DEVELOPMENT OF A MODEL FOR ACCUMULATION DAMAGE CALLED PROGRESSIVE DAMAGE FOR FATIGUE LIFE

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Abstract. *The incremental fatigue damage due to a loading is proportional to the amplitude of a cyclic load and the total fatigue damage is estimated by assuming a suitable damage accumulation rule. Was postulate Palmgren and Miner that the total damage caused by a stress history. Either a submitted material the cyclical tensions, exactly that these tensions are below of its limit of rupture, will have an accumulation damages affecting physical integrity the component, due to continuous cycles. This work considers a new methodology for the study damage and for this comparisons with other theories are carried through. This proposal after appeared to verify that the theoretical studies and experimental verifications theories accumulation damage of Palmgren-Miner, Corten-Dolan and Marin, it does not consider the history of the passed tensions, or either, each applied parcel.*

***Keywords**: fatigue, accumulation damage, model*

1. INTRODUCTION

The term fatigue can be defined as a gradual and located process with irreversible nature that occurs in material under tensions or floating deformations. These efforts can result cracks or complete failure materials. As consequence, they are formed local for nucleation cracks, for accumulating great located plastic deformation. These cracks propagate in each cycle of tension until instability (Gonçalves, 2003).

Time variant reliability assessment of deteriorating structures involves estimation of the fatigue damage due to random dynamic loads. In real life structures, the load effects causing fatigue damage, (Rychlik and Gupta, 2006)

When requesting a component the cyclical tension, has a phenomenon of accumulation of damages that depending on its intensity can harm in the integrity of the materials. This accumulation damages leads to the development of cracks being able to occur to the fracture of the material (Rocha, 2005).

All the solicitations that vary in intensity and/or in direction provoke variations on the structure tensions and can cause failure for fatigue. The live loads and environments are important in this association because they possess the biggest parcel load in structure (Kiepper, 2004).

The pioneering work that introduced the damage concept was elaborated by Kachanov in 1958 (Proença, 2000). It looked for to justify the rupture, precociously observed of metals, in regimen of slow deformation, as consequence of the existence of defects in the material. For consideration of the damage, it defined an variable to scale of the free material of defects, *D=0*. While, *D=1* corresponds to a state of complete loss of integrity of the internal structure of material, is given by Equation 1.

$$D = \frac{A_D}{A} \tag{1}$$

Where, A_D it is area with defects (cracks) and A is the total area (nominal) of the transversal section, as presented in the Figure 1.

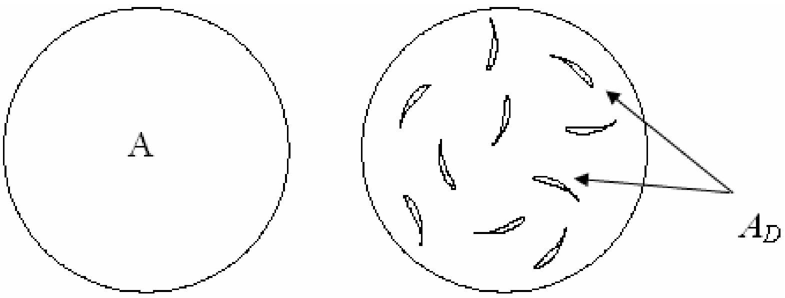

Figure 1. Damage model

The damage models have been accepted as alternative for the simulation of constituent behaviors of the materials that present loss of rigidity. Physically, the degradation of the mechanical properties of the material is resulted growth of micron defects, between them, micron pores and micron cracks. These factors associates with applied tensions cyclical are favorable to the increase of the damage for fatigue (Fatemi, Yang, 1998).

In general, a structure is projected to work as an appropriate safety and economy. The collapse of a structure can happen through two different mechanisms (Taier, 2002):

- The occurrence of high tension levels that exceeds the resistance capacity of the material, causing failures, for example, rupture or instability of a structural component;
- The structural collapse caused by accumulated damage produced by repetitive action of variable loads even for lower tension levels generating a fatigue process.

1.1. Elementary laws of the damages for fatigue

The mechanism formation imperfection for fatigue in metals is initiated with slipping bands formation (Suresh,1988). These are caused by movement disagreements in reticulate crystalline metal, leading the intrusions and extrusion formation. As consequence, they are formed local for nucleation cracks, for accumulating great located plastic deformation. These cracks propagate in each cycle of tension until instability.

For the fatigue degradation study it is necessary to develop essays that present typical cycles for each situation. The operating tensions in component must be known will be analyzed to regulate randomly.

While Kachanov (1958) assumed *D* as a nature variable to scale, posterior studies had led to the proposal of tensoriais amounts to describe the damage. More recently Lemaitre & Chaboche legalized the call on the basis of “Continuum Damage Mechanics” a methodology based on a irreversible process (Proença, 2000).

