ANALYSIS OF THE MOVEMENT OF A PARTICLE ON A DENSIMETRIC TABLE

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Abstract. *The densimetric table is a device designed for separating particles with different densities. It is broadly used in the agricultural industry in order to sort the grains produced into groups with a particular density. This machine is expensive but it is also considered the best at what it does. The problem resides in the fact that each type of cereal has a set of characteristics and with each one, the parameters of the device has to change in order to function properly. Currently the parameters are obtained experimentally. In this work we analyzed and modeled the movement of the grain on the table. It was made feasible by considering the geometric shape of the grain as a sphere and supposing that there was no collision. Using the basic characteristics of the grain such as density, diameter and friction coefficient, vibration parameters of the table and angle of elevation, it was possible to relate them and calculate, via a MATLAB code, the velocities in which the particles move on the table. Thus, we could establish functional zones to the velocities. The results achieved were quite satisfactory when compared to experimental results obtained in the literature.*

Keywords: *densimetric table; grain separation; simulation; MATLAB*

1. INTRODUCTION

The densimetric table has a very wide variety of applications, it is frequently used, on the agricultural industry, as a mean for beneficiation of several kinds of grains, it is also used to aid the process of solid waste processing. The capacity to induce specific gravity separation is used as well in the chemical and food industry. Nowadays , even the recycling industries use the densimetric table. This device consists essentially of a near horizontal surface that vibrates so that there is an in-plane and a normal component of vibration. It has a special surface that allows the passage of air produced by one or more ventilators.

No earlier work on the densimetric table was found, but on the other hand there is an interesting similarity between a densimetric table and a vibratory feeder, both devices work on the same principle. What differentiate them is the passage of air, on a vibratory feeder there is none. A fundamental overview of a feeder is given by Bothroyd *et al.* (1976). An introduction to the description of the micro ballistic transport process is documented by Wiendahl and Ahrens (1984). Lim (1993, 1995) and Wolfsteiner (1997) developed simulation tools for a vibratory feeder.

This paper, inspired by the work of Lim (1995), presents the dynamic analysis of a moving part on a densimetric table. The forcing agent was assumed to provide a simple harmonic motion. The factors that were taken into account were the angle frequency and amplitude of vibration, the coefficient of friction, inclination of the table, velocity of the air in the table and diameter and density of the particles. The developed model can be used to study the various parameters that influence motion on the densimetric table.

2. MECHANICAL MODEL

The separation process on a densimetric table is based on a micro ballistic principle that is driven by an oscillating track. A simplified basic model of the particle on an inclined densimetric table is shown in Fig.1. Only two bodies are considered in this model, the vibrating surface of the table and the particle that is being conveyed.

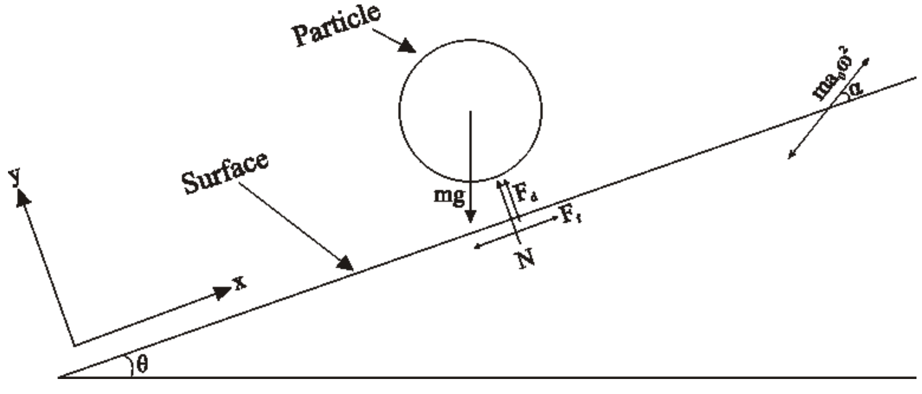

Figure 1. Simplified scheme of a densimetric table

In Fig. 1 it is possible to notice the forces that are actually acting upon the particle. *mg* is the gravitational force, where m is the mass of the particle and g is the local gravity, F_f is the frictional force, F_d is the force promoted by the air stream and N is the normal force at the surface.

