COMPARISON OF FINITE ELEMENT AND DIGITAL WAVEGUIDE FOR ACOUSTIC ROOM SIMULATION

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Abstract. *One of the main challenges for the computational modeling in acoustic environments is to choose a technique that can efficiently represent the reverberation. Geometrical methods are often used to model the room acoustics properties, but they are valid only for high frequencies. In this contest, two techniques wave-based are discussed to investigate theirs efficiency and accuracy: Waveguide and Finite Element Method (FEM). The Digital Waveguide can be characterized by wave equation solution; it means that in the lossless case, all solutions can be expressed in terms of travelling waves (right-going and left-going) using a two-directional digital delay line. A digital waveguide model is obtained by dispersion joints. To simulate FEM, was used the ANSYS® tool, element Acoustic 29 to obtain the results. The acoustic properties are defined by measuring the Room Impulse Response (RIR) at a specific listener location for an input signal applied at a given sound source location. In this paper, both techniques are used for modeling the RIR two-dimensional propagation of sound within an enclosed room.*

Keywords: *Digital Waveguide, Finite Element Method, Response Impulsive Room*

1. INTRODUCTION

Sound propagation analysis methods can be categorized by geometrical acoustics methods and wave acoustics ones. The geometrical acoustics methods, as ray-tracing, have some limitation, because they can't consider the wave nature of sound wave to assure the accuracy of results, furthermore, demand high computational cost. Otherwise, wave acoustics methods, which include Digital Waveguide and Finite Element Method (FEM) consider the wave nature and iteration of wave with obstacles.

Digital waveguide models have provided an accurate and efficient method of modeling the properties of many resonant structures, including acoustic spaces (Murphy and Mullen, 2002). This methodology can simulate different geometry easier than others techniques and theirs results are more precision.

This work aims in using digital waveguide mesh structures for reverberation and room acoustics modeling, these being fundamental tool in the field of audio processing. Therefore, briefly review numerical technique Digital Waveguide mesh and discus theirs efficiency and computational cost for modeling the RIR.

2. DIGITAL WAVEGUIDE

The waveguide is a device for guiding the propagation of waves and can be characterized by one-dimensional wave equation, using d'Alembert' solution, as "Eq. (1)". Considering a lossless case, one-directional, all solution of the Wave equation can be obtained in terms of left-going traveling waves by $f(x-ct)$ and right-going traveling waves by $g(x+ct)$, where f and g are arbitrary twice-differentiable functions (Moura *et all*, 2005):

$$u = g(x+ct) + f(x-ct) \tag{1}$$

Where x is spatial variable, t is temporal and c is sound velocity variable. This solution satisfies general solution of wave equation, which is a hyperbolic problem, whose solution must be a function of time as well a distance, (Blackstock, 2000). A digital waveguide is a bidirectional digital delay line as illustrates in "Fig.(1)".

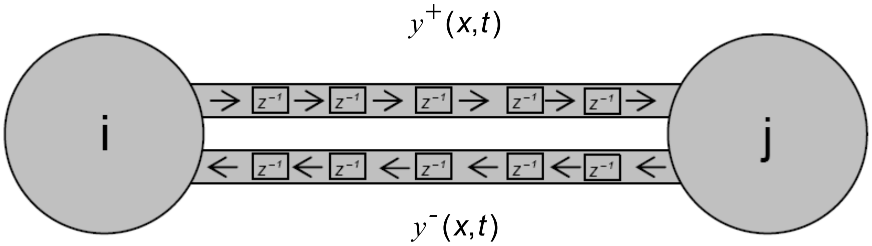


Figure 1. Representation of digital waveguide formed of two delay lines with opposite wave propagation directions.

On the scheme present, the letter Z represents the unitary delay that will conduct the wave from point I to point J. The signs "+" and "–", means respectively the input and output signal on nodes. A digital waveguide model is obtained by sampling, both in space and time, the one-directional travelling waves occurring in a system of ideal lossless waveguide (Van Duyne and Smith, 1993). The sampling points, scattering junctions, are connected by bi-directional unit delay digital waveguides (Smith, 1992). The "Fig.2", shows the general case of a scattering junction J with N neighbors (i=1…N).