In a fatigue tests, the evolution can be linear or not linear the damage, as presented in Figure 2.

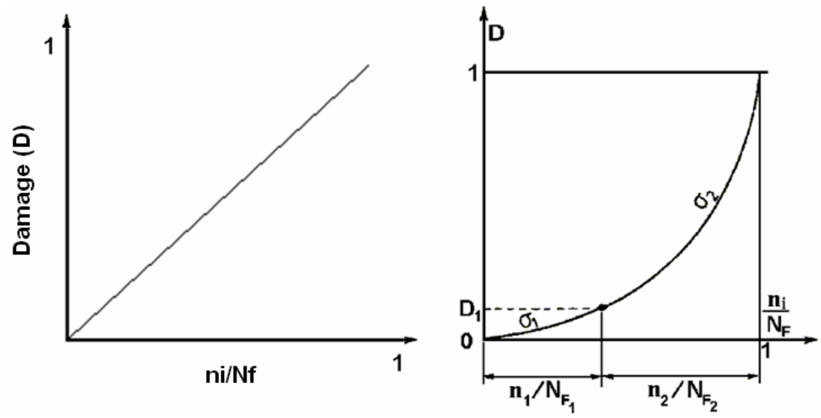

Figure 2. Linear and not linear model damage

1.2. Theories of accumulation damages

The accumulating processes of damage can produce imperfections in the material due the cyclical generating fatigue (Lennon et al, 2004). Since the decade of 40, the works and theories proposals are adding new concepts for a better detailing of the problem. In this aspect, some referring theories to the accumulation of damage are presented.

The processes considered for Palmgren-Miner admit that the referring damage to each request can be quantified in quotient enters the number of cycles (*n*) applied and the number of cycles (*N*) necessary to cause the failure (Domingues, 2003). After that, is admitted that the fracture for fatigue occurs when the addition of the quotients of the relative damages, to each request, equal to the unit, 1, as the Equation 2.

$$\sum D_i = \frac{n_i}{N_i} \geq 1 \tag{2}$$

Where, ni is the number of cycles applied to the component under a tension σ_i and N , is the number of cycles gotten of curve S-N-P.

Figure 3, illustrates this situation on curve S-N of material data. In this example, it assumes that already the two first requests without occurring rupture had existed. When applying the third request, the rupture would occur when the number of cycles satisfied the following equality, as Equation. 3:

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 1 \tag{3}$$

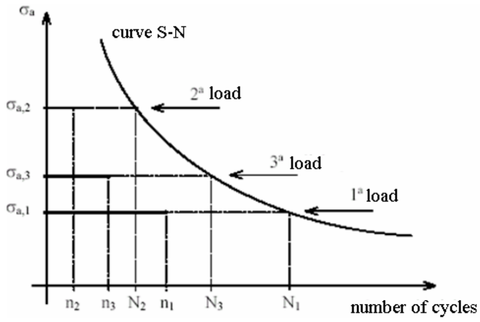

Figure 3. Theory Palmgren-Miner

Second (Mansur, 2003) the main deficiencies of this theory are independences of the level of tension, the sequence of the carrying and the disrespect of the interaction of the damages.

In the equation, verifies that it is not important to the order of application of the tension cycles, for example, would not be important to apply the cycle *n3/N3*, before the cycle *n1/N1*. However, the experience demonstrates that the order of application of the cycles of tensions is of great importance in the fatigue strength. The application of a cycle of tensions with bigger amplitude in first place cause more damage of what the initial application of a cycle with lesser amplitude of tensions (Correia, 2001).

Corten-Dolan a little more complex in relation to the theory of the damage had developed a theory proposal for Palmgren-Miner and incorporates six hypotheses based on the nucleation of the fatigue, the damage and its propagation (Farrar, 1999):

- The period of nucleation (possibly a small number of cycles) is necessary to initiate the damages in fatigue;
- The number of nucleation of damages grows with the increase of the tension;
- The damages, for one given amplitude of tension, increase with the growth of the number of cycles;
- The tax of damages for cycles grows with the increasing tension; The total damage that leads the component to the imperfection is a constant for all the descriptions that can be applied;
- The damage continues to be propagated in tension levels lesser that the minimum of tension necessary to initiate the same ones, as Equation. 4.

$$D = (\frac{n_1}{N_1}) + (\frac{n_2}{N_1}) \times (\frac{\sigma_{2a}}{\sigma_{1a}})^d + (\frac{n_3}{N_1}) \times (\frac{\sigma_{3a}}{\sigma_{1a}})^d + \cdots + (\frac{n_i}{N_1}) \times (\frac{\sigma_{ia}}{\sigma_{1a}})^d \tag{4}$$

Where, n_1 is the number of cycles applied to the component under a tension σ_{1a}, N_1 is the number of cycles gotten of curve *S-N-P* for the biggest amplitude of tension, and thus successively. Having $\sigma_{1a} > \sigma_{2a} > \ldots \sigma_{na}$, being d a constant of the material, equal the 6,67 for the steel. The imperfection of the component occurs when $D = 1$.

