OPTIMIZATION OF SOURCE TERMS IN BIOHEAT TRANSFER PROBLEMS

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Abstract. *This work aims to optimize the parameters of a laser during a refractive eye surgery by using some optimization methods. Considering that the human eye structures are highly sensitive to a laser emission, the process of optimization is realized in order to obtain temperatures above 85°C at the corneal surface and below 65ºC in endothelium (the innermost layer of the cornea). Consequently, it would be possible to avoid the thermal damage in the endothelium and promote the maximum shrinkage of the cornea during the surgery. The mathematical model that represents the heat transfer in the eye is governed by the Pennes'equation together with the damage function proposed by Henriques and Moritz. The bioheat transfer equation proposed by Pennes contains two terms, representing the effects of blood perfusion and a combination of metabolic heat generation of the tissue and the radiation emitted by an external device. The thermal damage function quantifies the damage accumulated by the tissue exposed to a high temperature promoted by the external device. In this work, we present the results for the optimization of the laser ranging time, applying the Differential Evoution and Particle Swarm Methods. A study is conducted in order to evaluate the performance of each method, to six proposed scenarios for the damage function, associated to three different exposure time intervals for the tissue to the laser irradiation.*

Keywords: Bioheat equation, optimization, damage function, parameter optimization.

1 NOMENCLATURE

h_{bl}	Blood convection coefficient
h_{∞}	Heat transfer coefficient
c	Specific heat
k	Thermal conductivity
S	Source term
L	Tissue length
T	Tissue temperature
T_{bl}	Blood temperature
T_{∞}	Ambient temperature
t	Time
P	Laser power
x	Tissue abscissa
A	Amplitude
f	Frequency
R	Universal gas constant
B	Pre-exponential factor
E_{vap}	Evaporation rate
E_0	Peak irradiance
E	Activation energy
F	Fresnel's reflectance
Ω	Damage function
υ	Decay constant
ε	Emissivity of the cornea
σ	Stefan–Boltzmann constant
μ	Laser absorption coefficient
ρ	Density
Subscripts	
bl	for the blood
i	for the tissue

2 INTRODUCTION

The process of bioheat transfer in organic soft tissues is a complex problem. Therefore, the numerical and computational tools available in engineering are being continuously employed in computer simulations of medical procedures, such as the use of laser in therapeutic treatments or as a tool in refractive eye surgeries. The user of a laser device has made significant progress in ocular surgery, since it is less invasive, thus reducing infection rates.

The laser device is an auxiliary equipment in medicine. Its first application was in ophthalmology. Lasers are used, for example, in photocoagulation of blood vessels, in treatment of tumors, in refractive surgery, for the treatment of some types of cataracts, glaucoma, and corneal ulcers.

In other medical fields, such as in neurosurgery, the laser is largely used, as it promotes the removal of tissue without bleeding, and no physical contact. This device is also used in urology, through optical fibers, such as the surgeries related to the vaporization of kidney stones. In the skin, the laser is applied to the removal of patches of skin, warts, benign tumors, skin rejuvenation and treatment of scars.

The increase of tissue temperature, caused by the laser, causes irreversible damage, associated to the denaturation of proteins, loss of biological functions of the molecules or to their evaporation. Therefore, its therapeutic effect depends on the absorption characteristic of the tissue, the wavelength of the radiation emitted, the density of energy and time of exposure.

Although being largely used in refractive surgery, the heating promoted by the laser can cause an inadequate temperature distribution during a refractive surgery, if not properly controlled. The present work aims to optimize the parameters of a laser by using some optimization techniques. To avoid an irreversible thermal damage in tissue, an idealized objective function is proposed in such a way that the temperature does not exceed 65ºC in the endothelium while remaining above 85°C at the corneal surface, in order to obtain the maximum shrinkage of the cornea (Ooi et al., 2008).

The mathematical formulation of the problem is governed by the Pennes' equation (1948) together with the thermal damage function proposed by Henriques and Moritz (1946). This model involves a heat diffusion process, which has two source terms representing the effects of blood perfusion and the volumetric heat generation of the tissue due to the radiation emitted by an external device. Moreover, the thermal damage function quantifies the damage accumulated in the tissue when exposed to a high temperature. The laser profile is supposed to be Gaussian and its mitigation in the intra-ocular tissue follows the Beer-Lambert Law (Ooi et al., 2008). The results for the optimization of the laser with temporal variation are achieved by means of the following heuristic optimization techniques: Differential Evolution (DE) and Particle Swarm (PS). The performance of each method is analyzed for six profiles of the damage function, together with three different intervals of time for exposure of the tissue to laser irradiation.

3 ANATOMY OF THE HUMAN EYE

The basic function of the eye is to capture light. This, in turn, is focused on the back of the globe (retina), which is converted into electromagnetic pulses, transmitted by the optic nerve and optic tract, to the visual centers of the brain. In these centers, it gives a visual perception, the image recognition and location of the targeted object (SBO, 2009). Figure 1 illustrates the structures involved during the perception of light by the eye.

The geometry of the eye is approximately spherical (about 25 mm diameter) and has three tunics (external, middle and internal), a lens and two liquids. Specifically, the outer layer is composed by the sclera and the cornea, the middle layer is formed by the choroid (which is a well vascularized connective tissue that produces melanin) and ciliary's body, the tunica which is the internal retina (innermost membrane of the eye). In Fig. 1 we can see that the cornea is located in front of the eyeball (with transparent appearance) along with the sclera, and forms the outer envelope of the eyeball. Its curve is sharp whose thickness varies from 0.6 mm to 1.3 mm in the periphery. Its mean diameter is equal to 12 mm, and can vary from 11 mm to 12.5 mm.

