Placement Optimization of Piezoelectric Actuators for Vibration Reduction of Beam Structures Using SVD Analysis

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Abstract. *Piezoelectric effect is based on the property of energy conversion from mechanical to electric one and vice versa. It was discovered by the Curie Brothers in 1880. Piezoelectric actuators and sensors have received a lot of attention from researchers, mainly related to its applicability in mechanic vibration active control. Nowadays these structures, which integrate sensors, actuators and controllers, are known as intelligent structures. The study of the placement and the number of actuators is a fundamental part for effective intelligent structure design. Misplaced and bad distributed actuators, as well as a poor number of actuators, can cause lack of controllability and observability of the system. The present paper deals with the optimum placement of more than one actuators attached to the host structure, in this case a simply supported beam. The study is based on the simulation of the dynamic behavior of structures with bonded piezoelectric elements. For that purpose, modal decomposition and a quantification index obtained by singular value decomposition of the control matrix [B] are used. Applying the finite element method (FEM), it is possible to verify the vibration modes of the beam and to simulate the optimal place for the piezoelectric actuators.*

Keywords: *: Actuators and sensors placement, smart structures, singular analysis value*

1. INTRODUCTION

A active vibrations control nowadays is real, its results are more effective than passive vibration control. According to new emphasis, the response of a structure can be minimized by using integrated active elements, such as: sensors, actuators and controllers. This integration allows to control system answer to outside excitation for specified modes. Hence, it is possible to compensate the effects that could move away its response of acceptable levels. Nowadays, these systems, integrating structures, sensors, actuators and controllers, are known intelligent structures (Lima Jr., 1999).

Several technologies and materials have been proposed for the development of the intelligent structures. Among these materials, there are the piezoelectric materials, especially the ceramics, PZT - piezoelectric zirconate titanate lead and polymer films, PVDF - piezoelectric vinylidene fluoride. The active control using piezoelectric materials is a topic of a lot of interest among the researchers. The reason of this interest is because the piezoelectric materials are small, lightweight and resilient against adverse working environments. Moreover piezoelectric materials have been used as both actuators and sensors, because they are owner of the ability to transform mechanical energy to electrical and vice versa (Crawley and De Luis, 1987). The ceramics have high stiffness, therefore are used as actuators. While that the polymer films more handler than ceramics and can be produced in complex geometric shapes, for this reason, they are used as sensors (Tzou and Fu,1994).

The intelligent structure design is dividing at least in three areas: modeling, actuators and sensors placement, and system controller (Oliveira, 2008). In a good intelligent structure design, actuators and sensors placement study is fundamental part to avoid undesirable effects in structure under active control, such as: lack of observability and controllability system and spillover (Costa e Silva and Arruda, 1997). In the present paper a study about optimum piezoelectric actuators and sensors placement bonded in a flexible structure, considering modal and spatial controllability measurements are present. To quantify the controllability index, we intend to use the singular value analysis through the [S] matrix (Wang, 2001).

2. MODELING OF BEAM WITH PIEZOELECTRIC ATTACHED

Euler-Bernoulli beam equation with piezoelectric actuator based on the Love's Postulates, appropriated Lamé parameters and piezoelectric effect is (Novozhilov, 1970 and Banks and Smith, 1995):

$$\rho h b \frac{\partial^2 w}{\partial t^2} + YI\frac{\partial^4 w}{\partial y^4} = bF_z + b\frac{\partial m_x}{\partial y} \quad (1)$$

where: ρ is the material density (kg/m^3), Y is the Young module (GPa), h is the thickness (m), b is the width (m), I is the inertia momentum (m^4), F_z is the force (N), m_x is the momentum (Nm/m) and w is the transversal displacement (m).

2.1 Actuator equation

The contribution of piezoelectric material can be divided in two classes: inside and outside contribution. The inside contribution is related to the structural material propriety, such as: mass, stiffness and damping and is although present when no electric potential is applied. In contrast, the outside contribution appears only when an electric potential is apply to the PZT. It results in forces and moments due to the induced deformation of the PZT. (Tzou and Fu, 1994; Banks and Wang, 1995).