This theory is based on the modification of curve S-N, that is simply a rotation in the original curves *N* around a corresponding point to the highest level of the description of the load.

The theory of Marin (1962) considers a relation enters the damages in function of the cycle number and the change in the curve of the *S-N* caused for accumulates damages. Observe that the theory has exponents similar the *d* to the theory of Corten-Dolan [Farrar, 1999]. The theory is based on a family of curves of constant damages, where curve *S-N-P,* for the free material of damages, is the curve of constant damages equal the *1* for the imperfection. The expression of the accumulated damage is given by Equation 5.

$$D = (\frac{n_1}{N_1}) + (\frac{n_2}{N_2}) \times (\frac{\sigma_{a2}}{\sigma_{a1}})^q + (\frac{n_3}{N_3}) \times (\frac{\sigma_{a3}}{\sigma_{a1}})^q + \cdots + (\frac{n_i}{N_i}) \times (\frac{\sigma_{ai}}{\sigma_{a1}})^q \tag{5}$$

Where, $q = y - x$, $y = d$ (of the theory of Corten-Dolan), the numbers of cycles are removed of curve *S-N-P* and the value of x is given by Equation 6.

$$x = \frac{(\log N_1 - \log N_2)}{(\log \sigma_{a2} - \log \sigma_{a1})} \tag{6}$$

Mansur considered a new methodology that considers the history of the last tensions. In its work it describes that the total damage is gotten by the addition of partial damages D_i. Each parcel of the D_i damage is gotten by the relation enters the number of cycles n_I under an alternating tension applied σ_I and the waited life N_I. From the second parcel, the number of cycles is multiplied by the arithmetic mean of the applied tensions. Its description this presented in Eq.(7) and (8).

$$D = \sum_{i=1}^{n} Di = D1 + D2 + D3 + \ldots \tag{7}$$

$$D = \frac{n_1 \times \sigma_1}{N_1 \times \sigma_1} + \frac{n_2 \times \left(\frac{\sigma_1 + \sigma_2}{2} \right)}{N_2 \times \sigma_2} + \frac{n_3 \times \left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \right)}{N_3 \times \sigma_3} + \frac{n_4 \times \left(\frac{\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4}{4} \right)}{N_4 \times \sigma_4} + \cdots \tag{8}$$

The proposal of this theory is to consider the arithmetic mean of the operating tensions in the damages of the materials, caused for fatigue. In this theory, n_I is the number of cycles that the samples was submitted to the alternating tension σ_I, N_I is the corresponding number of cycles to the life of the component under this tension, removed of curves *S-N-P*, and thus successively.

2. METHODOLOGY

For work, the steel chosen for the survey of curves S-N was ASTM A-36 (ASTM A 36/A 36M – 00a. 2000), acquired in market in circular bars diameter of 1/2 ". Where removed samples. This type steel very is used in under water structures, for treat with a material that is classified as a steel carbon of average resistance mechanics.

The samples had been essayed in fatigue machine bending tension to verify the number of cycles necessary to rupture. The assays occurred with sample rotating to one determined speed and with load application provoke fatigue.

The applied tension is directly related to the weight. The disposable weights in equipment are: 176, 197, 236 e 278 MPa.

Initially curves S-N-P of A-36 material had been known to verify its limit of fatigue well as its behavior in high and low tensions, as presented in fig. (4). After that an analysis of linear regression of the gotten points and the analysis of residues to verify if the behavior of the data gotten they were normal. It verified that the gotten values are in the standard.

For the tests of accumulation of damage, the numbers of applied cycles had corresponded 30% of waited it samples to rupture with one definitive tension, being considered a probability of failure of 1%, inside of curve S-N-P of A-36 material in study.