The refractive surgeries occur mostly in the cornea, which refracts the light and focuses them on the plane of the retina. During the surgery, the curvature of the cornea is modified. The cornea has six layers: epithelium (50 μm), Bowman's membrane (15 μm), stroma (500 μm), Descemet's membrane (the internal environment) and endothelium (inner layer). Figure 2 shows a diagram of the structures of the cornea.

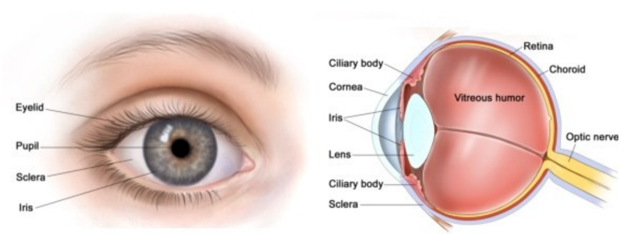

Fig. 1 Human eyeball section (in: MDSC, 2009)

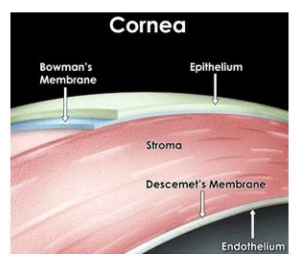

Fig. 2 Layers of the cornea (in: Lasik, 2009)

4 PHYSICAL PROBLEM AND MATHEMATICAL MODELS

The intra-ocular structure is composed of five layers: cornea, aqueous humor, lens, vitreous humor and sclera. In this work, the optic nerve is not included in the formulation, because its contribution to the temperature distribution inside the eye is small. Each layer is assumed homogeneous and thermally isotropic. The thermophysical properties and dimensions of each layer are given in Tab. 1.

In this work we will consider one-dimensional transient heat conduction in a human eye, where the Cartesian coordinate system will be adopted. We will also consider ideal thermal contact between the layers. The heat supplied by the laser is supposed to be absorbed only within the cornea, where over 95% of the energy is absorbed and the remaining percentage is reflected by the surface of the cornea. The loss of heat in the cornea arises from the combination of evaporation of the tear, with air convection and radiation, while in the opposite end (the most external layer of the sclera), convection occurs adjacent to the vascular system.

Tab. 1 Thermal properties of each layer of the human eye (Ooi, 2008)

Layer	Thickness [mm]	k [W m^{-1}K^{-1}]	ρ [kg m^{-3}]	c [J Kg^{-1} K^{-1}]
1. Cornea	0.6	0.58	1050	4178
2. Aqueous humor	3	0.58	996	3997
3. Lens	4	0.40	1050	3000
4. Vitreous humor	15	0.60	1000	4178
5. Sclera	0.1	1.00	1100	3180

According to the above mentioned hypothesis, the physical phenomena is modeled by the following partial differential equation (Pennes,1948), where the blood perfusion was not considered, since the vascularization within the eye is very small:

$$\rho_i c_i \frac{\partial T_i}{\partial t} = \frac{\partial}{\partial x}\left(k_i \frac{\partial T_i}{\partial x}\right) + S_i \qquad 0 < x < x_i \qquad t > 0 \qquad (1.1)$$

$$-k_1 \frac{\partial T_1}{\partial \vec{n}} = h_\infty (T_1 - T_\infty) + \varepsilon\sigma\left(T_1^4 - T_\infty^4\right) + E_{vap} \qquad x = 0 \qquad t > 0 \qquad (1.2)$$

$$-k_5 \frac{\partial T_5}{\partial \vec{n}} = h_{bl}(T_5 - T_{bl}) \qquad x = L \qquad t > 0 \qquad (1.3)$$

where i=1,...,5, represent each one for the structures presented in Table 1.

The source term, which has an exponential decay, is given by Eqs. (2) and (3),

$$S_i = \begin{cases} \phi(t)\mu(1-F)E_0 e^{-\mu x} & , i = 1 \\ 0 & , i = 2,3,4 \text{ and } 5 \end{cases} \qquad (2)$$

$$\phi(t) = \begin{cases} 1 & \text{, laser on,} \\ 0 & \text{, laser off.} \end{cases} \qquad (3)$$

The initial condition of Eq. (1) is given by the solution of the following equations:

$$\frac{d}{dx}\left(k_i \frac{dT_i}{dx}\right) = 0 \qquad 0 < x < L \qquad (4.1)$$

$$-k_1 \frac{dT_1}{d\vec{n}} = h_\infty (T_1 - T_\infty) + \sigma\varepsilon\left(T^4 - T_\infty^4\right) + E_{vap} \qquad x = 0 \qquad (4.2)$$

$$-k_5 \frac{dT_5}{d\vec{n}} = h_{bl}(T_5 - T_{bl}) \qquad x = L \qquad (4.3)$$

The interfaces between the layers must obey the continuity of heat flux and temperature:

$$T_i = T_{i+1} \qquad i = 2,3,4 \qquad (5.1)$$

$$k_i \frac{dT_i}{dx} = k_{i+1} \frac{dT_{i+1}}{dx} \qquad i = 2,3,4 \qquad (5.2)$$

The parameters adopted in Eqs. (1.1-3) and Eqs. (4.1-3) are presented in Tab. 2. During the surgery, the organic tissue is exposed to high gradients of temperature, which causes its thermal damage. Henriques and Moritz (1946) were the pioneers in quantifying the damage to organic tissue caused by an external source generating heat. Assuming that the heating of the organic tissue result in thermal denaturation of proteins, they postulated and tested a dimensionless criterion, based on the Arrhenius' equation to quantify the thermal damage accumulated. This damage function is given by Eq. (6):

$$\Omega(x) = \int_{t_i}^{t_f} B exp\left(-\frac{E}{RT(x,t)}\right) dt \qquad (6)$$

where B is a pre-exponential factor (measuring the frequency of molecular collision), E is the energy of activation for the reaction, R is the universal gas constant, T is absolute temperature, t_i is the time of initial exposure to laser, t_f is the final time of exposure to the laser, and T is the temperature of the tissue in the position where Ω is calculated (Cain and Welch, 1984).