The deformation amplitude induced in PZT is:

$$\varepsilon_{pe} = (\varepsilon_y)_{pe} = \frac{d_{31}}{h^a}\phi^a \tag{2}$$

where: ε_{pe} is the induced deformation, d_{31} is the piezoelectric constant (m/V) and ϕ^a is the electric potential applied to the actuator (V).

The individual stress, σ_y (GPa), in PZT is:

$$(\sigma_y)_{pe} = -Y_{pe}\varepsilon_{pe} = -Y_{pe}\frac{d_{31}}{h^a}\phi^a \tag{3}$$

Integrating the voltage over the face of a fundamental element it follows that external force and moment resultants due to the activation of the pzt patches, can be written as:

$$(bN_y)_{pe} = -Y_{pe}bh_{pe}\varepsilon_{pe} = -Y_{pe}bd_{31}\phi^a \tag{4}$$

$$(bM_y)_{pe} = -\frac{1}{8}Y_{pe}b\left[4\left(\frac{h}{2}+h^a\right)^2 - h^2\right]\varepsilon_{pe} = -\frac{1}{2}Y_{pe}b\left(h+h^a\right)d_{31}\phi^a \tag{5}$$

Equation 4 and Eq. 5 can be modified for finite piezoelectric length. Hence, for PZT with the lengths y_1 and y_2, the forces and moments are:

$$bF_y = (bF_y)_{pe} = -X_{pe}\left(y\right)\chi_{pe}\left(y\right)\frac{\partial\left(bN_y\right)_{pe}}{\partial y} \tag{6}$$

$$bm_x = (bm_x)_{pe} = \chi_{pe}\left(y\right)\frac{\partial\left(bM_y\right)_{pe}}{\partial y} \tag{7}$$

with

$$\chi(y)_{pe} = \begin{cases} 1 & y_1 \le y \le y_2 \\ 0 & otherwise \end{cases} \quad and\ X(y)_{pe} = \begin{cases} 1 & y < \left(y_1+y_2\right)/2 \\ 0 & y = \left(y_1+y_2\right)/2 \\ 1 & y > \left(y_1+y_2\right)/2 \end{cases} \tag{8}$$

2.2 Sensor equation

The piezoelectric sensor equation can be obtained throughout piezoelectricity property and relation between stress and beam deformation. The piezoelectric material thickness is smaller than beam thickness, so the piezoelectric sensor deformation is constant, equal structure surface deformation (Lima Jr. and Arruda, 1999).

The voltage through the electrodes is:

$$\phi^s = -\int_{h^s} E_3 dz = h^s\left(h_{31}\varepsilon_y^s + \beta_{33}D_3\right) \tag{9}$$

where: β_{33} is the electric impermeability (m/F).

Rewrite the Eq. 9, we get:

$$D_3^s = \frac{1}{\beta_{33}}\left(h_{31}\varepsilon_y^s - \frac{\phi^s}{h^s}\right) \tag{10}$$

where D_3^s is defined like electric charge per area unit.

Integrating Eq. 10 over the electrode surface, we get the total surface load. The voltage open circuit condition can be obtained thorough zero charge, so:

$$\phi^s = \frac{h^s}{S^e}\int_{S^e}\left(h_{31}\varepsilon_y^s\right)dS^e = \frac{h^s}{y_2-y_1}\int_{y}\left(h_{31}\varepsilon_y^s\right)dy \tag{11}$$

So the Euler-Bernoulli beam sensor equation, is:

$$\phi^s = -\frac{h^s}{L^s}\int_{y_1}^{y_2}\left(h_{31}h_r^s\frac{\partial^2 w}{\partial y^2}\right)dy \tag{12}$$

Where: h_r^s is the distance from beam neutral line to sensor medium plane (m) and the h_{31} is the constant that relation open circuit voltage, given an input voltage (C/Fm).