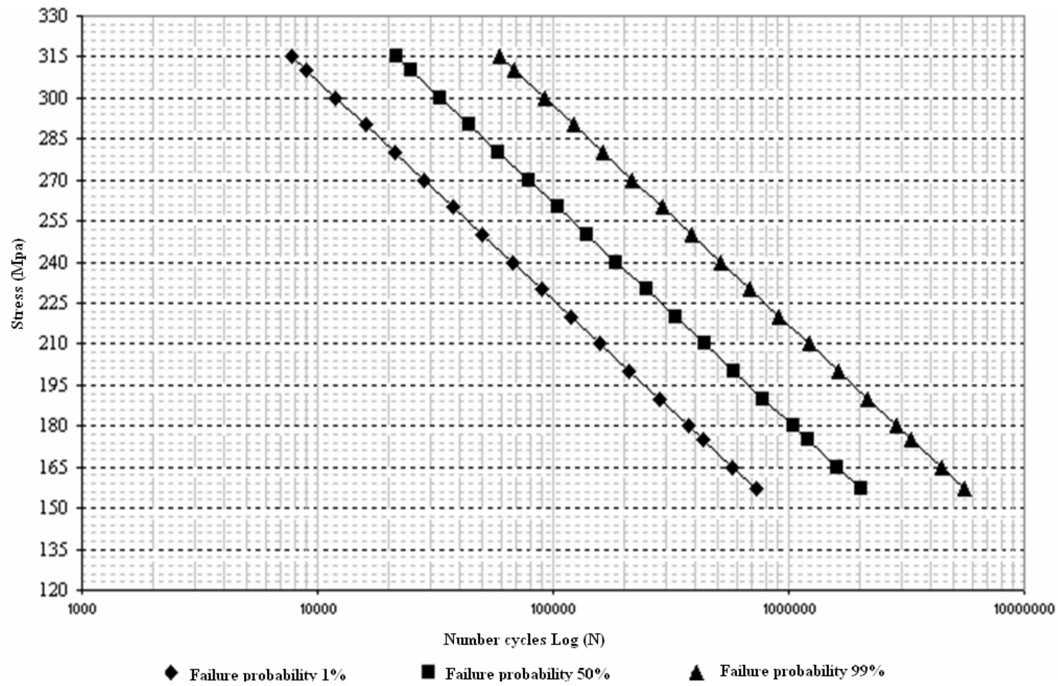

Figure 4. Curves *S-N-P*

In Tables 1 and 2, the applied tensions and the numbers of cycles are presented to damage the material. In the last applied tension the sample is taken until its rupture, after to pass for the three initial tensions, are they in the direction increasing or decreasing.

Table 1: Accumulation of damage using increasing tensions.

Samples	*Number cycles for the applied tension*			
	176 MPa	197 MPa	236 MPa	278 MPa
1	122735	67260	22009	28543
2	122733	67261	22014	35089
3	122733	67264	22012	98034
4	122738	67266	22008	68552

Table 1: Accumulation of damage using decreasing tensions.

Samples	*Number cycles for the applied tension*			
	278 MPa	236 MPa	197 MPa	176 MPa
1	6614	22012	67258	882087
2	6607	22020	67264	953921
3	6615	22016	67260	562056
4	6610	22010	67267	691525

With these data it will be possible to compare the theories, as well as, a new methodology for to study accumulation damage.

3. RESULTS

The work considered to study the theories of Palmgren-Miner, Corten-Dolan, Marin and the theory of the averages of the operating tensions proposal for Mansur. The results are presented in figures 5 the 8.

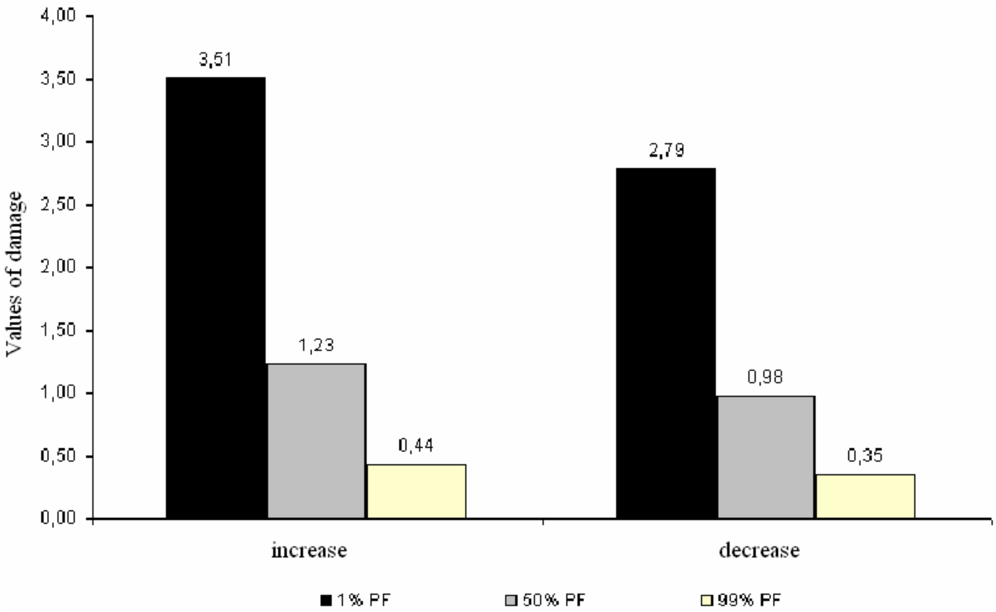

Figure 5. Values of the damage - theory of Palmgren-Miner.

