A NON-LINEAR RAINFALL-RUNOFF MODEL APPLIED TO AN AMAZON CATCHMENT TO SIMULATE THE FLOW DURATION CURVES

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Abstract. An alternative to attend the energy demand from isolated communities is the use of the water resources of catchments, close to these communities through the setting up of micro and mini hydropower. The first difficulty is determining the potential, as most of these catchments have no data flow. These data serve to obtain the flow duration curves of small rivers. These curves are necessary for the design of hydroelectric power. The main of this work is the implementation of a rainfall-runoff model for the simulation of flow. Due to the rainfall-runoff process being known non-linear, a modification is applied to this model, based on the residuals relationship between a simple linear model and the observed discharge. Was adopted a new procedure for calibration of the modified model. To analyze the performance of the model were used two evaluation criteria, the classic RMS and Nash criterion, where the RMS criterion was used in the calibration process and the Nash criterion was used to analyze the accuracy of the model in calibration and validation periods. For calibration were used seven years and for validation were used six years of rainfall and flow data. The simulated discharge are compared to observed discharge, showing that after the modification of the model, it has a nonlinear behavior, with satisfactory results in flow duration curves simulations, where they are used for these purposes on the catchments targets without discharge data. These curves together with the values of gross heads are used for estimating the installed power of the sites, demonstrating that the small rivers can serve for the setting up of micro and mini hydro to the supply the small isolated communities.

Keywords: rainfall-runoff model, Amazon catchment, flow duration curve.

1. INTRODUCTION

The Amazon region has a vast network of small watersheds. These watersheds can be used to attend the energy demand from small isolated communities in the region through the setting up of micro and mini hydro power (Blanco et al. 2008). In this case, the plants would have a power less than 1000 kW (DNAEE / Eletrobrás, 1985). However, most of these catchments do not have flow data, complicating the potential determination. As the rainfall data are only data type available for these small catchments, this work has as main objective the implementation of a rainfall-runoff model to simulate the discharges and consequently the flow duration curves for a small Amazon catchment. Models that follow the rainfall-runoff relationship applied to an Amazon catchment to simulate the flow duration curves already exist in the literature, for example, Blanco et al., (2005, 2007 and 2008-b), obtaining satisfactory results.

In this present work, the model used was based on developed by Kachroo and Natale (1992). This model, although linear, it quest to treat the problem of non-linearity existing in hydrological cycles. This model was used to simulate the flow duration curves. This curve is an important methodological tool to the estimate of the power of hydrological sites. The estimation of power will allow the analysis of the small catchments hydroelectric exploitation, answering the energy demands of small isolated communities.

2. RAINFALL-RUNOFF HYDROLOGICAL MODEL

The model developed by Kachroo and Natale (1992) is based on the modification of a simple linear model, due the relationship between rainfall and runoff be known non-linear and variable in time (Kachroo and Liang, 1992). The model modification was fundamented on the relationship between residuals of a simple linear model and observed discharge as evidence of non-linearity. This non-linearity is mainly due the existence of physical phenomena such as infiltration, evapotranspiration and spatial variations of the catchment subsoil. The variability in time is explained by the seasonality of rainfall and discharges that characterize the hydrological systems. Nevertheless, the simplifying hypothesis of the model is the invariability in time of the rainfall-runoff relationship, justified by the small size of the catchment studied.

The simple linear model, with the error output, as vector / matrix is given by:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \\ y_{m+1} \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 & 0 & \cdots & 0 \\ x_2 & x_1 & \cdots & 0 \\ \vdots & \vdots & \vdots\vdots\vdots & \vdots \\ x_m & x_{m-1} & \cdots & x_1 \\ x_{m+1} & x_m & \cdots & x_2 \\ \vdots & \vdots & \vdots\vdots\vdots & \vdots \\ x_n & x_{n-1} & \cdots & x_{n-m+1} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_m \end{bmatrix} + \begin{bmatrix} e_1 \\ e_1 \\ \vdots \\ e_n \end{bmatrix} \quad (1)$$

Where n is the number of observations of y and m is the memory length. The equation (1) can be written by the compact form:

$$Y=XH+E \quad (2)$$

The equation 2 is the simplest representation of a casual, time-invariant rainfall-runoff relationship, where the discharges are represented by the convolution between the rainfall end the impulsional response. That impulsional response, also known as transfer function, is the mathematical representation of the relationship between inputs and outputs of a linear and time-invariant system (figure1).