3. MATHEMATICAL MODELING

The piezoelectric linear equation is given:

$$\begin{array}{l}\{\sigma\} = \left[c^E\right]\{\varepsilon\} - [e]\{E\} \\ \{D\} = [e]^T\{\varepsilon\} + [\xi^\varepsilon]\{E\}\end{array} \tag{13}$$

where:

$$\begin{array}{l}[e] = \left[c^E\right][d] \\ [\xi^\varepsilon] = [\xi^\sigma] - [d]^T\left[c^E\right][d]\end{array} \tag{14}$$

where: $\{\sigma\}$- stress tensor; $\{\varepsilon\}$ - deformation tensor; $\{E\}$ - electric field vector; $\{D\}$ - electric displacement vector; $\left[C^E\right]$ - elasticity matrix for constant electric field; $[e]$ - piezoelectric constants matrix; $[\xi^\varepsilon]$ - dielectric constants tensor for constant deformation $[\xi^\sigma]$ - dielectric constant matrix for constant stress; $[d]$ - constant matrix of piezoelectric deformations.

The variational principle equation for piezoelectric material (Lima Jr, 1999), is given by:

$$\begin{array}{l}\iiint\limits_V \rho\{\delta u\}^T\{\ddot{u}\}dV + \iiint\limits_V \{\delta\varepsilon\}^T\left[c^E\right]\{\varepsilon\}dV - \iiint\limits_V \{\delta\varepsilon\}^T[e]^T\{E\}dV - \iiint\limits_V \{\delta E\}^T[e]\{\varepsilon\}dV \\ -\iiint\limits_V \{\delta E\}^T[\xi]\{E\}dV = \iiint\limits_V \{\delta u\}^T\left\{\bar{f}_V\right\}dV + \iint\limits_{S_f}\{\delta u\}^T\left\{\bar{f}_S\right\}dS - \iint\limits_{Sq}\delta\phi\sigma_q dS\end{array} \tag{15}$$

3.1 The finite element method

The structure has been modeling with isoparametric beam element, therefore with three degree of freedom by node. The shape function for horizontal displacement is linear, while for vertical displacement is cubic. So the node approximations are (Bathe, 1996):

$$\left\{\begin{array}{c} u \cong \bar{u} = [N_u]\{q_i\} \\ w \cong \bar{w} = [N_w]\{q_i\} \\ \phi \cong \bar{\phi} = [N_\phi]\{q_i\}\end{array}\right. \tag{16}$$

where:

$$\{q_i\} = \left[\begin{array}{cccccc}\bar{u}_i & \bar{w}_i & \bar{\theta}_{y_i} & \bar{u}_j & \bar{w}_j & \bar{\theta}_{y_j}\end{array}\right]^T \tag{17}$$

3.2 Strain Energy

Strain energy of piezoelectric materials in the matrix form, is:

$$\delta U = \iiint\limits_V \{\delta\varepsilon\}^T\{\sigma\}dV - \iiint\limits_{V_{pe}}\{\delta E\}^T\{D\}dV_{pe} \tag{18}$$

In the beam model proposed, there are two domains. The first is structure material domain, V_{st}, and the second is piezoelectric material, domain, V_{pe}. For the second domain, the Euler-Bernoulli beam model equation, is:

$$\begin{array}{l}\{\varepsilon\} = e_x, \quad \left[C^E\right] = E_{pe}, \quad \{\sigma\} = \sigma_x \\ \{e\} = e_{31}, \quad [\zeta^\varepsilon] = \zeta_{33}^\varepsilon, \quad [D] = D_3, \quad \{E\} = E_3\end{array} \tag{19}$$

Substituting Eq. 19 and Eq. 16 into Eq. 18 yields:

$$[k_{qq}] = L\left(E_{st}A_{st} + E_{pe}A_{pe}\right)\int_0^1 [B_u]^T[B_u]d\xi + L\left(E_{st}I_{st} + E_{pe}I_{pe}\right)\int_0^1 [B_w']^T[B_w']d\xi \tag{20}$$

$$[k_{q\phi}] = E_{pe}A_{pe}d_{31}L\int_0^1 [B_u]^T [B_{\phi u}]d\xi - \left(h + \frac{h_{pe}}{2}\right)E_{pe}A_{pe}d_{31}L\int_0^1 [B'_w]^T [B_{\phi w}]d\xi \tag{21}$$

$$[k_{\phi\phi}] = \frac{\zeta_{33}^{\varepsilon}A_{pe}L}{h_{pe}^2}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \tag{22}$$

where the index *st* refers structure material and the index *pe* refers piezoelectric material.