In the Figure 6 are presented values of the damages when is used the theory of Corten-Dolan.

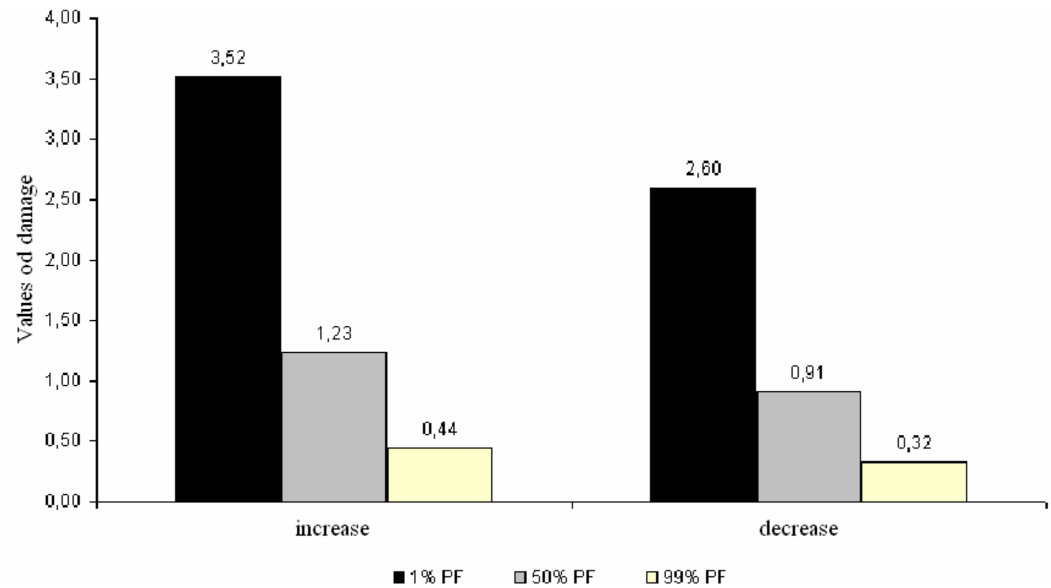

Figure 6. Values of the damage - theory of Corten-Dolan.

The Figure 7 are presented values of the damages when is used the theory of Marin.

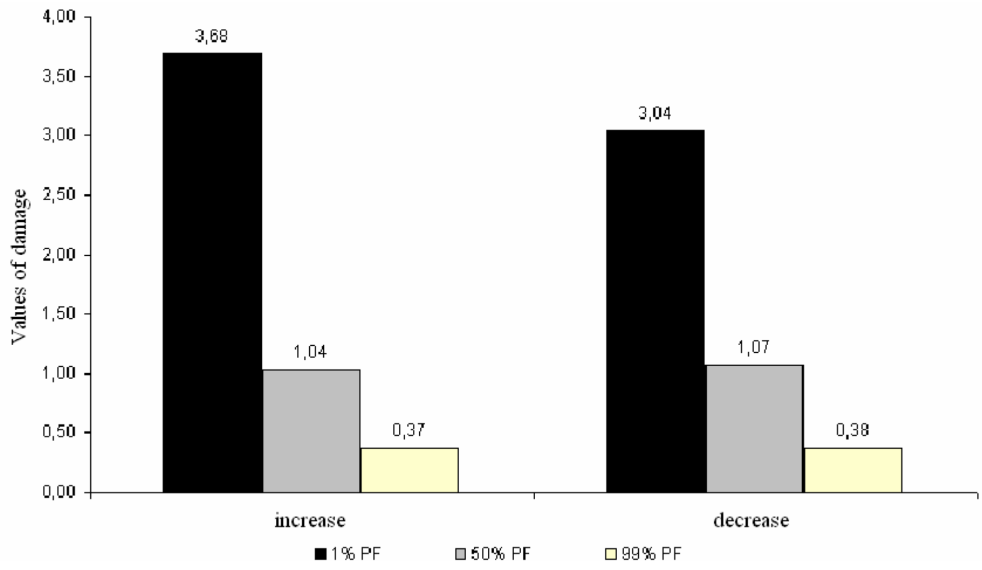

Figure 7. Values of the damage - theory of Marin.

The Figure 8 is presented the values of the damages using the theory of the averages proposal for Mansur.