The optimum solution for impulsional response H is given by:

$$H=[X^TX]^{1}X^TY \quad (3)$$

where Y is an [n,1] column vector of output series, X is an [n,m] matrix of input series, H is an [m,1] column vector of impulsional response and E is an [n,1] column vector of residuals.

2.1 The modificated model

The modification of the model was proposed due the different behaviors shown by the results of the simple linear model for regions of low, medium and high discharges in the same catchment (Kachroo and Natale, 1992). It showed that the impulsional response of the system should be adequate for each of these regions.

The idea was to input gain factors (*W*) into the impulsional response in order to suit the linear system (Fig. 1 with the blue box) to the non-linear behavior for low, medium and high discharges.

After that modification, the system has a new impulsional response suited for each discharge region, called weighted impulsional response (Fig. 2 with the red box).

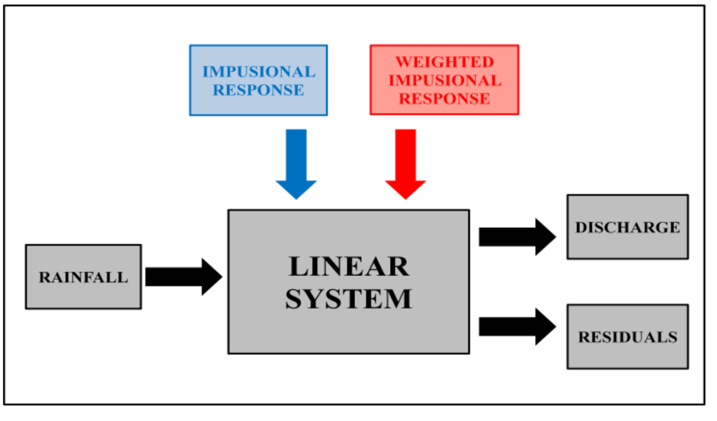

Figure 1. Scheme of the rainfall-runoff system (simple/modified).

The new optimum solution for impulsional response H is given by:

$$H=[X^TWX]^{1}[X^TWY] \quad (4)$$

where *W* is the gain factor suitable to each flow region.

2.2 Model evaluation criteria

Two criteria were adopted to evaluate the performance of the model. The first criterion is used to optimize the value of m and is based on the sum of squares of differences between observed and estimated discharges, i.e., the classic root mean square (*RMS*) given by:

$$RMS=\sum_{j=1}^{n}\frac{\sqrt{(Q_j-\hat{Q}_j)^2}}{n} \tag{5}$$

where Q is the observed discharges, $\hat{Q}$ is the estimated discharges and n is the number of observations.

The second criterion used is based on the comparison of the residual variance with the initial variance. That criterion evaluates the model performance, independently of the length of vector or the scale of discharges. This criterion is called Nash or R^2.

$$R^2=1-\sum_{j=1}^{n}\frac{(Q_j-\hat{Q}_j)^2}{(Q_j-\bar{Q})^2} \tag{6}$$

where Q is the observed discharges, $\hat{Q}$ is the estimated discharges, $\bar{Q}$ is the mean observed discharge and n is the number of observations.

2.3 Selected small catchment

The Igarapé da Prata is located 160 km east of Belém, Pará (Fig. 2) and has a drainage area of 82 km^2. It serves as a source catchment, because it is the only region that has consecutive flow data necessary to calibrate and validate the rainfall-runoff model, but has no direct rainfall data. It was used the nearest rainfall data station from near the hydrometric station, distant about 15 km to the north (Table 1). Both the flow data as the rainfall data are daily. These data are available at ANA - Water National Agency of Brazil.