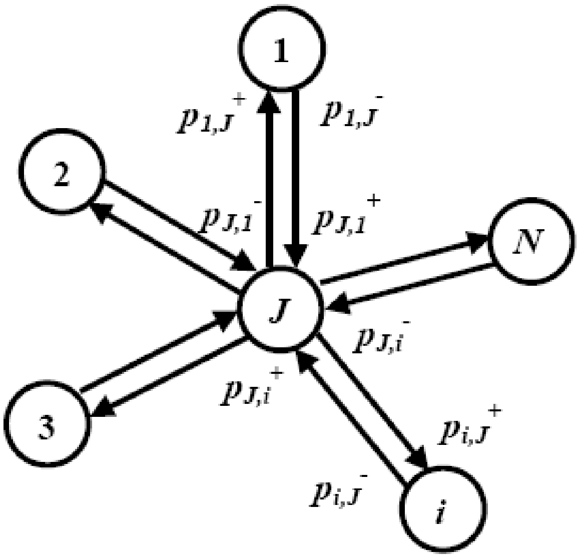


Figure 2. A general scattering junction J with N connected waveguides for i=1,2,..,N.

The sound pressure in a waveguide is presented by p_i the volume velocity by v_i and the impedance of the waveguide by Z_i . The input to a waveguide is termed p_i^+ and the output p_i^- . The signal $p_{i,j}^+$ therefore represents the incoming signal to junction I along the waveguide from the opposite junction J. Similarly, the signal $p_{i,j}^-$ represents the outgoing signal from junction I along the waveguide to the opposite junction J (Murphy and Howard, 2001). The volume velocity v_i is equal ration between pressure p_i and impedance Z_i . The delay elements are bi-directional and so the sound pressure is defined as the sum of its input and output, as solution of d'Alembert, see "Eq. (2)":

$$p_i = p_i^+ + p_i^- \tag{2}$$

Two conditions must be satisfied at a lossless junction connecting lines of equal impedance. (Samir, 2000).

1. The sum of input volumes velocities, v^+ equals the sum of the output volume velocities, v^- (flows add to zero), represented by "Eq.(3)";

$$\sum_{i=1}^{N} v_i^+ = \sum_{i=1}^{N} v_i^- \tag{3}$$

2. The sound pressures in all crossing waveguides are equal at the junction, as "Eq.(4)";

$$p_1 = p_2 = \ldots = p_i = p_N \tag{4}$$

Using these conditions the sound pressure at a scattering junction can be express as "Eq.5":

$$p_j = \frac{2\sum_{i=1}^{N} \frac{p_i^+}{z_i}}{\sum_{i=1}^{N} \frac{1}{Z_i}} \tag{5}$$

The input to a scattering junction is equal to the output from a neighboring junction is equal to the output from a neighboring junction, into the connecting waveguide at previous time step. This can be expressed as "Eq. (6)":

$$p_{j,i}^+ = z^{-1} p_{i,j}^- \tag{6}$$

2.1. Digital Waveguide Mesh

The digital waveguide mesh is an extension of the one-dimensional digital technique. The mesh can be used for simulation of wave propagation in acoustic spaces and have advantage to simulate different kind of geometry. The 2-D digital waveguide mesh is a regular array of 1-D digital waveguides interconnected. Applying the conditions of "Eq.(2)", "Eq.(5)" and "Eq.(6)" a difference equations can be derived for a junction that has N neighbors at unit distance, as "Eq.(7)":

$$p_J(n) = \frac{1}{N}\left[\sum_{i=1}^{2N} p_l(n-1)\right] - p_J(n-2) \tag{7}$$

Where N is the number of neighboring, $p_J(n)$ represents the signal value at the junction at time step n, $p_l(n-1)$ represents the signal value at the junction's neighboring at time step n-1 and $p_J(n-2)$ represents the signal value at the junction at time step n-2. According (Savioja and Valimaki, 2000) this equation is valid for mesh of any topologies, e.g., for a 2-D rectangular mesh (N=2), 3-D rectangular mesh (N=3).

