.

Langley, R.S., 2007a, "On the diffuse field reciprocity relationship and vibrational energy variance in a random subsystem at high frequencies" Journal of the Acoustical Society of America, Vol. 121, pp. 913-921.

Langley, R.S., 2007b, "Numerical evaluation of the acoustic radiation from planar structures with general baffle conditions using wavelets", Journal of the Acoustical Society of America, Vol.121(2), pp. 766–777.

Langley, R.S. and Cotoni, V., 2007, "Response variance prediction for uncertain vibro-acoustic systems using a hybrid deterministic-statistical method" Journal of the Acoustical Society of America, Vol.122(6), pp. 3445-3463.

Langley, R.S. and Cordioli, J.A., 2009, "The analysis of structural-acoustic coupling at medium and high frequencies", Journal of Sound and Vibration, Vol. 321, pp. 893–912.

Shorter, P.J. and Langley, R.S., 2005a, "Vibro-acoustic analysis of complex systems" Journal of Sound and Vibration, Vol. 288(3), pp. 669-699.

Shorter, P.J. and Langley, R.S., 2005b, "On the reciprocity relationship between direct field radiation and diffuse reverberant loading" Journal of the Acoustical Society of America, Vol.117(1), pp. 85-95.

Shorter, P.J., Cotoni, V. and Langley, R.S., 2005, "Numerical and experimental validation of the hybrid FE-SEA method" Proceedings of the International Congress on Noise and Vibration Emerging Methods (NOVEM2005), Saint-Raphael, France.

Shorter, P.J., 2005, "Hybrid area junctions". ESI US R&D Inc. Internal Report.

Shorter, P., Zhang, Q. and Parrett, A., 2007, "Using the Hybrid FE-SEA Method to predict and diagnose component transmission loss", SAE Technical paper 2007-01-2172.