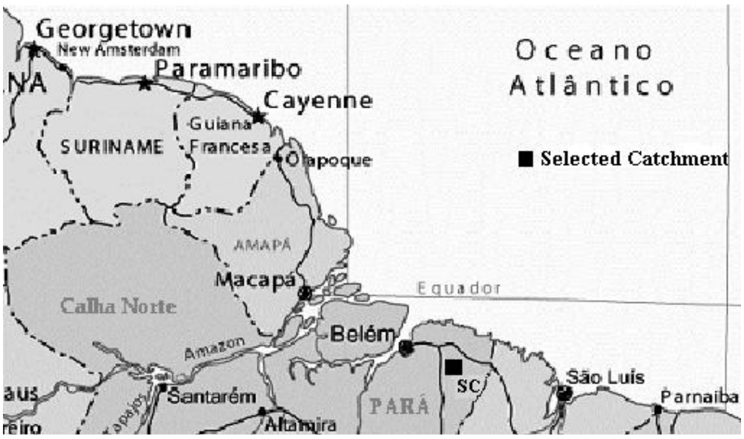

Figure 2. Localization of the selected small catchment.

Table 1. Hydrologic stations.

Station	ANA code	Latitude	Longitude
Hydrometric	31600000	-1°39’06’’	-47°07’03’’
Pluviometric	00147016	-1°33’02’’	-47°07’01’’

2.4 Model calibration

For the model calibration were used 7 years (1992-1999) of rainfall and flow data from the pluviometric station Ourém and the hydrometric station Marambaia (Table 1). It was used the RMS criterion in this period, because this criterion relates the model residuals with the memory length used in the impulsional response of the system.

The results of RMS criterion were disposed under the graphic form in function of the memory length m to the calibration period (Fig. 3), in order to obtain the optimized value of the parameter *m*. Also was constructed the Table 2 that presents the results RMS and Nash criteria and the optimized value of m obtained in the calibration period, considering the permanence discharges.

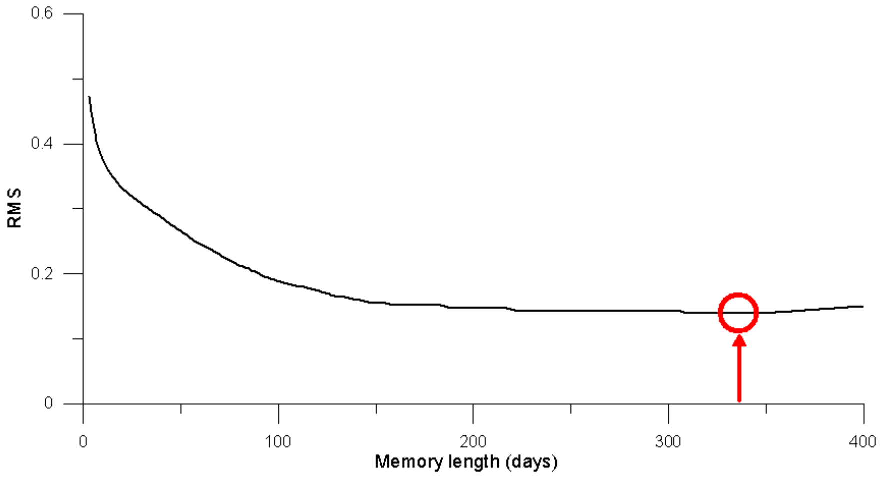

Figure 3. Graphic of RMS residual (calibration).

Table 2. Performance parameters of the model.

Period	RMS	Nash (R^2)	Optimized m
Calibration	0.138	0.893	338
Verification	0.108	0.922	338

After analyzing the results (Fig. 3 and Table 2), we can observe that the model presented good accuracy in the calibration process, however the parameter *m* presented a relatively elevated value, almost one year of rainfall data (Fig. 3 - red circle).

3. RESULTS

3.1. Calibration

For the implementation of the model were used thirteen years (1992-2005) of rainfall and flow data from the pluviometric station Ourém and the hydrometric station Marambaia (Table 1), while for the calibration of the model were used seven years of data to calibration period.

The results were plotted as hydrographs and flow duration curves for both periods.