The angle of vibration is measured from the surface of the table, and is known as α, the amplitude of vibration is a_0 and the frequency is ω. The equations of motion of the table are:

$$x_{table} = a_0 \cos\alpha \sin\omega t \tag{1}$$

$$y_{table} = a_0 \sin\alpha \sin\omega t \tag{2}$$

$$vx_{table} = \omega a_0 \cos\alpha \cos\omega t \tag{3}$$

$$vy_{table} = \omega a_0 \sin\alpha \cos\omega t \tag{4}$$

$$ax_{table} = \omega^2 a_0 \cos\alpha \sin\omega t \tag{5}$$

$$ay_{table} = \omega^2 a_0 \sin\alpha \sin\omega t \tag{6}$$

The coordinate system is also shown in Fig. 1. The X-axis is parallel and the Y-axis is perpendicular to the table. This coordinate system is used for all displacements, velocities and accelerations.

2.1 The body forces affecting motion

For small vibration amplitudes, the part will remain stationery on the plate because the parallel inertia force acting on it will be too small to overcome the frictional resistance force F_f, between the part and the table. It must be also noted that the part moves during vibration due to frictional effect, Lim (1993).

For the particle to move, we may write

$$ma_0\omega^2 \cos\alpha > mg\sin\theta + F_f \tag{7}$$

Where

$$F_f = \mu N = \mu(mg\cos\theta + ma_0\omega^2 \sin\alpha \sin\omega t - F_d) \tag{8}$$

The drag force F_d is obtained through experimental data from Donley (1991), it relates the drag coefficient with the Reynolds number of the flow. In Fig. 2, it is shown the experimental data.

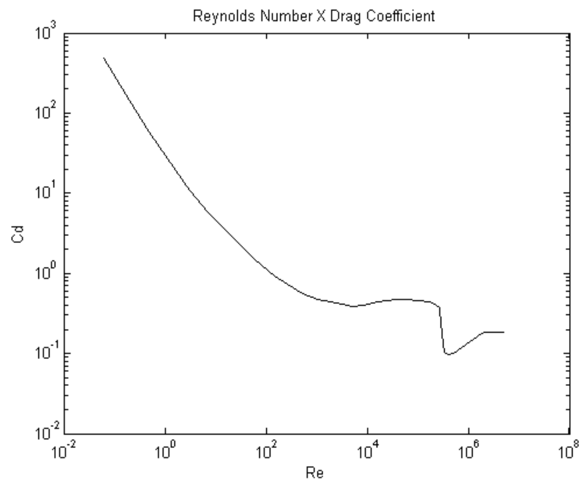

Figure 2. Experimental data from Donley (1991)

Using the properties and the velocity of the air and the properties of the grain it is possible to calculate the Reynolds number and consequently the drag coefficient. The drag coefficient, C_d, relates to the drag force by Eq. (9), where A is the area of the particle, ρ is the density of air and v is the velocity of air. This force is what makes the densimetric table unique, when the air stream is turned off the table behaves as a vibratory feeder and no separation is made.

$$F_d = C_d \cdot \rho \cdot v^2 \cdot A/2 \tag{9}$$

Using the properties and the velocity of the air and the properties of the grain it is possible to calculate the Reynolds number and consequently the drag coefficient. To obtain this coefficient it was necessary to assume the particle as a sphere, but knowing that the grains are not spheres the effect of rolling was not considered

If the amplitude is sufficiently large the particle will leave the table and hop forward during each cycle. This happens when the normal force becomes zero:

$$N = mg\cos\theta + ma_0\omega^2 \sin\alpha \sin\omega t - F_d = 0 \tag{10}$$

And that will only happen if

$$ma_0\omega^2 \sin\alpha_d > mg\cos\theta - F_d \tag{11}$$

It is very helpful to visualize the motion that is likely to result. For small amplitudes and low frequencies, it is likely that the body will be on the same relative position due to the frictional force. This means no movement relative to the table, which is not a desired situation. However, if the vibration is more significant the forces on the y-direction will overcome gravity, in the same direction, and the body will leave the table and travel in free flight. This results in a substantial motion along the table.