Tab. 2 parameters of the mathematical model (Ooi, 2008)

Parameters	**Value**	**Dimension**
Blood temperature	37	ºC
Environment temperature	25	ºC
Convective blood coefficient	65	W m^{-2} K^{-1}
Convecive environment coefficient	10	W m^{-2} K^{-1}
Evaporation rate	40	W m^{-2}
Emissivity of the cornea	0.975	---
Stefan–Boltzmann constant	5.67×10^{-8}	W m^{-2} K^{-4}
Fresnel's reflectance	0.024	---
Coefficient of laser absorption (assumed as the same as for water)	1900	---

The values of the pre-exponential constant and activation energy for heating the skin at low temperatures, determined by Henriques and Moritz (1946), have been widely used in literature. The values adopted for the parameters of Eq. (6) are displayed in Tab. 3.

Tab. 3 Parameters of damage function (Cain e Welch, 1984)

B **[s^{-1}]**	***E*** **[cal M^{-1}]**	***R*** **[cal $M^{-1}K^{-1}$]**
1.0×10^{44}	7.0×10^{4}	2.0

For skin burns, the values of the damage function are equivalent to 0.53, 1 and 1×10^4 for burns of first, second and third degree, respectively. Medical analyses of burns are less quantitative, establishing the first degree for epidermis, second degree for dermis and third degree for subcutaneous tissues.

In this paper we use six different profiles for an ideal damage profile of the cornea. The laser parameters are thus optimized in order to achieve such ideal damage. Note that the damage function is calculated only within the cornea, according to the assumption that absorption of energy occurs only in this layer.

Equation (7) is the mathematical representation of the first ideal profile for thermal damage in the epithelium, while the other layers of the cornea are supposed to be free of damage. In this work, we considered that the damage starts when $D_1(x)$ is greater or equal to 0.5.

$$D_1(x) = 1.5\,exp\left(-\frac{1250016743}{5\times10^4}x\right),\ 0 \le x \le 6\times10^{-4} \tag{7}$$

The two equations below represent the second and third ideal profiles for thermal damages. These curves are more difficult test cases, since the solution of the damage function is a curve with exponential decay. The D_2 relation, as described in Eq. (8), idealizes the maximum thermal damage in the epithelium and a minimum damage in the other layers of the cornea. Equation (9) represents a thermal damage with a linear decay in the epithelium and a minimum damage in the other layers of the cornea.

$$D_2(x) = \begin{cases} 1.5 & ,\ 0 \le x \le 5\times10^{-5} \\ 0.5 & ,\ 5\times10^{-5} < x \le 6\times10^{-4} \end{cases} \tag{8}$$

$$D_3(x) = \begin{cases} -10^4 x + 1 & ,\ 0 \le x \le 5\times10^{-5} \\ 0.5 & ,\ 5\times10^{-5} < x \le 6\times10^{-4} \end{cases} \tag{9}$$

The next three ideal thermal damage functions represent an irreversible damage in 25%, 50% and 75% from the surface of the cornea (the origin of the system), corresponding to 0.15 mm, 0.3 mm and 0.45 mm. In these cases the behavior of the functions is exponential, as the solution of the damage function. The indices indicate the depth of the damage; the lowest is 25% while the highest 75%.

$$D_4(x) = 1.5\,exp\left(-\frac{1831020481}{25\times10^4}x\right),\ 0 \le x \le 6\times10^{-4} \tag{10}$$

$$D_5(x) = 1.5\,exp\left(-\frac{1831020481}{5\times10^5}x\right),\ 0 \le x \le 6\times10^{-4} \tag{11}$$

$$D_6(x) = 1.5\,exp\left(-\frac{2441360641}{1\times10^6}x\right),\ 0 \le x \le 6\times10^{-4} \tag{12}$$

In this work, the laser power is modeled as expressed in Eq. (13), where A is the amplitude, f is the frequency, t is the time and υ is a decay constant.

$$S(t) = A\,sen(f\,2\pi t)\,exp(-\nu t) \tag{13}$$

5 RESULTS AND DISCUSSION

Optimization deals with maximization or minimization of some objective functions, attempting to find the best solution for a specific problem. It is an area of research that aims to find points of maximum or minimum of a function in a Euclidean space of finite dimension, and it is an effective methodology to solve problems. Several numerical methods have been developed, being usually classified as deterministic or heuristic, as mentioned earlier. In this work heuristic methods will be employed.

In this paper the objective function is the squared difference between the idealized thermal damage function and a calculated one, by optimizing the laser parameters appearing in Eq. (13) (Feng, 2009). The optimization methods used for the optimization of the source term in this study are categorized into heuristic methods: Differential Evolution (DE) and Particle Swarm (PS) (Colaço et al., 2004). Note that the thermal damage function is calculated by Eq. (6), which requires the knowledge of the temperature field within the tissue.

For the solution of Eqs. (1.1-3) and Eqs. (4.1-3), we used the Finite Volume Method (FVM) with a fully implicit formulation (Maliska, 1995). The algebraic equations system obtained, which provides a matrix of coefficients, was solved by the TDMA (TriDiagonal Matrix Algorithm).

The approach employed in this work consists in optimize the laser parameters appearing in Eq. (13), such that the thermal damage function approximates the idealized ones, represented in Eqs. (7)-(12). The optimization algorithms stop if the difference between the calculated and the idealized values have a difference of less than 10^{-5}, or if the number of iterations is greater than 1000.