3.3 Kinetic energy

With the kinetic energy variational equation, applying for proposed beam element, we get:

$$[m_{st}] = \rho_{st}A_{st}L\int_0^1 [N_q]^T [N_q]\,d\xi, \qquad [m_{pe}] = \rho_{pe}A_{pe}L\int_0^1 [N_q]^T [N_q]\,d\xi \tag{23}$$

where: $[m_{st}]$ is the structure mass matrix and $[m_e]$ is the piezoelectric mass matrix.

3.4 The work

Applying the work variational, done by outside loads and forces, we get:

$$\{f_q\} = \int_0^1 [N_w]^T \{\bar{f}_s\}\,Ld\xi \tag{24}$$

$$\{q_s\} = -\int_0^1 [N_\phi]^T \sigma_q Ld\xi \tag{25}$$

3.5 Equation global system

Global system equations of displacement of Euler-Bernoulli beam model, is given by:

$$\begin{cases} [M_{qq}]\{\ddot{q}_i\} + [k_{qq}]\{q_i\} + [k_{q\phi}]\{\phi_i\} = \{F_s\} \\ \qquad\qquad [k_{\phi q}]\{q_i\} + [k_{\phi\phi}]\{\phi_i\} = \{Q_s\} \end{cases} \tag{26}$$

In the piezoelectric sensor there isn't voltage apply ($Q_s = 0$). So the electric potential yield by sensor is:

$$\{\phi_i\} = -[K_{\phi\phi}]^{-1}[K_{\phi q}]\{q_i\} \tag{27}$$

Taking Eq. 27 into Eq. 26, we get the global system equation for a beam with actuator attached, that is:

$$[M_{qq}]\{\ddot{q}_i\} + [K^*]\{q_i\} = \{F_s\} + \{F_{el}\} \tag{28}$$

where:

$$[K^*] = [K_{qq}] - [K_{q\phi}][K_{\phi\phi}]^{-1}[K_{\phi q}] \tag{29}$$

$$\{F_{el}\} = -[K_{q\phi}][K_{\phi\phi}]^{-1}\{\phi_a\} \tag{30}$$

3.6 Controllability index

Controllability and observability are concepts related to control theory. A system is controllable if it can be conducted into a particular state by applying an adequate controller. The efficiency of the controller is related to the control input needed to realize a desired performance criterion. The objective is to determine the optimal placement of the piezoelectric actuators on the beam structure, that is applying the actuators most effective. Thus optimal placement means that the control input needed to realize a predefined performance criterion is minimal, or vice versa, that the control energy supplied by the piezoelectric actuators for a given control input is maximal. Wang and Wang (2001) suggest a controllability index that indicates the amount of energy supplied by the piezoelectric actuators for a given control input.

From the state equation of the system the control force $\{f_c\}$ applied to the system in vector form is given by (Benaroya, 2004):

$$\{f_c\} = [B]\{u\} \tag{31}$$

Multiplying Eq. 31 with the transposed control force vector yields

$$\{f_c\}^T\{f_c\} = \{u\}^T[B]^T[B]\{u\} \tag{32}$$

Using the singular value decomposition, the matrix $[B]$ can be written as

$$[B] = [U][\Sigma][V]^T \tag{33}$$

where $[U]^T[U] = [I]$ and $[V][V]^T = [I]$.