2.2. The states of motion

The typical motion results of an alternating pattern of contact and non-contact states. It is possible to see more clearly this pattern on Fig.3.

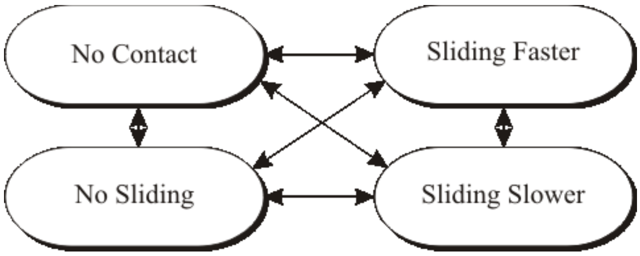

Figure 3. The four states of motion

The condition for the particle to leave the table is that the normal reaction becomes zero. Then the particle will travel in free flight on a parabolic trajectory until it makes contact with the table again. After landing, the component will slide forward relative to the track and may, after remaining stationary relative to the track for a period, slide forward prior to leaving the track on the next cycle. It is also possible for the part to slide backward relative to the track between the two forward sliding regions. Therefore a number of distinct types of motion are possible and no fixed sequence of state transitions will occur for all possible combinations of the parameters Lim (1995).

The method of choice in this paper is quite simple using the values from a time t we calculate the acceleration in both directions, using Newton's second law of motion, in a time $t+dt$. The acceleration in an interval of time dt is considered constant, that assumption allows the use of the simple equations of constant acceleration in each interval of time.

During state transitions, sudden changes in the applied forces can occur. The most critical transition state is from the non-contact state to one of the contact state. In practice, the landing will cause some kind of elastic effect. However, on account of the difficulty to model impacts, it is assumed that the particle does not bounce upon impact with the table.

2.2.1. No contact state

This state occurs when the body is in free flight. By resolving forces in both directions:

$$ma_x = -mg\sin\theta \tag{12}$$

$$ma_y = -mg\cos\theta \tag{13}$$

In this state the drag force is not considered, because when the particle leaves the table the flow of the air does not act upon it. The particle enters this state when the acceleration of the table is in the same direction of the gravity and the normal force becomes zero. This state ends when the y-position of the body becomes zero, by which time surface contact is regained.

2.2.2. No sliding state

This state is an intermediate state between the sliding faster and the sliding slower states. By resolving forces in both directions:

$$ma_x = m \cdot ax_{table} \tag{14}$$

$$ma_y = N - mg\cos\theta \tag{15}$$

This state is entered when the velocities, in the x-direction, from the particle and the table are the same and

$$\mu N > m \cdot ax_{table} + mg\sin\theta \tag{16}$$

This state is ended when Eq. (16) is no longer satisfied.

2.2.3. Sliding slower state

When the particle is in contact and is sliding with a less positive x-velocity than the table, it results in this state. By resolving forces in both directions:

$$ma_x = -mg\sin\theta + \mu N \tag{17}$$

$$ma_y = N - mg\cos\theta \tag{18}$$

This state is entered when,

$$\mu N < m \cdot ax_{table} + mg\sin\theta \tag{19}$$

2.2.4. Sliding faster state

When the particle is in contact and has a more positive x-velocity than the table, it results in this state. By resolving forces in both directions:

$$ma_x = -mg\sin\theta - \mu N \tag{20}$$

$$ma_y = N - mg\cos\theta \tag{21}$$

This state is entered when,

$$\mu N < m \cdot ax_{table} + mg\sin\theta \tag{22}$$

3. SIMULATION

Using the model of motion formulated above, a script, on MATLAB, was developed to simulate the motion of the particle and obtain its displacement, velocity and acceleration on any instant of time.

The first step is to calculate the drag force, afterwards it is necessary to start the time analysis, the initial values are assumed and then the loop that evaluates all desired instants of time is begun. In each loop, the first step is to analyze the results from the last one and discover in which state the particle is. After the accelerations are computed at the end of the loop, the new velocity and displacement are found using the equations for uniform acceleration,

$$v = v_0 + a \cdot dt \tag{23}$$

$$s = s_0 + v_0 \cdot dt + a \cdot dt^2 / 2 \tag{24}$$

The results obtained for a simulation using a frequency of 25 Hz, a vibration angle of 10°, inclination angle of 4º amplitude of 0.005m, coefficient of friction of 0.36, grain diameter of 0.01m, air velocity of 4m/s and grain density of 800kg/m^3. Figure 4 and Figure 5 are the x-position and the x-velocity of the simulation.