The optimized parameters for the first ideal thermal damage function (D_1) are presented in Tab. 4. In this and all subsequent tables, DE represents the Differential Evolution Algorithm, while PS stands for the Particle Swarm Algorithm. The number between parentheses, close to the method name, represents the population size used in the method. Also, in all test cases, we considered three different durations for the medical treatment: 10 s, 20 s and 60 s.

From Tab. 4, it can be verified that the PS method, for 10 s of treatment, converged to an objective function equals to 1.7632 after 168 iterations, when a population of 180 individuals was used. When the population was reduced to a population with one third of the initial size, the performance was slightly lower. However, the number of iterations increased, taking 326 more iterations to converge. For 20 seconds of treatment, the PS method with a population size equals to 180 showed a small increased in the objective function, compared to the case with 10 seconds of treatment. Also, the number of iterations increased to 286. For 60 seconds of treatment, with a population size equals to 180 individuals, the final value of the objective function was 1.8205 for the PS method, obtained after 64 iterations. From this table, it can be seen that the shorter the time of treatment, the lower is the value of the objective function. Also, the PS method obtained a better solution than the DE method.

Tab. 4 Optimal solution to D_1

Method	t [s]	A [s^{-1}]	f [$\times10^{14}$ Hz]	υ	Functional	Iterations
$DE_{(60)}$	10 s	2.1833×10^4	1.6043	1.0000×10^{-2}	2.2924	67
	20 s	1.0295×10^5	1.6043	6.3285×10^{-1}	2.1732	81
	60 s	4.5531×10^4	1.6043	2.0654×10^{-1}	2.4752	67
$DE_{(180)}$	10 s	1.8322×10^5	1.6043	1.4388	2.0849	54
	20 s	1.5936×10^5	1.6043	1.1955	2.0826	52
	60 s	1.8705×10^5	1.6043	1.5986	2.5437	51
$PS_{(60)}$	10 s	2.2181×10^5	1.6216	1.7500	1.8243	494
	20 s	1.1946×10^5	1.6213	8.5432×10^{-1}	2.0346	301
	60 s	1.3635×10^5	1.6055	1.0428	1.9676	213
$PS_{(180)}$	10 s	2.6561×10^5	1.6099	1.7500	1.7632	168
	20 s	2.4423×10^5	1.6109	1.7496	1.8105	286
	60 s	2.2406×10^5	1.6043	1.7500	1.8205	64

Figures 3 and 4 show the results obtained with $PS_{(180)}$ method, associated with an interval of 10s for the medical treatment. In Fig. 3, one can notice that the higher temperatures are reached in early times. The surface of the cornea does not exceed 70ºC and the endothelium does not reach 55ºC. From Fig. 4, which represents the temporal evolution of the thermal damage function, it can be seen that no damage is performed in any layer of the cornea, since $\Omega < 0.5$. Figure 5 shows the ideal damage function and the optimized, and it can be observed that, for this particular test case, the obtained solution produced a poor approximation.

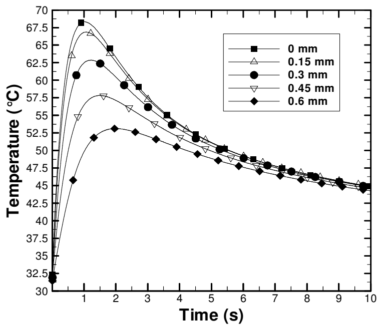

Fig. 3 Temperature profile at fixed point in the cornea to D_1

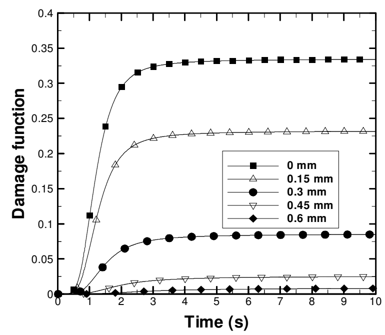

Fig. 4 Damage function profile at fixed point in the cornea to D_1

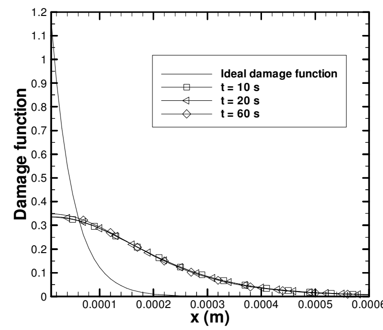

Fig. 5 Damage function profile to the end of time intervals adopted to D_1

Table 5 presents the results for the ideal damage function D_2. For the smallest interval of time (10 s) the best result was obtained by the $PS_{(180)}$ method, with a value for the functional equals to 5.7553. This is slightly below the performance of the $DE_{(180)}$ method, which also required a significantly less amount of iterations. For the treatment intervals of 20 s and 60 s, the $DE_{(60)}$ method obtained the best solutions. The order of magnitude of the amplitude of the laser varies from 10^4 to 10^5 and the decay constant varies from 0.05 to 1.75, which corresponds to the upper limits of the range of search.