Taking the singular value composition (svd) of the matrix $[B]$, Eq. 33 into Eq. 32 yields

$$\{f_c\}^T\{f_c\} = \{u\}^T[V][\Sigma]^2[V]^T\{u\} \tag{34}$$

where

$$[\Sigma]^2 = \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \dots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_k^2 & \dots & 0 \\ 0 & 0 & \dots & 0 & \dots & 0 \end{bmatrix} \tag{35}$$

By defining a new control input vector $\{v\}$

$$\{v\} = [V]^T\{u\} \tag{36}$$

Equation 34 can be written as

$$\{f_c\}^T\{f_c\} = \{v\}^T[\Sigma]^2\{v\} = \sum_{i=1}^{k} \sigma_i^2 v_i^2 \tag{37}$$

As the amount of control energy supplied by the piezoelectric actuators is proportional to the scalar product of the control force vector $\{f_c\}^T\{f_c\}$, the objective of the optimization is to maximize the scalar product $\{f_c\}^T\{f_c\}$. Wang and Wang (2001) suggest introducing a controllability index Ω based on σ_i:

$$\Omega^2 = \prod_{i=1}^{k} \sigma_i^2 \tag{38}$$

It follows that, Eq. 37 and Eq. 38, the control energy supplied by the piezoelectric actuators is proportional to the controllability index Ω. The higher controllability index Ω is, then higher will be the scalar product magnitude $\{f_c\}^T\{f_c\}$ and the energy supplied by the piezoelectric actuators. As the singular values of the control matrix $[B]$, σ_i, and the controllability index Ω depend on the location of the piezoelectric actuators on the beam, the proposed index can be used to determine the optimal placement of the actuators.

4. NUMERIC SIMULATION

In the following, the optimal placement of up to three piezoelectric actuators for vibration control of the first three mode shapes of simply supported beams will be examined (Oliveira, 2008).

First, a simply supported beam with the geometrical dimensions and material properties given in Tab. 1 is considered.

Table 2 shows the geometrical dimensions and material properties of the piezoelectric actuators bonded to the beam structure.

4.1 Case 1: Vibration control of 1st mode shape

Initially, a simply supported beam with only one actuator is considered, Fig. 1.

Furthermore, only the first mode shape of the beam should be controlled. Figure 2 shows the value of the controllability index against the location of the piezoelectric actuator bonded to a simply supported beam. It can be seen that the controllability index is the highest when the actuator is placed at 50 % of the beam length. This result is expected as the bending moment is maximum at the midspan of the beam.

Table 3 shows the optimal actuator position values considering beam length and beam length percentual.

In the Fig. 3 is possible to see, in red color, that FRF of the closed loop of the first mode is less than the open loop FRF to same mode, in blue color. It was shown that the actuator was efetive to reduce the vibration of beam.

Table 1. Geometrical dimensions and material properties of the simulated beam.

Variable	***Value***	***Unit***
Length L	1,500	m
Width b	0.075	m
Thickness h	0.075	m
Density ρ	7800	kg/m^3
Elastic modulus Y	210x10^9	N/m^2
Cross sectional area A	5.625x10^{-3}	m^2
Moment of inertia I	2.6367x10^{-6}	m^4

Table 2. Geometrical dimensions and material properties of the piezoelectric actuators.

Variable	***Value***	***Unit***
Length L_a	0.150	m
Width b_a	0.075	m
Thickness δ	0.010	m
Density ρ_a	7600	kg/m^3
Reduced elastic modulus C_{11}	139x10^9	N/m^2
Cross sectional area A_a	7.500x10^{-4}	m^2
Moment of inertia I_a	1.089x10^{-6}	m^4
Piezoelectric constant e_{31}	-6.800	C/m^2