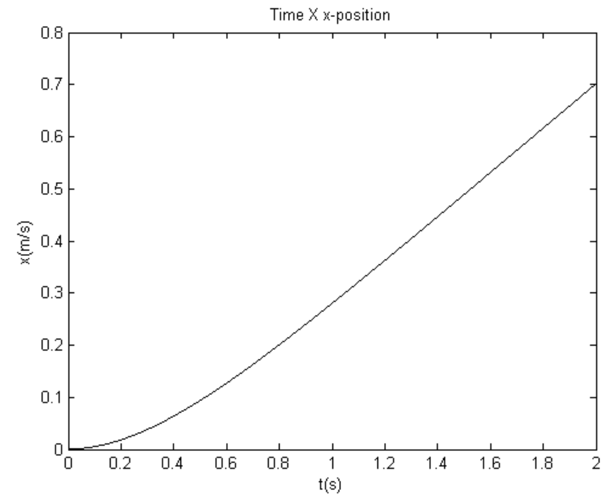

Figure 4. The x-position of the solution found

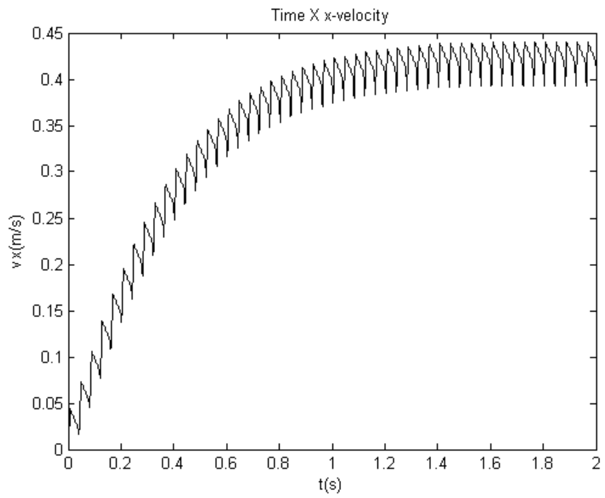

Figure 5. The x-velocity of the solution found

It is possible to notice, at Fig.5, that after one second the velocity oscillates cyclically around a mean value. Figure4 shows that after a second the slope of the displacement becomes constant. When these characteristics appear, the steady state has been reached. In order to evaluate the final mean velocity, the slope of Fig. 4 was calculated. The found velocity was 0.5763m/s.

The densimetric table may be treated as a compound, formed from two schemes from Fig.1 ninety degrees apart, as shown in Fig. :

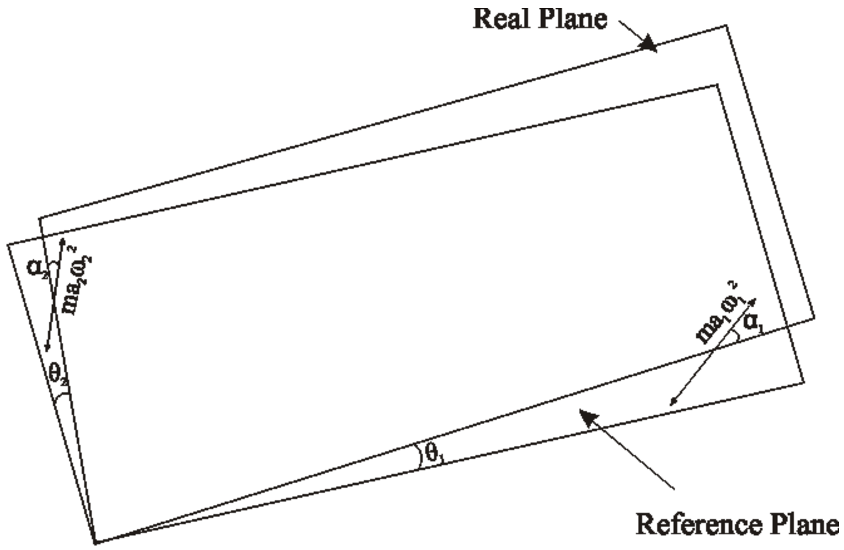

Figure 5. Three-dimensional model of the densimetric table

To analyze the movement in the 3d model, the velocities in both faces are calculated. That becomes two problems from the previous type. After evaluating the final mean velocities, a vector sum is calculated making it possible to obtain the resultant velocity.