A Square Waveguide mesh is a rectangular structure illustrated in "Fig. 3":

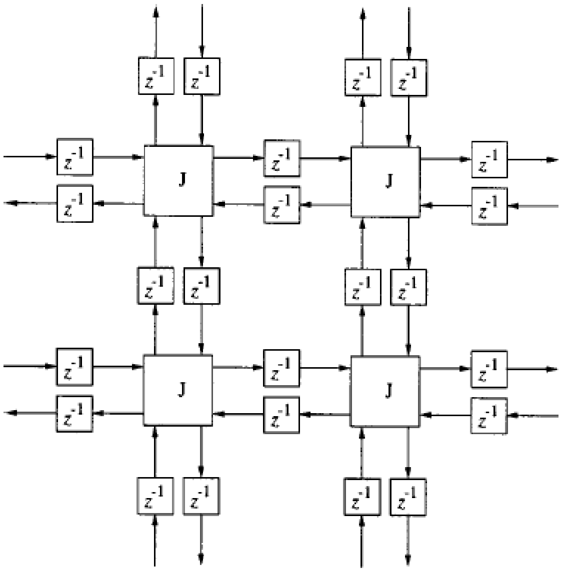


Figure 3. A general scattering junction J with N connected waveguides for i=1,2,..,N.

The mathematical formulation is represented in “Eq. (8)”:

$$p(n,i,j) = \frac{1}{2}[p(n-1,i-1,j) + p(n-1,i+1,j) + p(n-1,i,j-1) + p(n-1,i,j+1)] - p(n-2,i,j) \tag{8}$$

This equation is equivalent to a difference equation derived from the Helmhotz Equation by discretizing time and space (T. Lokki and L. Savioja, 2008). The update frequency of an N-dimensional mesh is:

$$f_S = \frac{c\sqrt{N}}{\Delta x} \tag{9}$$

Where c represents the speed of sound in the medium, N is a number of directions and Δx is the special sampling interval corresponding to the distance between two neighboring nodes.

2.2. ROOM IMPULSE RESPONSE (RIR)

The Room Impulse Responses (RIRs) is generate from a 2-D representation of an enclosed space using some parameters which were choose to simulate a real world variables such as room size, source location and receiver position, coefficients of absorption.

The behavior at boundaries are represented by a set of a boundary junction as having only one other neighbor (Murphy *et all*, 2001). The effect of a boundary in a real room is to produce a reflection of an incident sound wave, usually with some frequency dependent absorption of wave energy at the boundary itself. In a digital waveguide structure a reflection is caused by a change in the impedance of the waveguide. This can be conceptualized by connecting a dummy junction on the other side of the boundary junction, essentially within the boundary itself, as in “Fig. 4”.

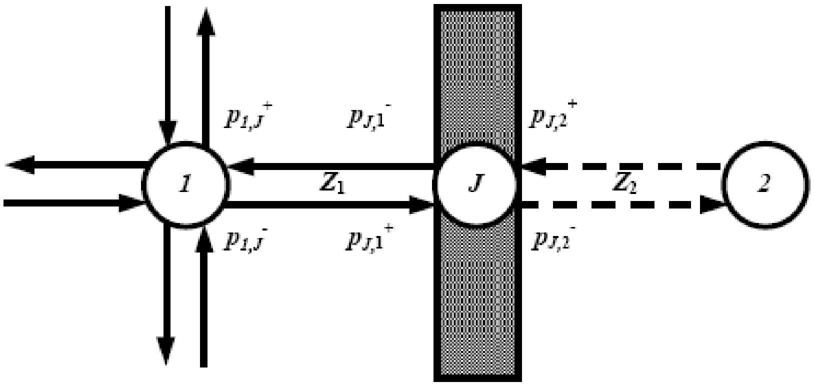


Figure 4. Modeling effect of a boundary condition.

At obstacle, the connecting waveguides on either side of the boundary will have different characteristics impedances, Z_1 and Z_2, respectively. The reflection coefficient r is defined as:

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{10}$$

In recent work, Murphy *et all*, (2001) says that there is no contribution into the boundary junction, J, from the dummy junction 2 and using "Eq.(5)" and "Eq.(6)", the sound pressure for the boundary junction can be calculated as a function of the sound pressures of the incident travelling waves, p_i^{+}, which result in "Eq. (11)":

$$p_j = (1 + r)p_{j,1}^{+} \tag{11}$$

The amount of energy reflected at the boundary is determined by setting r equal to a value between 0 and 1.