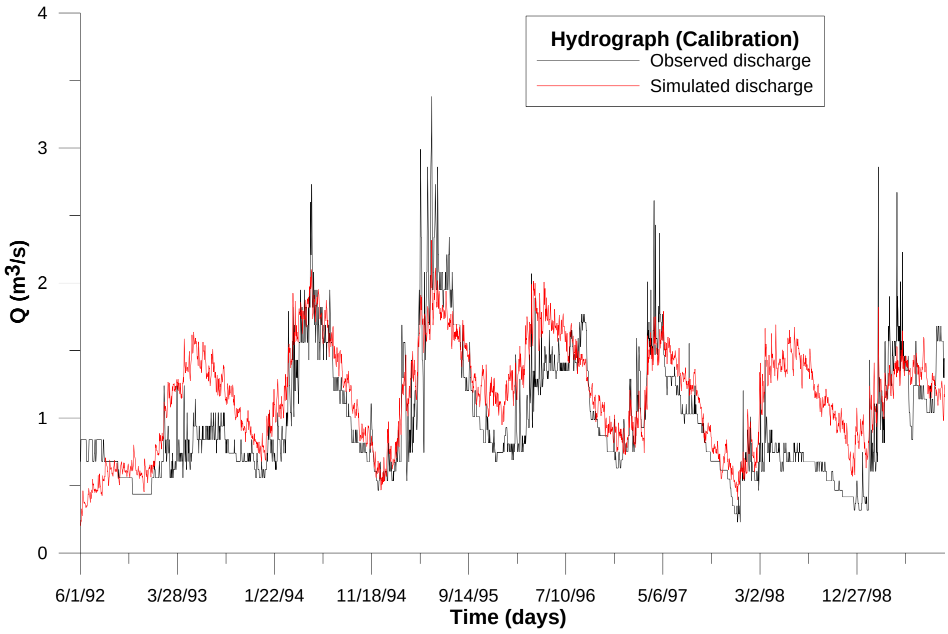

Figure 4. Hydrograph of calibration period.

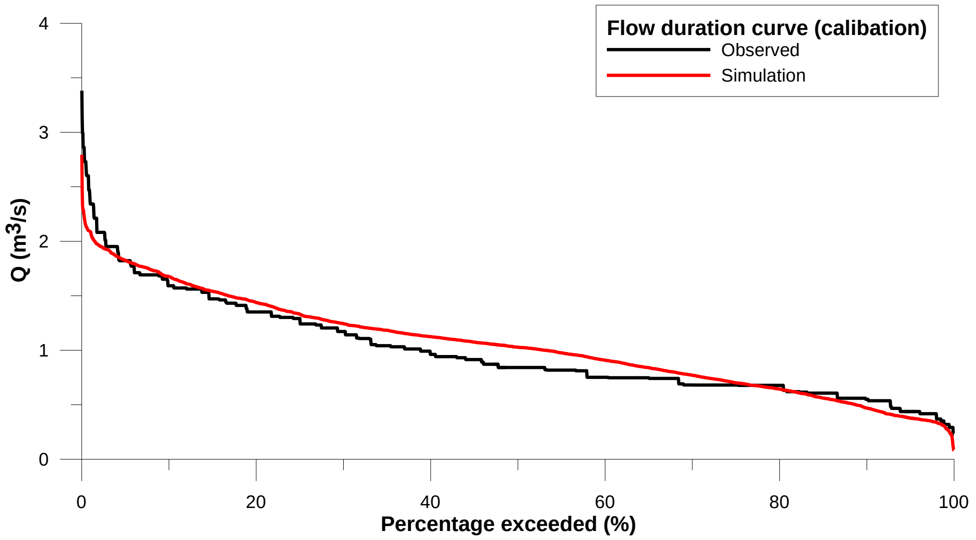

Figure 5. Flow duration curve of calibration period.

3.2. Validation

For the validation period of the model were used six years (1999-2005) of rainfall and flow data from the pluviometric station Ourém and the hydrometric station Marambaia (table 1).

To estimate the discharges in this period, It was used the estimated impulsional response H obtained with the gain factors W and memory length m suited for each flow region in the calibration period.