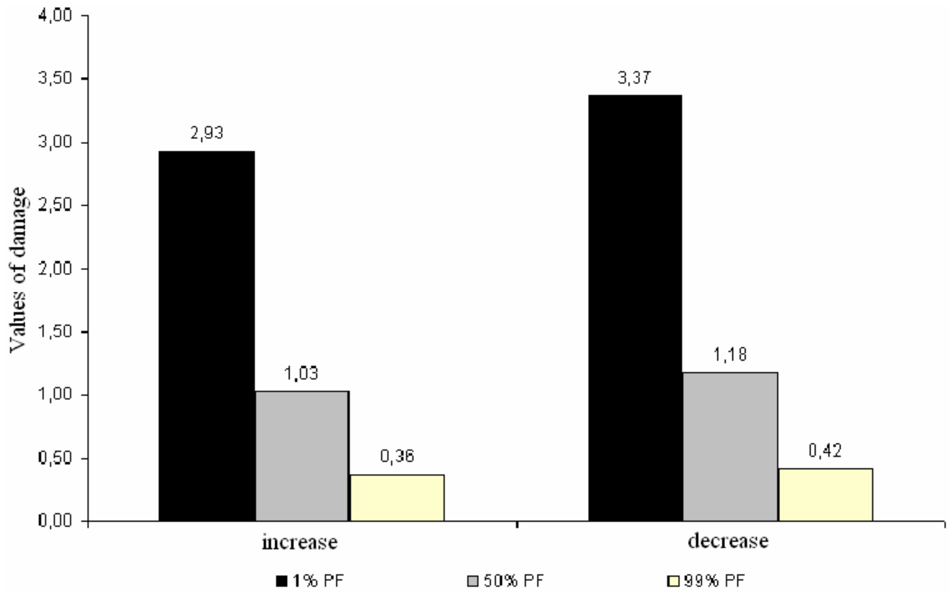

Figure 8. Values of the damage - theory of Mansur.

3.1. Proposal of a new Methodology for calculation of Damage: Study of the Progressive Damage

After studies and experimental verifications of the theories of accumulation of damage of Palmgren-Miner, Corten-Dolan and of Marin, was observed that none of them considered the history of the passed tensions, that is, of each applied parcel.

The methodology proposal for Mansur, to predict the life of materials submitted to the fatigue, considers the description of the accumulation of the damage, from the applied mean stresses for each parcel, during the life of the component in study.

The proposal of this new methodology (Castro, 2007) presents that the total damage is gotten of the addition of partial damages D_i of each parcel of the applied tensions. Each parcel of the D_i damage is gotten by the relation enters the number of N_i cycles under one tension applied σ_i and the waited life N_i. From the second parcel, the number of cycles is multiplied by exponent of the differences of the tensions (actual and passed), divided for the tension actual and multiplied by the accumulated damages of the previous parcels and thus successively. In the attempt to consider a study that considers the descriptions of the damages involved in each parcel, they consider the following one, in accordance with the Eq. 9 and 10.

$$D = \sum_{i=1}^{n} Di = D1 + D2 + D3 + \dots \tag{9}$$

$$D = \frac{n_1 \times Exp\left((\frac{\sigma_1 - \sigma_0}{\sigma_1}) \right)}{N_1} + \frac{n_2 \times Exp\left((\frac{\sigma_2 - \sigma_1}{\sigma_2}) \times D_1 \right)}{N_2} + \frac{n_3 \times Exp\left((\frac{\sigma_3 - \sigma_2}{\sigma_3}) \times D_1 \times D_2 \right)}{N_3} + \tag{10}$$

$$+ \frac{n_4 \times Exp\left((\frac{\sigma_4 - \sigma_3}{\sigma_4}) \times D_1 \times D_2 \times D_3 \right)}{N_4} + \frac{n_n \times Exp\left((\frac{\sigma_n - \sigma_{n-1}}{\sigma_n}) \times D_{n-1} \times D_{n-2} \times D_{n-3} \times \dots \right)}{N_n}$$

In this theory, n_1 is the number of cycles that the sample submitted to the tension σ_1, N_1 is the corresponding number of cycles to the life of the component under this tension, removed of curves *S-N-P*, and thus successively. The Figure 9, is presented the results of the Progressive Damage, submitted to the fatigue.