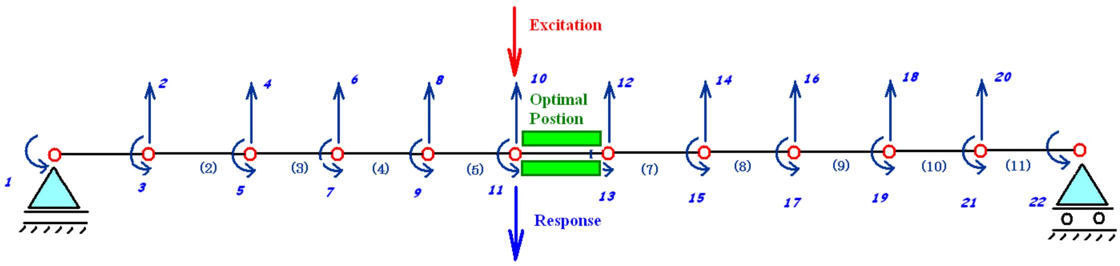

Figure 1. Beam setup with one array of piezoelectric actuators.

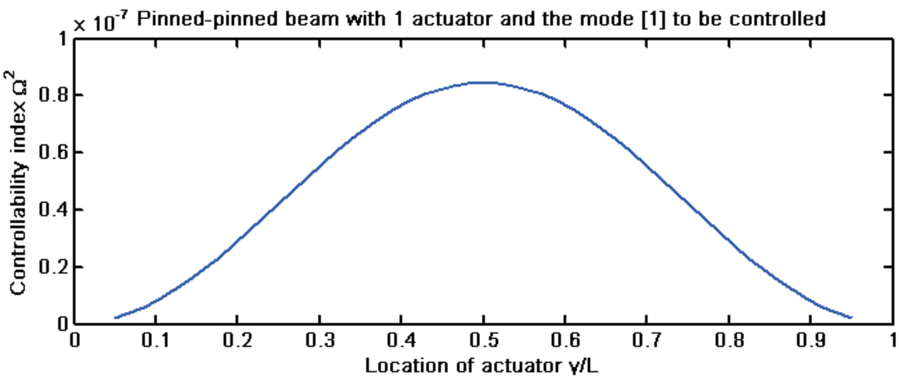

Figure 2. Controllability index for first mode shape of simply supported beam.

Table 3. Optimal actuator position for first mode shape of simply supported beam.

	***Optimal position* (m)**	***Optimal position* (%)**
Actuator 1	0.750	50.00

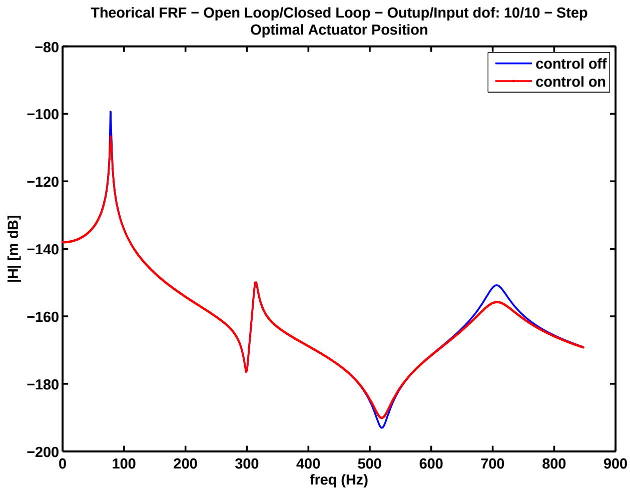

Figure 3. FRF to beam control on and beam control off for first mode.

4.2 Case 2: Vibration control of 1st and 2nd mode shape

From the second mode shape on, more than one actuator are necessary to control the bending vibrations of the beam. Considering the first and second mode shape of a simply supported beam, two piezoelectric actuators should be used to decrease the beam vibrations, Fig. 4.

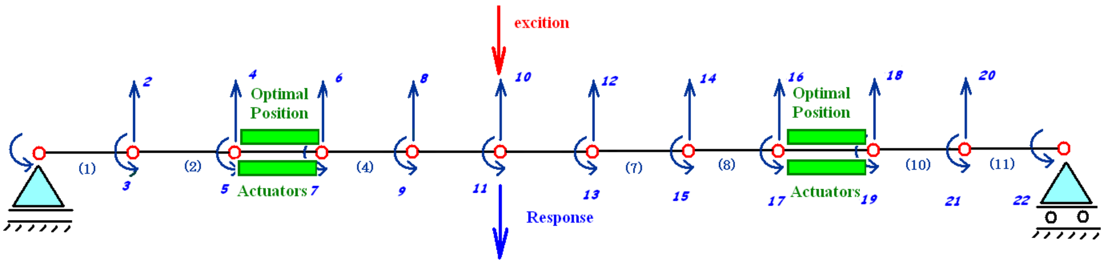

Figure 4. Beam setup with with two array of piezoelectric actuators.