The following factors were used to simulate a 3d model of the densimetric table: a frequency of 25Hz, a friction factor of 0.6, an air velocity of 15m/s, two angles of vibration: 5º and 20º, two vibration amplitudes: 0.005 and 0.006m, two inclinations: 5º and 4º, a grain of 0.01m of diameter and two densities: 800 and 400 kg/m3. Using the MATLAB script, one velocity for each direction with each density was calculated, totalizing 4 velocities. For each different density a resultant velocity was calculated, the simulated directions of movement are shown at Fig. 6.

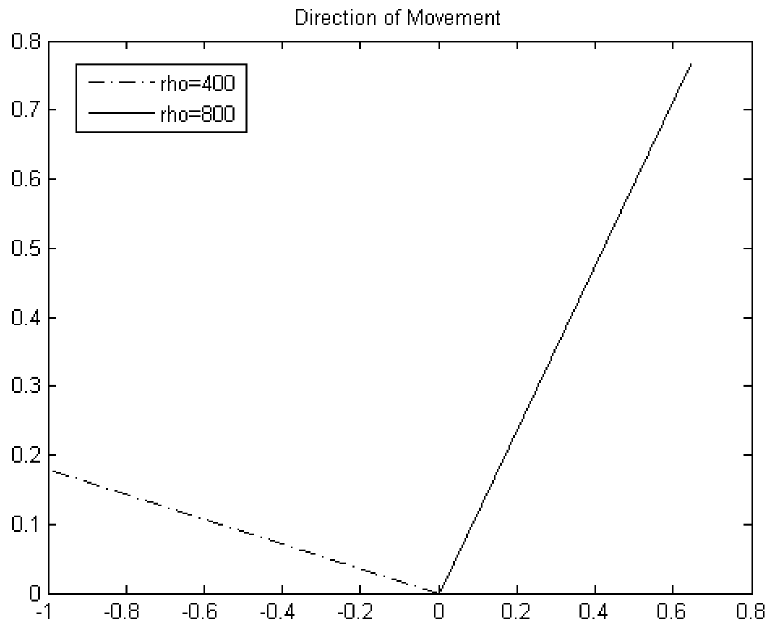

Figure 6. The different directions for particles of different densities

Figure 7 shows very clearly how the particles are separated according to its density. For each density a singular direction is provided, therefore the grains can be collected at separated points on the side of the table and each collector will only receive one kind of density, or at least a group with very similar densities.

Figures 8, 9 and 10, shows the comparison of the experimental results obtained by Lim (1993) for vibratory feeder and the results of the simulation presented in this article. All comparisons use a frequency of 10 Hz, a vibration angle of 30 degrees, each comparison works with a different inclination angle and within each comparison the amplitude is changed.

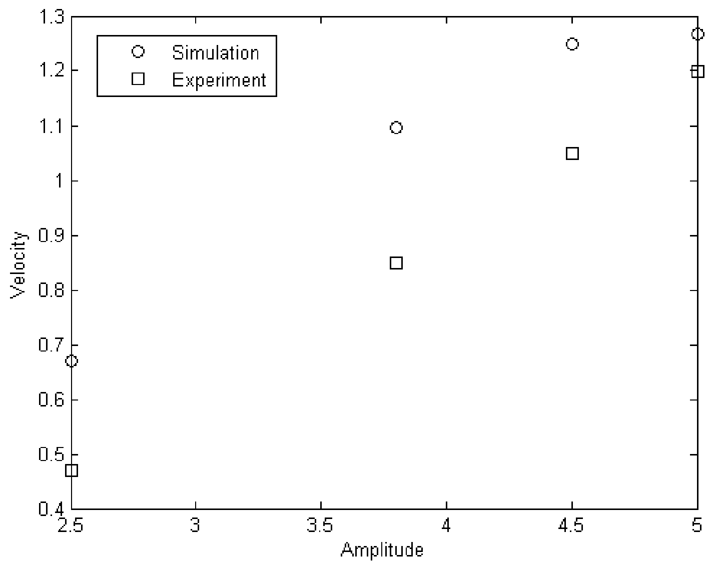

Figure 8. Velocity against vibration amplitude (table angle = 0).