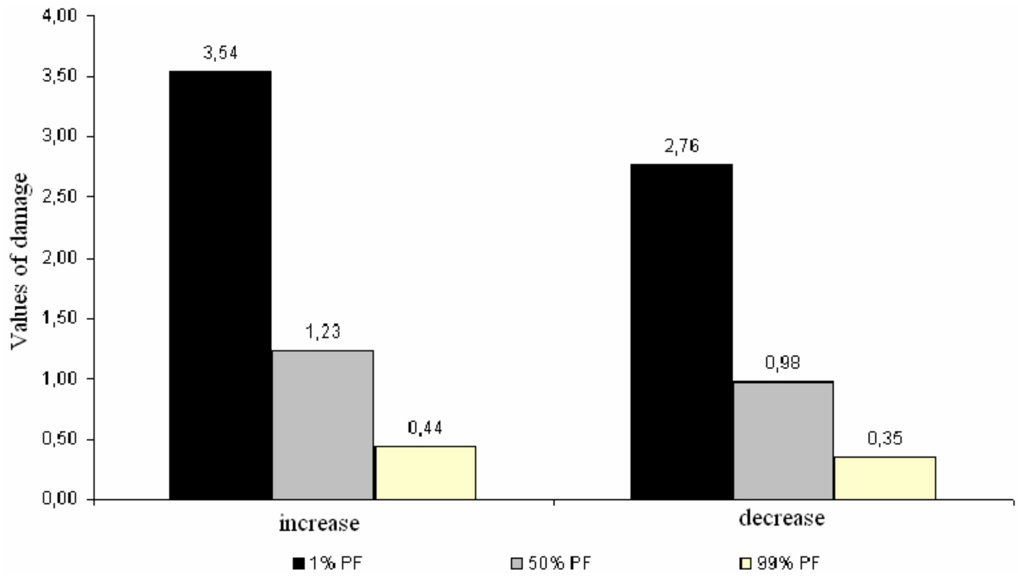

Figure 9. Values of the damage - theory of progressive damage

In the Figures 10, 11 and 12, all are presented the theories with the purpose to compare its results, considering the failures probabilities of 1%, 50% and 99%.

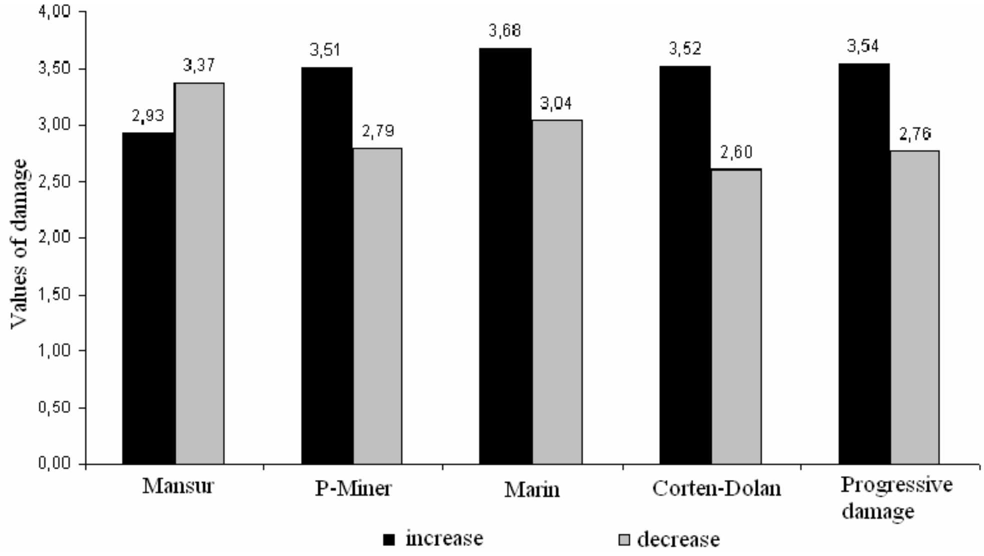

Figure 10. Comparison of the theories for a failure probability of 1%.