Tab. 5. Optimal solution to D_2

Method	*t* [s]	*A* [s^{-1}]	*f* [×10^{14} Hz]	υ	Functional	Iterations
DE$_{(60)}$	10 s	2.5324×10^4	1.6043	1.1570×10^{-2}	6.2690	229
	20 s	3.3028×10^4	1.6216	1.0565×10^{-1}	4.7211	102
	60 s	2.1226×10^4	1.6043	4.5410×10^{-2}	4.0802	63
DE$_{(180)}$	10 s	3.9065×10^4	1.6043	1.0854×10^{-1}	5.8212	81
	20 s	1.7512×10^4	1.6043	1.0000×10^{-2}	4.9815	77
	60 s	1.0579×10^4	1.6043	1.0000×10^{-2}	4.0981	91
PS$_{(60)}$	10 s	2.4741×10^5	1.6049	1.7500	8.3076	187
	20 s	2.1404×10^5	1.6139	1.4565	8.0593	289
	60 s	1.2586×10^5	1.6043	7.1466×10^{-1}	7.0415	364
PS$_{(180)}$	10 s	4.8177×10^4	1.6101	1.8044×10^{-1}	5.7553	623
	20 s	5.0086×10^4	1.6066	1.9355×10^{-1}	5.7512	400
	60 s	7.6193×10^4	1.6051	3.6294×10^{-1}	6.0825	283

Figure 6 illustrates the transient temperature field through a period of 60s, for the optimal solution obtained by the $DE_{(60)}$ method. Note that the increase in temperature occurs in the early 20s, reaching a peak around 60ºC on the surface of the cornea. Moreover, it does not exceed 58ºC in the innermost layer of endothelium. At the end of exposure time all three layers (epithelium, stroma and endothelium) of the cornea are in thermal equilibrium. To complement this analysis, Fig. 7 shows the thermal damage accumulated in the same positions showed in Fig. 6. As expected, the most external layer of the epithelium has a more significant damage. Furthermore, the endothelium remains uninjured, as desired. Figure 8 depicts the curves obtained for the thermal damage in the functional step form for the methods that showed better performance in each time interval. None of the methods obtained a thermal damage function greater than 1.

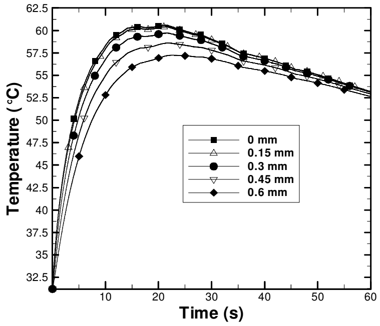

Fig. 6 Temperature profile at fixed point in the cornea to D_2

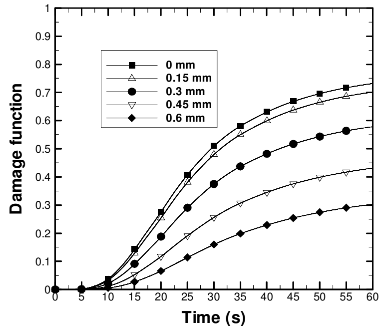

Fig. 7 Damage function profile at fixed point in the cornea to D_2

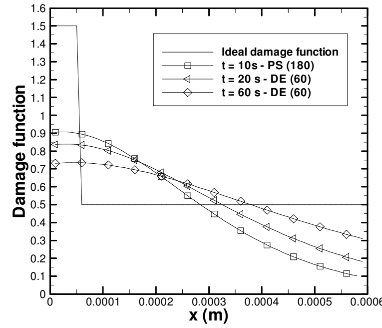

Fig. 8 Damage function profile to the end of time intervals adopted to D_2

Table 6 shows the optimized parameters for the ideal thermal damage curve D_3. The results indicate a variation with a order of magnitude between 10^4 and 10^5 for the laser amplitude, while the decay constant appears to be in the range of 0.02 - 1.7. For this case, the Particle Swarm simulation did not obtain a satisfactory result.

Tab. 6 Optimal solution to D_3

Method	*t* [s]	*A* [s^{-1}]	*f* [$\times 10^{14}$ Hz]	υ	Functional	Iterations
DE$_{(60)}$	10 s	2.5116×10^4	1.6043	1.3442×10^{-2}	4.2378	106
	20 s	7.1203×10^4	1.6043	3.2671×10^{-1}	4.0546	52
	60 s	1.0414×10^4	1.6043	1.0000×10^{-2}	9.5988×10^{-1}	84
DE$_{(180)}$	10 s	5.0258×10^4	1.6043	1.8668×10^{-1}	3.6504	52
	20 s	3.5588×10^4	1.6043	9.0706×10^{-2}	3.7542	58
	60 s	1.3302×10^4	1.6043	1.9259×10^{-2}	8.6183×10^{-1}	135
PS$_{(60)}$	10 s	1.4855×10^5	1.6112	9.0875×10^{-1}	7.3653	337
	20 s	3.5004×10^5	1.6185	1.6672	7.9411	55
	60 s	1.5767×10^5	1.6146	9.8060×10^{-1}	7.4877	118
PS$_{(180)}$	10 s	8.2345×10^4	1.6112	4.0476×10^{-1}	6.2012	407
	20 s	4.9129×10^4	1.6169	1.8669×10^{-1}	5.7486	428
	60 s	4.8298×10^4	1.6205	1.8419×10^{-1}	5.7554	743

Figure 9 presents the evolution of temperature in the cornea over 60s, for the $DE_{(180)}$ method. The largest increase was approximately 28ºC in the epithelium, with a final value close to 60 ºC. One may notice that the damage function reaches its limit, when it starts the irreversible thermal damage of 0.6, at the end of the application of laser, in the regions near the surface of the cornea. Moreover, the more internal layers suffer less disruption due to the heat generated by the laser, and consequently the values for damage function are lower.

Figure 11 shows the behavior of the optimized thermal damage. An examination of the results indicates that regardless of the time interval used, the method was not successful in recovering the ideal damage function.