A rectangular room (4m long and 3 m wide) has been modeled using the square mesh (SWG – Squared Waveguide) and Finite Element Method (FEM). The sound source, represented by circle in "Fig.5," is situated in down left corner, 0.5m from each wall. A RIR is measured around a point situated 0.5 from the top wall and 0.5 from the right wall. Wave phenomena such, absorption and reflection are natural consequences of the model. The "Figure (5)" illustrates the physical simulation:

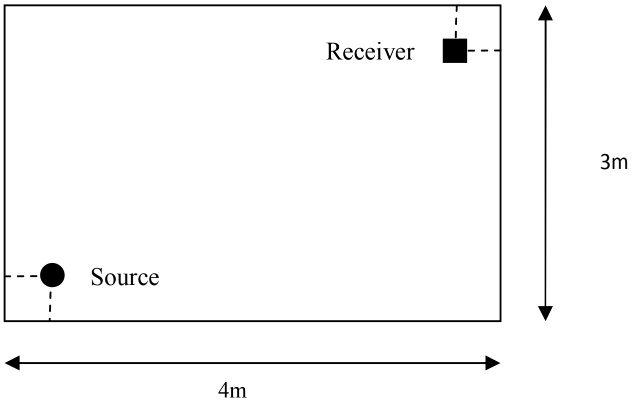
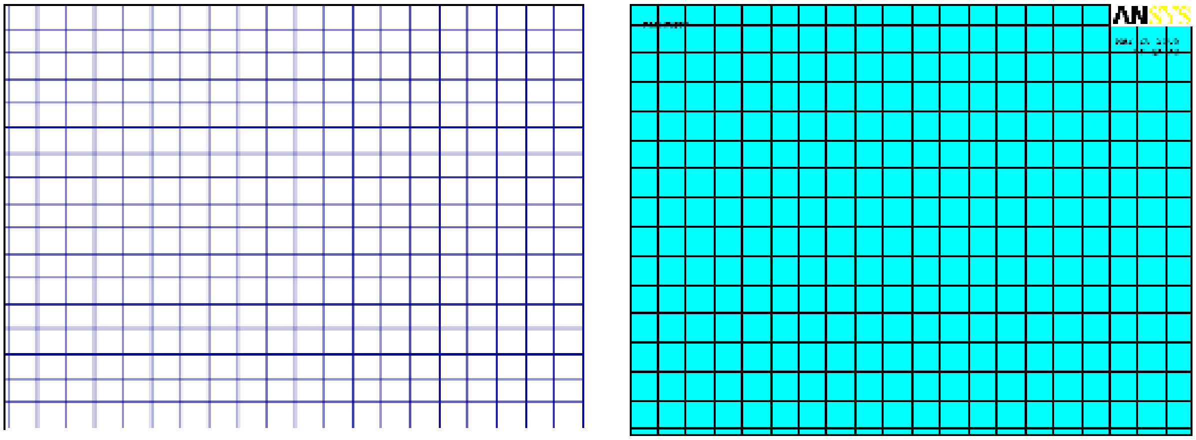


Figure 5. Plan view of the modeled room showing positions of input sound source and read sound receiver.

To simulate waveguide technique, the impedance of the borders of the mesh was set to allow a rigid condition that is rigid wall and have been implemented using both coefficient of 0.1. Te frequency of the mesh, in the others words, update frequency, was set to 1000 Hz, the distance between nodes 0.02 meters. The input signal has a senoidal form with frequency set to 100 Hz. The FEM was simulated using Ansys®, element Acoustic 29 with squared mesh to obtain closer results in both techniques. The "Fig.6" shows the mesh used in booth techniques.