In this case, the optimal locations of the actuators are not as obvious as in the preceding example. Figure 5 and Tab. 4 show the controllability index with respect to the location of the two actuators. When is used two actuators the controllability index can be represented by a 2D graph. Figure 5 is a contour plot from the surface controllability index. It can be seen that optimal placement is founded when the controllability index is the highest and the actuators must be placed at about 30% and 70% from the left end of the beam. Further, it can be observed from the contour plot, Fig. 5, that the controllability index is sensitive to the locations of the two actuators.

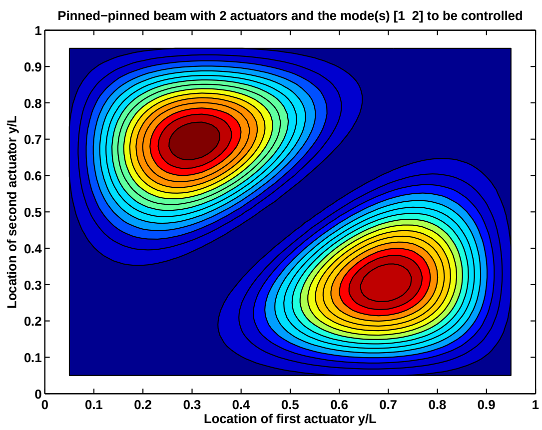

Figure 5. Controllability index for first two mode shapes of simply supported beam.

Table 4. Optimal actuator positions for first two mode shapes of simply supported beam.

	Optimal position (m)	*Optimal position (%)*
Actuator 1	0.445	29.67
Actuator 2	1.055	70.33

Figure 6 show the FRF to open loop and closed loop control using two piezoceramic actuator to control de first and second mode shape. It is possible to see that the amplitude of two modes shape was reduced.

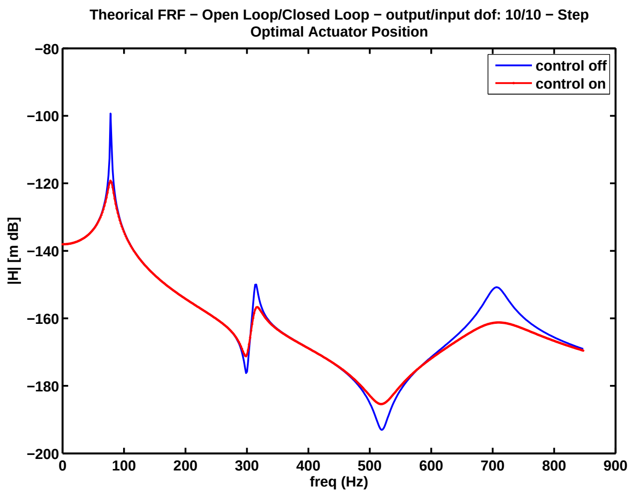

Figure 6. FRF to beam control on and beam control off for first two modes.

4.3 Case 3: Vibration control of 1st, 2nd and 3rd mode shape

Assuming that the first, second and third mode shape of a simply supported beam should be controlled, it is advisable to use two or three piezoelectric actuators, Fig. 7.