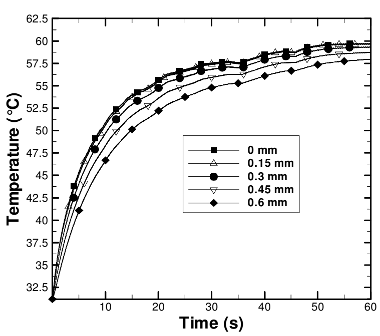

Fig. 9 Temperature profile at fixed point in the cornea to D_3

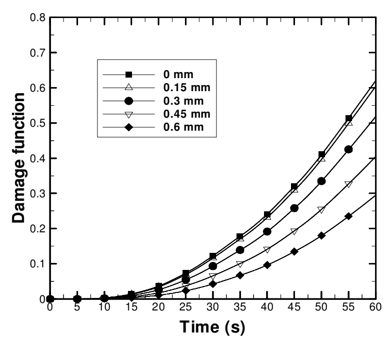

Fig. 10 Damage function profile at fixed point in the cornea to D_3

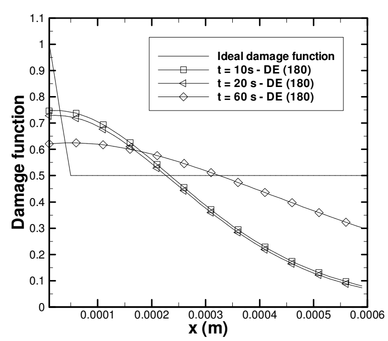

Fig. 11 Damage function profile to the end of time intervals adopted to D_3

Table 7 presents the optimal solutions for the ideal curve representing the thermal damage in 25% of the cornea from the most external layer of the same (curve D_4). The lower value of the functional was equal to 0.064, after 151 iterations, using the $PS_{(180)}$ method. For all time intervals, the PS method with 180 individuals obtained the best performance. These figures suggest that values of the amplitude have an order of magnitude ranging from 10^4 to 10^5.

Tab. 7 Optimal solution to D_4

Method	*t* [s]	*A* [s^{-1}]	*f* [×10^{14} Hz]	υ	Functional	Iterations
DE$_{(60)}$	10 s	2.6302×10^{4}	1.6043	2.4781×10^{-2}	2.3034	57
	20 s	1.5384×10^{4}	1.6043	1.0000×10^{-2}	3.2525	83
	60 s	1.1102×10^{5}	1.6043	6.7410	2.5213	56
DE$_{(180)}$	10 s	1.5606×10^{5}	1.6043	9.7285×10^{-1}	1.0841	52
	20 s	1.8507×10^{5}	1.6093	1.2622	2.8981	52
	60 s	2.2310×10^{5}	1.6043	1.7136	2.0899	54
PS$_{(60)}$	10 s	1.9138×10^{5}	1.6098	1.3019	8.5630×10^{-1}	186
	20 s	2.4662×10^{5}	1.6043	1.7500	6.7225×10^{-1}	66
	60 s	2.1435×10^{5}	1.6125	1.4475	7.9039×10^{-1}	196
PS$_{(180)}$	10 s	2.5144×10^{5}	1.6200	1.7500	6.6913×10^{-1}	171
	20 s	2.7353×10^{5}	1.6096	1.7500	6.4369×10^{-1}	151
	60 s	2.4463×10^{5}	1.6174	1.7500	6.7562×10^{-1}	176

Figure 11 presents the thermal field at certain points of the cornea for the optimal solutions obtained by the PS$_{(180)}$ method, during 20s of laser application. Variations in temperature are more pronounced compared to those obtained in previous simulations. At the surface, the temperature reached a maximum value equals to 72.5ºC in the cornea and less than 55ºC in the endothelium, at 2s. Since the temperature in the endothelium does not exceed 65 º C, it is expected that this region will not be damaged by the laser energy absorbed. In fact, it is apparent from Fig. 12 that the behavior of thermal damage in each layer is below 0.5 in this region. However, at the surface of the cornea there is a considerable increase in the amount of thermal damage, compared to values previously obtained. Figure 13 shows the profile for the damage function obtained by the optimal solutions generated by the PS$_{(180)}$ method. From this figure, it can be noticed that damage function obtained is very close to the ideal one.

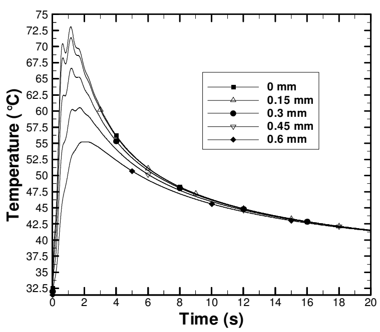

Fig. 11 Temperature profile at fixed point in the cornea to D_4

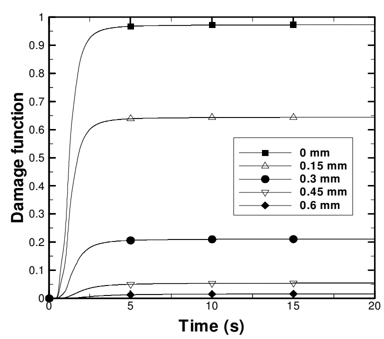

Fig. 12 Damage function profile at fixed point in the cornea to D_4

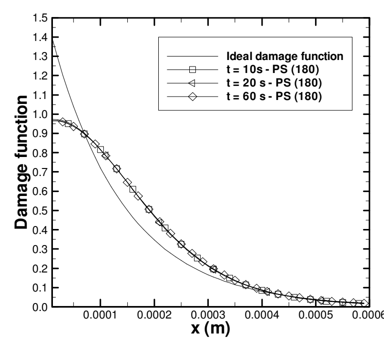

Fig. 13 Damage function profile to the end of time intervals adopted to D_4

Table 8 shows the results for the optimization of the parameters that aim to represent the equivalent thermal damage to the cornea at a depth of 50% from the surface (curve D_5). Note that the magnitude of the amplitude, as in previous test cases, varies in the range from 10^4 to 10^5, while the decay constant is between 10^{-2} and 1.71. The best performance was obtained for the PS method with 60 individuals, with a functional equivalent to 0.3839 at 458 iterations, for the case of exposure time equals 60 seconds. The number of individuals is a relevant factor in these methods, since an increase in this parameter generates a better solution.