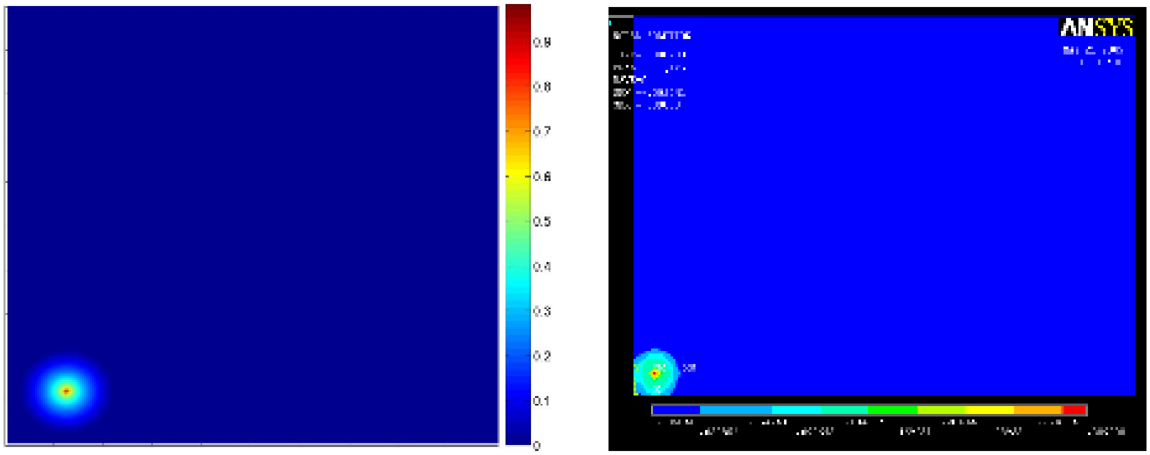


A B

Figure 6: Zoom of the mesh generated for the room representation. Side A represents simulations using SWG and side B represents simulations using FEM.

4. RESULTS AND DISCUSSION

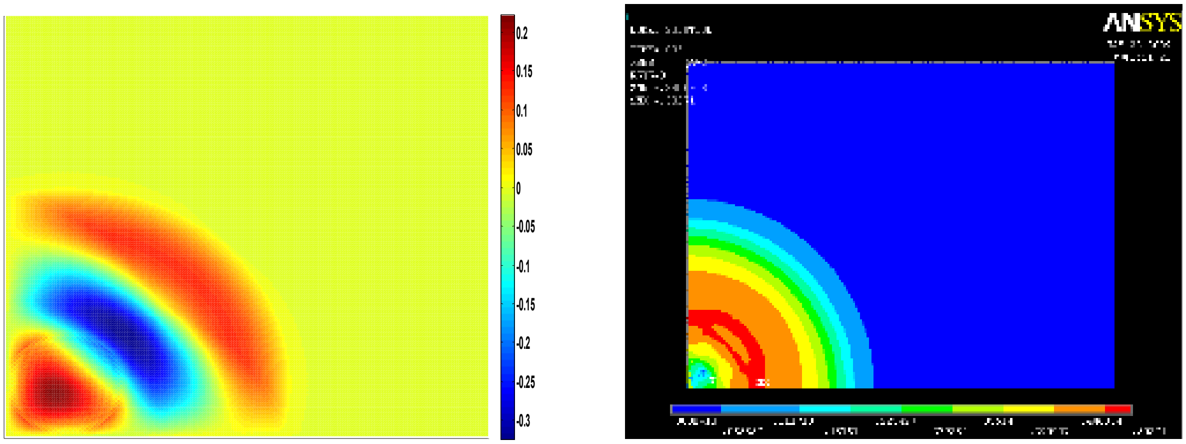
In the following figures, there are demonstrations of the sound propagation through a room, with booth technique.



A B

Figure 7: The mesh generated for the room representation. Side A represents simulation using SWG and side B represents simulations using FEM.

In the "Figure 7", it is shown the wave formed from source, which has turned off, after 0.01s.



A

B

Figure 8: Representation of a room. Side A represents simulation using SWG and side B represents simulations using FEM.

In “Figure 8”, it is shown the wave hitting the walls and beginning to reflect to the medium. As it was expected, the receiver will capture both original sound of the sources and the reflected waves that move along the room, with the same behavior of the incident wave. Consequently, the sound heard by the receiver came from the source and structures of the room. The advantage of waveguide can be seen in the side A that has more precision in these results and illustrates correctly the high and low pressure. Otherwise, the side B shows some distortions, probably the mistakes was caused by methodology

The response impulsive is representing at “Fig.9” and “Fig.10”.

