INVERSE DYNAMICS OF A REDUNDANTLY ACTUATED FOUR-BAR MECHANISM USING A DIFFERENTIAL ALGEBRAIC EQUATION APPROACH

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Abstract. *This paper presents a method for estimating joint torques in closed-chain mechanisms with a prescribed kinematics and redundant actuation, i.e., with more actuators than degrees of freedom. The dynamics of the multibody system is described by a set of Differential Algebraic Equations (DAE). Inverse dynamics of the system does not have an unique solution, since there are more unknowns (actuator torques) than equations. This problem is traditionally treated by using the Moore Penrose pseudo-inverse matrix. Here, an alternative formulation based on a variation of Valasek's Transmission Matrix approach is proposed. A four bar mechanism, with a crank rotating at constant velocity, was analyzed to serve as benchmark. In addition, the regular case with one torque actuator is solved and compared to two, three and four actuators case.*

Keywords: *Differential Algebraic Equations, Inverse Dynamics, Closed-Chain Mechanisms, Redundant Mechanisms*

1. INTRODUCTION

Parallel manipulators with actuator redundancy have been studied in the last few years by several authors. These mechanisms are characterized by closed kinematic chains with more actuators than degrees of freedom. This configuration offers several advantages over their serial counterparts when used in industrial robots and machine tools, since: have higher mechanical stiffness (Miller, 2001), higher trajectory and positioning accuracy (Nakamura and Ghodoussi, 1989), higher load capacity (Dasgupta and Mruthyunjaya, 1998) and smaller mobile mass (Miller, 2001). The existence of kinematical singularities is a key problem in the analysis of closed chain mechanisms. It becomes critical when a mechanism reaches the boundaries of the space. According to Cheng et al. (2003), when a parallel manipulator moves towards a singular configuration, its stiffness and accuracy properties quickly deteriorates.

Liu et al. (2001) studied the undesired effects of singularity over parallel manipulators. To solve this problem, they suggested the introduction of three types of redundancy: (i) kinematic redundancy, where the number of manipulator Degrees of Freedom (DOF) is greater than end-effector's; (ii) over constraining, i.e., increase the number of closed kinematic chains and (iii) over actuation, when the number of actuators is greater than DOFs. This opinion is supported by Cheng et al. (2003), who agree that redundant actuation provides effective means for eliminating singularities of parallel manipulators, thereby improving its performance. Valasek et al. (2004) argues that Parallel Kinematic Machines (PKM) have advantages over their serial counterparts because: (i) machine tool frame loading by bending is replaced by tension/compression, (ii) large moving masses are reduced and (iii) backslashes and inaccuracies in serial kinematic chains are reduced. However, when PKM are exactly actuated (i.e, the number of actuators is equal to the number of end-effector DOFs), they suffer of limited workspace, have non-uniform mechanical properties and problems with accuracy and calibration. The principle of redundant actuation applied to PKMs can solve all this problems as was shown by Valasek et al. (2005) with the development of *Trijoint 900H* machining center .

In addition, redundant actuation often arises in biomechanical models. It is present in musculo-skeletal systems: (i) when individual muscles are considered as single actuators and (ii) when the net effect of all muscles that crosses a joint is considered a single torque actuator. An example of the first case, as presented by Hatze (2000), considers 42 DOFs, with 240 musculo-skeletal actuators for a three dimensional human body model. In the second case, the simplifying assumption of torque actuators may turn the model redundant or not.

While in human posture models Menegaldo and Weber (1997), Barin (1989), Cahouet et al. (2002) the number of actuators is equal to the DOFs, in pedaling (Hull et al., 1985), rowing (Lee et al., 2005) and double support phase of gait (Pandy and Berme, 1988) – where a closed kinematic chain is present – the number of actuators outcomes the number of degrees of freedom. In short, redundancy in biomechanics is a challenging problem because, as Hatze (2000) says, inverse dynamics does not have unique solution. Therefore, different control histories can reproduce the same (specified) kinematics. In particular, some relevant questions may be addressed in this context. For instance, how actuator torques or forces are distributed along the additional redundant actuators? If the number of actuators is changed, is it possible to choose an specific set with smaller maximum torque outputs, which are, therefore, more lightweight?

In this paper, a four bar mechanism with one, two, three and four torque actuators is studied, as decribed in Fig. 1. The inverse dynamics of the non-redundant mechanism is numerically calculated with the method presented by Haug (1989). Using the framework presented by Haug, a modification in the generalized forces term, based on a variation of Valasek's Transmission Matrix (Valasek et al. 2005), is introduced to account for additional torques. The resultant system of equations is solved via pseudo-inverse approach.