The results were also plotted as hydrographs and flow duration curves for both periods.

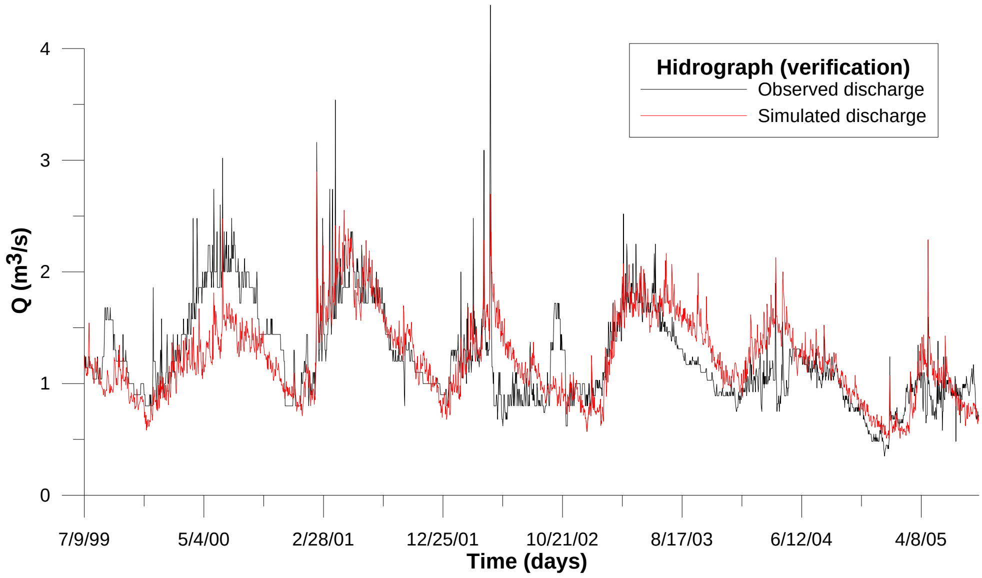

Figure 6. Hydrograph of verification period.

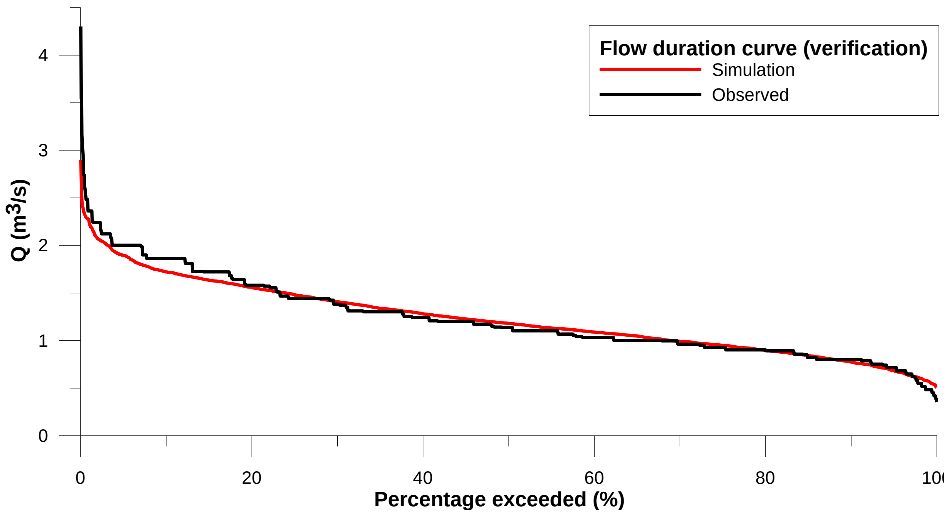

Figure 7. Flow duration curve of verification period.