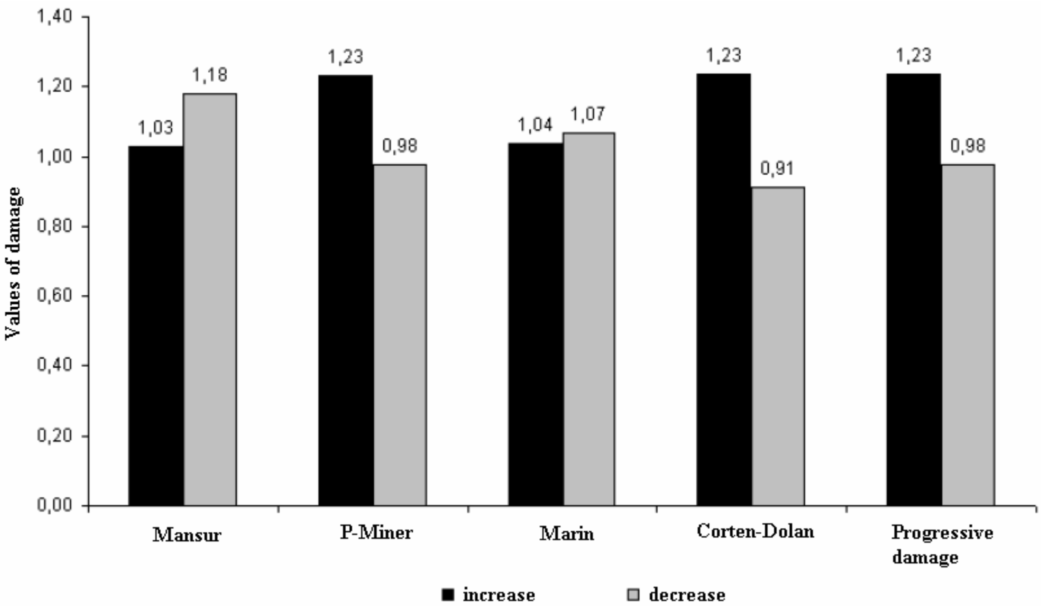

Figure 11. Comparison of the theories for a failure probability of 50%.

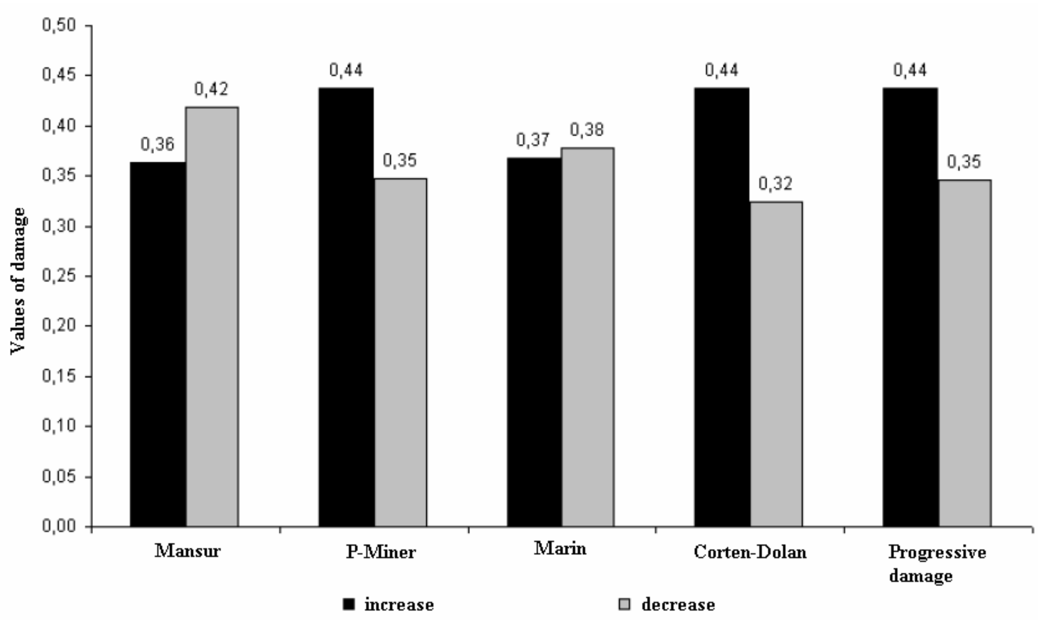

Figure 12. Comparison of the theories for a failure probability of 99%.

4. CONCLUSION

In Figure 10, it is verified that the values gotten for the theories of Mansur, Palmgrem-Miner, Corten-Dolan, Marin and the study of the Progressive Damage, present bigger values that 1. This indicates that the samples had failed with the probability of 1% in relation curves S-N-P.

In Figure 11, the values found with failure probability of 50%. It observes that the theories of Mansur, Palmgren-Miner, Marin, Corten-Dolan and Progressive Damage had resulted satisfactory. But in the theories of Mansur and Marin the values had been more satisfactory for assays with increase tensions. With decrease tension, the values gotten for the theories of Palmgrem-Miner, Marin and Progressive Damage had been next to *1*. The theory of Corten-Dolan got the biggest oscillation between the increase and decrease values.

The results gotten with failure probability of 99%, Figure 12, in accordance with the curves *S-N-P*, theories Mansur, Palmgren-Miner, Corten-Dolan, Marin and the study of the Progressive Damage had supplied lesser values that the *1*. This indicates that the fracture in the samples to the fatigue did not occur, what is not true, therefore the same ones had fractures.

In relation to the loads, it evidences that it has a small difference in relation its application, are increasing or decreasing they for the theories of Mansur, Palmgren-Miner, Marin and Gradual Dano. For the theory of Corten-Dolan, the gotten values had been more dispersed, in relation to the failures probabilities of 1%, 50% and 99%, that is, were more sensible to the considered loads.

5. ACKNOWLEDGEMENTS

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