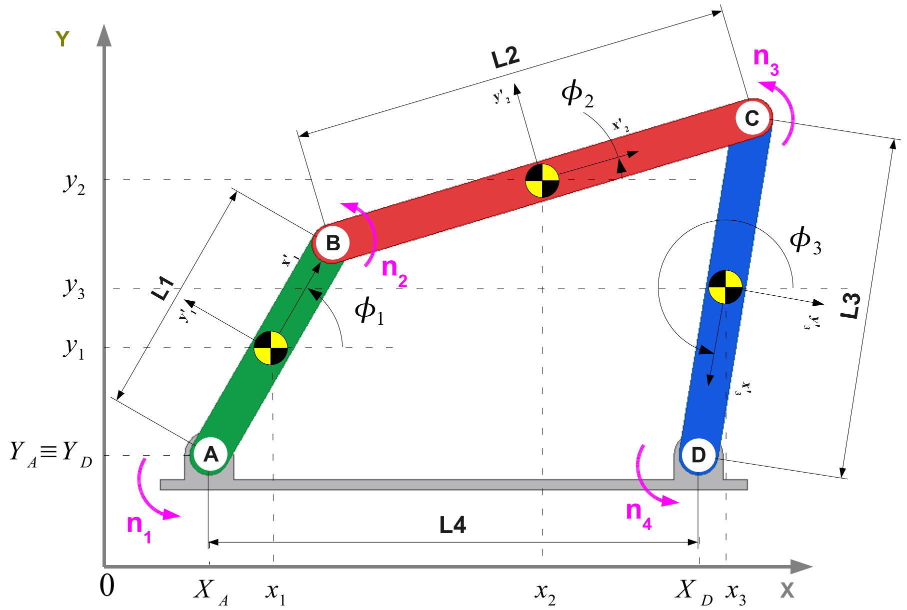


Figure 1. Diagram of four bar mechanism showing reference frames, Cartesian coordinates and actuator positions.

2. THEORETICAL FRAMEWORK

2.1 Kinematic Analysis

The first step in kinematic analysis is to write the expressions of constraint vector $\mathbf{\Phi}(\mathbf{q}, t)$. Constraint vector is composed of the *kinematic constraints* $\mathbf{\Phi}^K(\mathbf{q}, t)$ and *driving constraints* $\mathbf{\Phi}^D(\mathbf{q}, t)$. Kinematic constraints represents physical connections between bodies, which are functions system coordinates, but do not depend explicitly on time. They are be graphically expressed in Fig. 2.

In addition to kinematic constraints, the motion of the mechanism is described by driving constraints, which are the time dependent equations that prescribes the active coordinates. Kinematic and driving constraints can be assembled together as in Eq. (1). This equation can be solved with a Newton-Raphson algorithm.

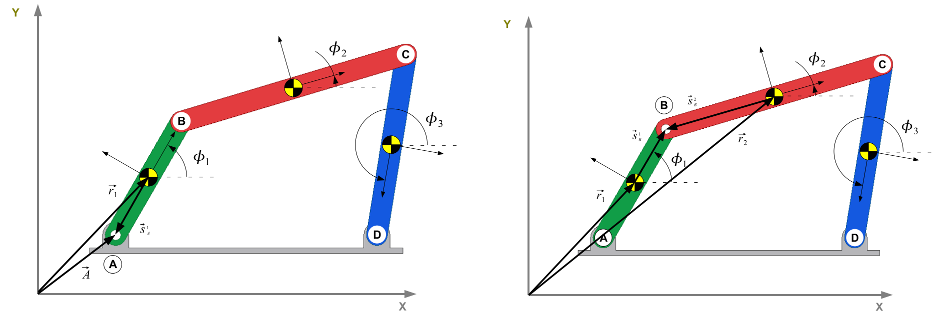
$$\mathbf{\Phi}(\mathbf{q}, t) = \begin{bmatrix} \mathbf{\Phi}^K(\mathbf{q}, t) \\ \mathbf{\Phi}^D(\mathbf{q}, t) \end{bmatrix} = [\mathbf{0}] \tag{1}$$

Differentiating Eq. (1) once we have the expression shown at Eq. (2), where $\mathbf{\Phi_q}$ is the Jacobian matrix and $\mathbf{\Phi}_t$ is the derivative of $\mathbf{\Phi}$ with respect to time. If $\mathbf{\Phi_q}$ is non-singular, Eq. (2) can be solved for velocities $\dot{\mathbf{q}}$.