Analyzing the hydrographs we can observe that great part of the calibration period as of the verification period the model simulates reasonably the discharges, however, the model did not present efficient to simulate the great peaks of discharges, what was better evidenced in hydrograph of the calibration period. This problem can be traduced by the fact that the optimal response impulsional of the model H had an elevated memory length m. This greater memory length is more appropriate for estimating low flow regions, which has higher frequencies and have most relevance in dimensioning process of hydro powers. Therefore, the model has lesser sensitivity to great gradients of input series, i.e., the model answers smoothly for abrupt variations of rainfall data. These great variations are presents especially in periods of high discharges, because this period coincides with the period of higher precipitation in the region. Also this period is the higher ground saturation, causing a reduction in the phenomenon of infiltration.

Already analyzing the flow duration curves, we can notice that the model generated reasonable results, mainly in the verification period. That better result presented by the verification period is due to the fact that in the calibration period, the results of calibration beginning presents more significant residuals, as well as other rainfall-runoff models, the model to be still passing for a process of adaptation to the behavior of the hydrological phenomenon. Other fact for this sensitive difference in the results is due probably the bigger non-linearity from behavior hydrologic in the calibration period.

3.3. Installed power estimation

After obtaining the flow duration curves, together with the values of head (H), it becomes possible to determine the power of the analyzed catchment.

The power of the catchment can be determined by the equation below:

$$P = \eta Q \rho g H \tag{7}$$

where P is the power η is the efficiency of the hydro power, Q is the rated discharge, ρ is the water density, g is the gravity acceleration and H is the nominal head.

The Table 3 shows the estimated power and energy in function of the rated discharge (Q = $Q_{95\%}$, $Q_{75\%}$ and $Q_{50\%}$) obtained from the simulated flow duration curve. In this estimation, was adopted $\eta = 0.8$, $\rho = 1000\ kg/m^3$ and $g = 9.81\ m/s^2$ (DNAEE/ELETROBRÁS, 1985). For the gross head, was adopted the value of 4.0 m (Blanco, 2005-b apud Mesquita et al., 1999).

Table 3. Estimated power

Rated discharge	Discharge (m^3/s)	Head (m)	Power (kW)	Energy (MWh/year)	Firm energy (months/year)
$Q_{95\%}$	0.72	5.0	22.4	187	12
$Q_{75\%}$	0.93	5.0	31.2	205	9
$Q_{50\%}$	1.13	5.0	35.2	154	6

Observing the Table 3, it can be noted that choice of the rated discharge to dimension the energy production depends directly on the energy demand. For example, for a hypothetical small community that has a domestic energy demand of 187 MWh and a productive energy demand of 87,6 MWh, the dimensioned power in function of the rated discharge $Q_{95\%}$ would attend the domestic demand or the productive demand during the 12 months of the year. For the higher discharges ($Q_{75\%}$ and $Q_{50\%}$), the generated power would attend completely both demands, but the periods of energy production would decreases to 9 and 6 months/year respectively. In this case, the rated discharge definition will depend on period of energy demand, that for the increase of the discharge values, obtaining higher powers, it generates lower durations discharges and consequently, the decrease of the of energy supply period. Therefore it is very important to consider the energy demand period in the dimensioning of the energetic production.

4. CONCLUSIONS

The results analysis demonstrates that the model presented higher performance in the flow duration curve prediction than the hydrographs prediction. This lower performance in the hydrographs appears mainly in the difficulty to simulate great peaks of discharge. This difficulty, as already discussed previously, is due the fact that the optimal impulsional response of the model had an elevated value of memory length, appropriated to estimate low flow regions in detriment of higher flow regions forecasting. It makes the model has lesser sensitivity for abrupt variations of the rainfall data.

The obtained results for the flow duration curves forecasting were good, attending thus the main objective of this work. The model presented better results especially for the verification period, due in the calibration period the model is adapting itself on the behavior system, generating higher residuals during this period.

After the result analysis of this present work it can be notice that the model proved to be a good alternative to estimate the flow duration curves of Amazon catchments, which is a fundamental hydrological tool for the dimensioning hydro powers, as well as to be useful at the feasibility analysis of its implantation.

Also is important to emphasize that the dimensioning of the energetic production is direct function of the energy demand period and the flow duration curve is very important is this relationship.

However, the results also demonstrate that to estimate the flow duration curves and consequently the estimation of installed power of small Amazon catchments is necessary to have flow data to calibrate the rainfall-runoff model.

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