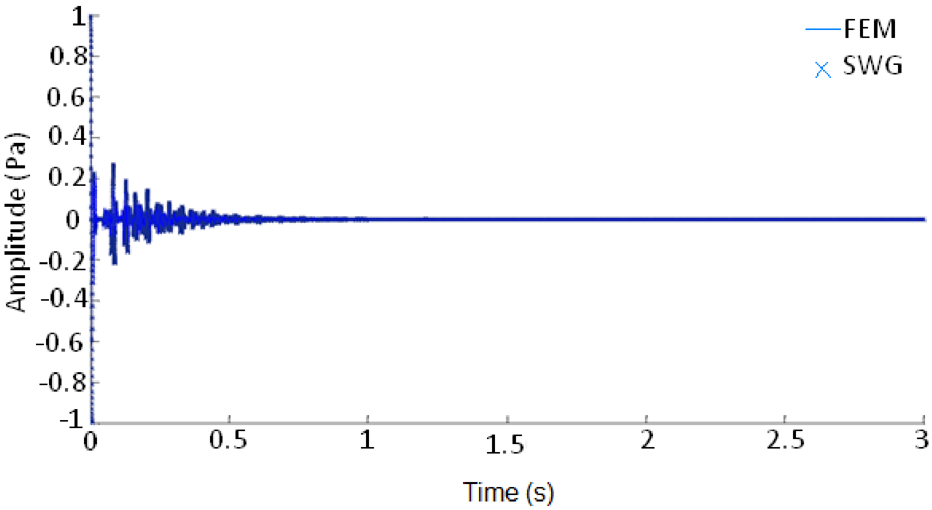


Figure 9: Sensor placed at source.

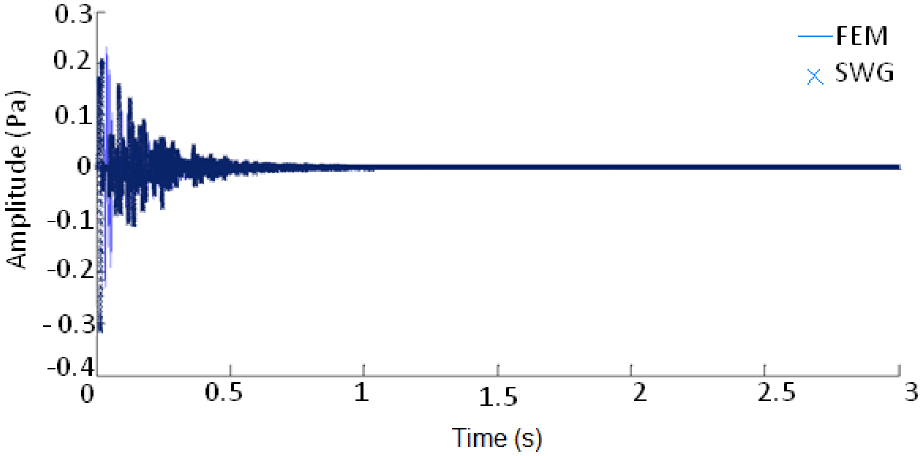


Figure 10: Sensor placed at receiver.

As expected, the graphic of acoustic energy at source, in “Fig.9”, has a pick at begin and after it has been attenuated. In “Fig.10”, the acoustic energy have been came decrease, because it has been attenuated by walls. The results obtained illustrated the advantage of using digital waveguide, principally, to simulate in low-frequency acoustic. The results in ANSYS®, could be improved if mesh was refinement, but it has a high computational coast and, consequently, expend more time.

5. CONCLUSION

The digital waveguide mesh is well known on the modeling of physical systems due to its efficiency and accuracy. This work made a comparison between two techniques for simulate the Room Impulsive Response. The results were satisfactory, obtained in a shorter time and more accurate using Digital Waveguide than ANSYS®. Although the boundary conditions (impedance) are satisfactory for most situations using Digital Waveguide, they are clearly not ideal. The direction dependent characteristics of the mesh structures due to its topology have an influence on any wave propagation through it. In simulations using SWG, the wave has been measured at preferential directions, an angle of 45° to the incident direct sound, but in this work, the frequency update was set to allow highest number of neighbors for compare with results from FEM.

Our ongoing research it to simulate Triangular Waveguide Mesh and another excitation, more complex than a pure tone, building a simple and efficient system able to solve a great range of problems.

6. ACKNOWLEDGEMENTS

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8. RESPONSIBILITY NOTICE

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