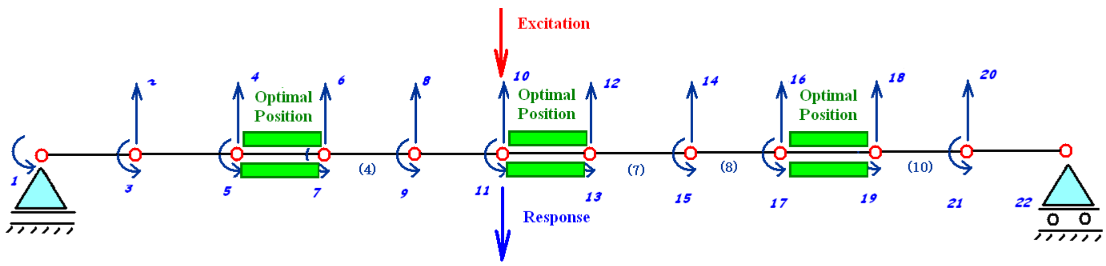

Figure 7. Beam setup with three arrays of piezoelectric actuators.

Figure 8 shows the controllability index when are used three actuators. Different from Fig. 5 it is not possible to represent the controllability index using a 3D graph. In this case was used a 1D graph with three curve, one to each actuator. In case that three actuators are used to control the 1st, 2nd and 3rd mode shape of a simply supported beam, the actuators should be placed at around 21%, 50% and 79% of the beam length (see Fig. 8 and Tab. 4).

Figure 9 show the FRF to beam control on and beam control off for first three modes. The amplitude of first three mode shape was reduced.

5. SUMMARY

Optimal placement of piezoelectric actuators should be consider as precondition for effective reduction of the bending beam structures vibration that minimizes the effort of the controller. In this paper, the optimal placement, analytic and numerical models, for piezoelectric actuators on beam structures have been proposed.

The optimal placement and the number of piezoelectric actuators used in flexible structure, depends on the mode that will be controlled and on the boundary conditions. The singular value decomposition of the control matrix $[B]$ yields a index that quantifies numerically the actuator optimum placement to control the vibration.

The index is shown to be sensitive to appoint the optimal actuator positions.

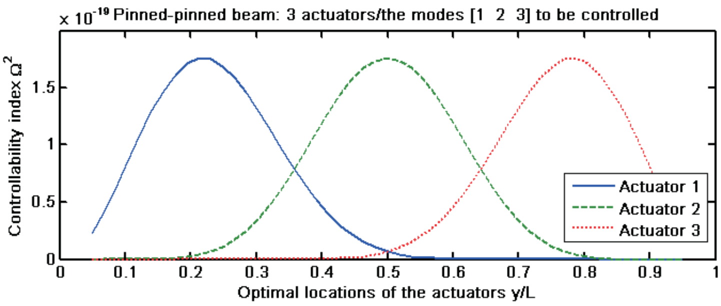

Figure 8. Controllability index for first three mode shapes of simply supported beam with three bonded actuators.

Table 5. Optimal actuator positions for first three mode shapes of simply supported beam with three bonded actuators.

	***Optimal position* (m)**	***Optimal position* (%)**
Actuator 1	0.315	21.00
Actuator 2	0.750	50.00
Actuator 3	1.185	79.00

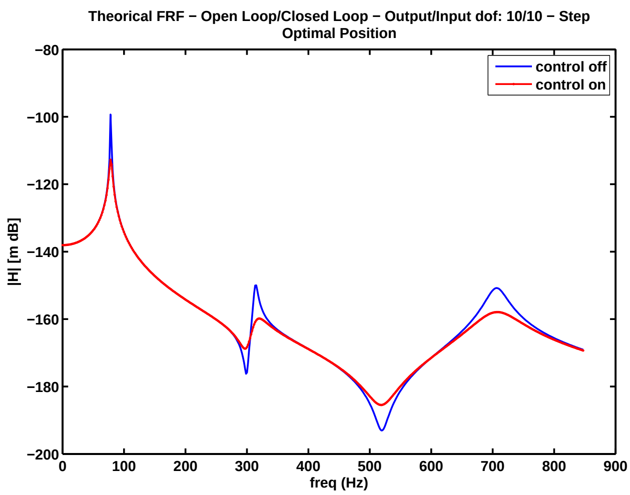

Figure 9. FRF to beam control on and beam control off for first three modes.

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