Tab. 8 Optimal solution to D_5

Method	*t* [s]	*A* [s^{-1}]	*f* [×10^{14} Hz]	υ	Functional	Iterations
DE$_{(60)}$	10 s	3.3691×10^{4}	1.6043	9.0051×10^{-2}	3.8193	54
	20 s	1.8498×10^{4}	1.6043	1.4575×10^{-2}	1.1169	65
	60 s	1.0232×10^{5}	1.6043	5.1829×10^{-1}	4.8450×10^{-1}	58
DE$_{(180)}$	10 s	2.4205×10^{4}	1.6043	1.0000×10^{-2}	2.6420	96
	20 s	6.5323×10^{4}	1.6043	2.7187×10^{-1}	4.8168×10^{-1}	60
	60 s	4.7673×10^{4}	1.6043	1.6757×10^{-1}	9.7101×10^{-1}	58
PS$_{(60)}$	10 s	2.4849×10^{5}	1.6080	1.7101	1.6074	134
	20 s	2.3215×10^{5}	1.6207	1.5675	1.4685	221
	60 s	1.8951×10^{5}	1.6103	1.2097	1.1188	247
PS$_{(180)}$	10 s	1.5121×10^{5}	1.6100	9.0335×10^{-1}	8.2399×10^{-1}	214
	20 s	1.6174×10^{5}	1.6078	9.8429×10^{-1}	8.9261×10^{-1}	360
	60 s	8.4762×10^{4}	1.6216	4.2123×10^{-1}	3.8386×10^{-1}	458

The behavior of the temperature over the 60s of laser application, for the $PS_{(180)}$ method, is presented in Fig. 14. There is a significant increase in temperature at the surface of the cornea around 5s. After this, there is a decay of temperature in all layers of the cornea. This phenomenon is explained by the way the source term is formulated, given by Eq. (13). This is because at early times the laser power is maximum, which justifies the high temperatures up to 5s. Moreover, as time progresses the power decreases, and also the temperature field. Thus, it is expected a greater thermal damage at the surface of the cornea, as seen in Fig.15. Moreover, from this figure is apparent that the tissue is damaged during the period of 5s, where the power source is more intense. Figure 16 shows the profile of the damage function for the optimal solutions obtained. The methods that provided the best solutions are the PS and DE method, for 60 and 20 s, respectively.

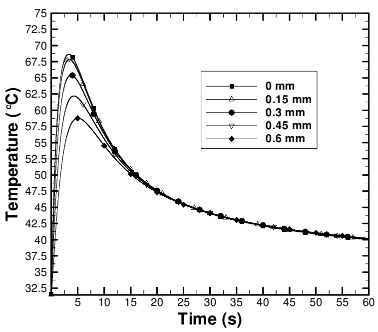

Fig. 14 Temperature profile at fixed point in the cornea to D_5

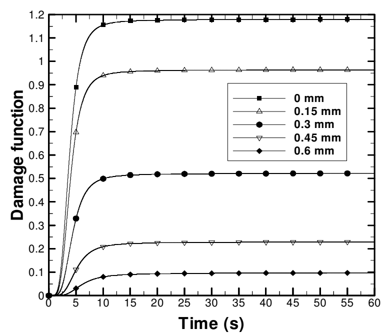

Fig. 15 Damage function profile at fixed point in the cornea to D_5

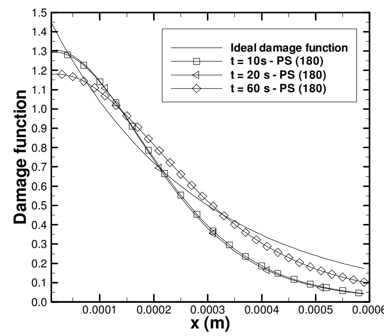

Fig. 16 Damage function profile to the end of time intervals adopted to D_5

Table 9 presents the optimal solutions for the ideal curve representing the thermal damage in 75% of the cornea from the most external layer (curve D_6). The lower value obtained for the functional is 7.1238 after 733 iterations, using the $PS_{(180)}$ method. For all time intervals, the PS method with 180 individuals obtained the best performance, except in the range of 1 min, where the $DE_{(60)}$ method was better. The figures for the amplitude present an order of magnitude varying from 10^4 to 10^5.

Tab. 9 Optimal solution to D_6

Method	t [s]	A [s^{-1}]	f [$\times10^{14}$ Hz]	υ	Functional	Iterations
$\mathbf{DE_{(60)}}$	10 s	2.7397×10^4	1.6043	1.4957×10^{-2}	4.1634×10	75
	20 s	2.0286×10^4	1.6043	1.6932×10^{-2}	2.4409×10	84
	60 s	7.5497×10^4	1.6043	3.7043×10^{-1}	3.2681	53
$\mathbf{DE_{(180)}}$	10 s	3.2384×10^4	1.6043	5.0104×10^{-2}	4.0216×10	74
	20 s	2.0898×10^4	1.6043	1.9873×10^{-2}	2.4369×10	110
	60 s	1.3924×10^4	1.6043	1.6385×10^{-2}	7.3098	92
$\mathbf{PS_{(60)}}$	10 s	1.2659×10^5	1.6134	6.6356×10^{-1}	5.1394×10	203
	20 s	2.2359×10^5	1.6043	1.4132	6.3996×10	329
	60 s	1.6085×10^5	1.6183	9.1725×10^{-1}	5.6654×10	273
$\mathbf{PS_{(180)}}$	10 s	5.0379×10^4	1.6172	1.7212×10^{-1}	3.6138×10	489
	20 s	3.0900×10^4	1.6153	8.0766×10^{-2}	2.1658×10	433
	60 s	1.3906×10^4	1.6044	2.2506×10^{-2}	7.1238	733

Figure 17 shows the temperature field for the $PS_{(180)}$ method during 20s. As mentioned earlier, the higher temperatures are reached at the beginning of the application of laser. However, it is noteworthy that the maximum temperature does not exceed 66ºC (surface of the cornea). Thus, the maximum shrinkage of the cornea is not obtained, because the temperature does not reach 85ºC. In contrast, the temperature of the endothelium (more internal layer of thecornea) did not exceed 65ºC, i.e., the tissue is not damaged as can be seen from the thermal history of the damage indicated in Fig. 18 ($\Omega < 0.5$).