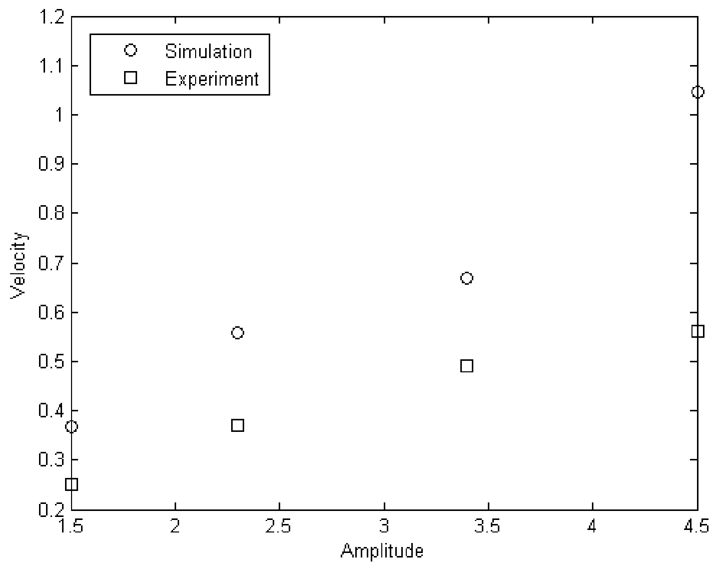

Figure 8. Velocity against vibration amplitude (table angle = 0).

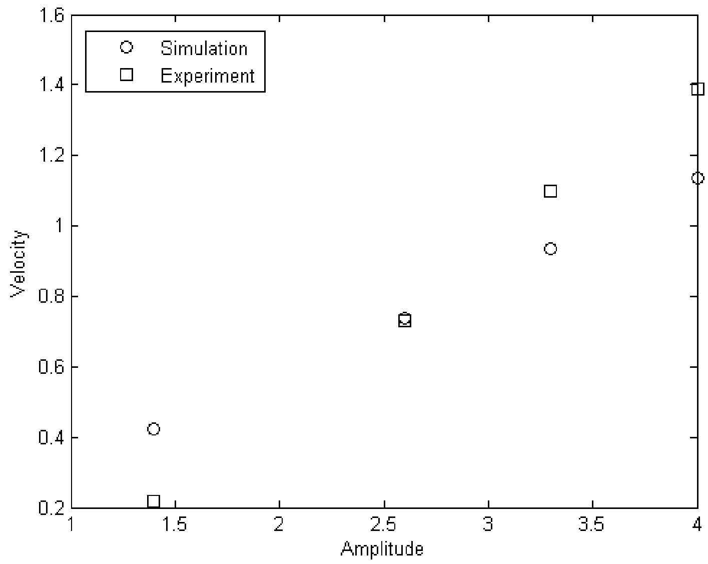

Figure 8. Velocity against vibration amplitude (table angle = 0).

The results obtained were quite satisfactory when the fact that a very simple computational model was used is taken in consideration. Even though the values for the velocity were good, for a densimetric table the most important value is the direction of the velocity and not its magnitude. It is direction that shows whether the particles will be separated or not.

4. CONCLUSION

This paper presents the dynamic analysis of a densimetric table and develops a model that incorporates various system and operating parameters that could affect the separation of the particles. When compared with the experimental results operating with similar parameters, approximate results were found. The model will be useful to designers concerned with the functioning of a densimetric table and can be used as an aid in the analysis of how the separation of grains may be optimized. The model has a great potential to support the experiment-based design, reducing significantly the number of necessary experiments.

5. REFERENCES

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