The curves obtained for the damage function with the best solutions of Tab. 9 are given in Fig. 19. It appears that the curve that is closest to the ideal curve is generated by the optimal solution of $DE_{(60)}$ method in 60s. In the more internal layers of the cornea the curves are similar. However, in more external layers that does not occur, since the temperatures obtained in the regions near the surface not to exceed 85ºC.

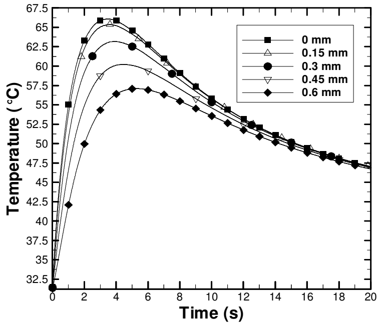

Fig. 17 Temperature profile at fixed point in the cornea to D_6

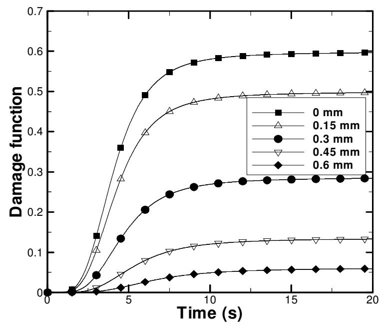

Fig. 18 Damage function profile at fixed point in the cornea to D_6

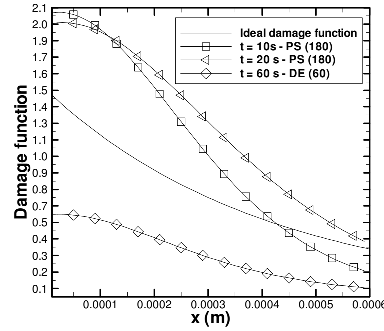

Fig. 19 Damage function profile to the end of time intervals adopted to D_6

7. CONCLUSION

The research reported in this paper employed heuristic optimization methods to obtain the optimal parameters of a function that aimed to represent the application of a laser refractive surgery in the human eye. The parameters were optimized to represent ideal profiles of some idealized damage function, whose objective was to test the methods and represent the desired thermal damage after the medical procedure. The purpose of optimization is to improve the quality of treatment and to provide a better manipulation of the laser.

The results of parameter optimization of the function with temporal variation allow us to conclude that, the exposure time of the tissue to external source of heat, and quantity of possible solutions in the iterative process, are relevant factors in the process of optimization. Moreover, as a general observation, the Particle Swarm method presented a better performance in view of the value of the functional.

Given the solutions for each case studied, it was noted that the curves of thermal damage in 25 and 50% of the cornea, were relatively well reproduced by the PS method with 180 individuals in the population. Therefore, the methodology is appropriate to simulate the refractive surgery when damage to the heat is generated at half of the cornea.

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9. REFERENCES

Cain, C. P. and Welch, A.J., 1984, “Measured and predicted laser-induced temperature rises in the rabbit fundus”, Bio-Medical Engineering Program, College of Engineering, The University of Texas at Austin, pp. 60-70.

Colaço, M.J., Orlande, H.R.B. and Dulikravich, G.S, 2004, “Inverse and optimization problems in heat transfer, Invited Lecture”. In: 10th Brazilian Congress Of Thermal Sciences and Engineeing – ENCIT, Rio de Janeiro, RJ.

Feng, Y., Fuentes, D.Hawkins, A., Bass, J.M. and Rylander, M.N., 2009, “Optimization and real-time control for laser treatment of heterogeneous soft tissues”, Computer. Methods Application. Mechanical . Engineering, pp. 1742–1750.

Henriques, F.C.and Moritz, A.R. 1946, “The conduction of heat to and through skin and the temperaures attained therein. A theoreticl and an experimental investigation”, American Journal of Pthology, pp. 53-549.

Lasik MD Laser Eye Surgery. Accessed in: http://www.lasikmd.com: February, 5th 2009.

Lopes, C.F., Souza, M.V.C., Scofano Neto, F., Colaço, M.J. Caldeira, A.B. and Guedes, R.O.C., 2008, “Inverse Problem of Estimation of the Perfusion Coefficient in Cancerous tissues Subjected to Hyperthermic Treatment”. In: International Conference on Engineering Optimization – ENGOPT, Rio de Janeiro, RJ.

Maliska, C.R., 1985, “Transferência de calor e mecânica dos fluídos computacional”, LTC- Livros Técnicos e Científicos, Editora S.A, Rio de Janeiro, RJ.

MSDC - Macular Degeneration Support Canada. Accessed in: http://www.amdsupport.ca: July, 22nd 2009.

OOI, E., ANG, W. and ANG, E., 2008, “A boundary element model of the human eye undergoing laser thermokeratoplasty”. Computers in Biology and Medicine, pp.727-737.

Pennes, H.H., 1948, “Analysis of Tissue and Arterial Blood Temperatures in the Resting Human Forearm”, Journal of Applied Physiology, pp. 93-122.

SBO - Brazilian Society of Ophthalmology. Accessed in: http://www.sboportal.org.br: April, 30th 2009.

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