$$\mathbf{\Phi_q}\dot{\mathbf{q}} + \mathbf{\Phi}_t = \mathbf{0} \tag{2}$$

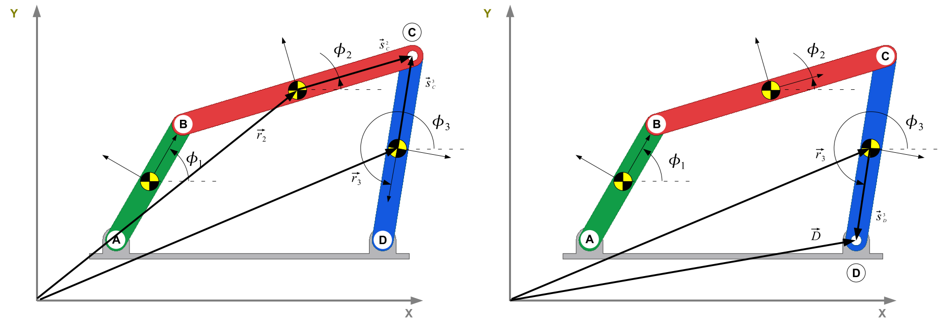
Differentiating Eq. (1) twice, Eq. (3) is obtained, where $(\mathbf{\Phi_q} \cdot \dot{\mathbf{q}})_\mathbf{q}$ is the same as $\partial(\mathbf{\Phi_q} \cdot \dot{\mathbf{q}})/\partial\mathbf{q}$. The term $\mathbf{\Phi}_{t\mathbf{q}}$ is a matrix whose elements are the time derivatives of Jacobian matrix and $\mathbf{\Phi}_{tt}$ is the second time derivative of Eq. (1). Again, if $\mathbf{\Phi_q}$ is non-singular it is possible to solve Eq. (3) for accelerations $\ddot{\mathbf{q}}$.

$$(\mathbf{\Phi_q} \cdot \dot{\mathbf{q}})_\mathbf{q}\dot{\mathbf{q}} + 2\mathbf{\Phi}_{t\mathbf{q}}\dot{\mathbf{q}} + \mathbf{\Phi_q}\ddot{\mathbf{q}} + \mathbf{\Phi}_{tt} = \mathbf{0} \tag{3}$$



(a) Absolute distance constraint to determine the position of point A.

(b) Revolute joint constraint at point B



(c) Revolute joint constraint at point C

(d) Absolute distance constraint to determine the position of point D

Figure 2. Graphical representation of constraint equations present at vector $\mathbf{\Phi}^K$.

2.2 Dynamic Analysis

After the accelerations of each coordinate are found in the kinematic analysis, the variational equation of motion of a system with nb bodies, can be written follows. It is derived from the principle of virtual work.

$$\sum_{i=1}^{nb} \delta\mathbf{q}_i \left[\mathbf{M}_i\ddot{\mathbf{q}}_i - \mathbf{Q}_i\right] = 0 \tag{4}$$

In Eq. (4), $\delta\mathbf{q}_i = [\delta x_i, \delta y_i, \delta\phi_i]^T$ are the virtual displacements, and $\ddot{\mathbf{q}}_i = [\ddot{x}_i, \ddot{y}_i, \ddot{\phi}_i]^T$ the accelerations of the Cartesian coordinates associated to the body i. $\mathbf{M}_i = diag(m_i, m_i, J_i)$ is a diagonal mass matrix of body i, and $\mathbf{Q}_i$ is a three element vector with all forces and torques (internal and external) that act over body i in x, y e ϕ, respectively. Finally, nb is the number of bodies in the system. Defining composite vectors as:

$$\mathbf{q} = [\mathbf{q}_1^T, \mathbf{q}_2^T, \ldots, \mathbf{q}_{nb}^T]^T \tag{5}$$
$$\mathbf{M} = diag(\mathbf{M}_1, \mathbf{M}_2, \ldots, \mathbf{M}_{nb}) \tag{6}$$
$$\mathbf{Q} = [\mathbf{Q}_1^T, \mathbf{Q}_2^T, \ldots, \mathbf{Q}_{nb}^T]^T \tag{7}$$

it is possible to re-write the variational form of system equations of movement in a compact form:

$$\delta\mathbf{q}^T \left[\mathbf{M}\ddot{\mathbf{q}} - \mathbf{Q}\right] = \mathbf{0} \tag{8}$$

At this point it is convenient to split the vector $\mathbf{Q} = \mathbf{Q}^C + \mathbf{Q}^A$ into 2 other vectors, one with internal (constraint) and the other with external (active) forces.

$$\delta\mathbf{q}^T \left[\mathbf{M}\ddot{\mathbf{q}} - \mathbf{Q}^C - \mathbf{Q}^A\right] = \mathbf{0} \tag{9}$$

If $\mathbf{q}$ is a vector with $3nb$ Cartesian coordinates and the system has n degrees of freedom, it is necessary to introduce $3nb - n$ kinematic constraints with its associated Lagrange multipliers. The theorem of Lagrange multipliers Haug (1989) states that if $\delta\mathbf{q}^T\mathbf{Q}^C = 0$ for all $\delta\mathbf{q}$ such that $\mathbf{\Phi_q}\delta\mathbf{q} = \mathbf{0}$, then there exists a vector $\boldsymbol{\lambda}$, called *Lagrange multiplier*, such that $\delta\mathbf{q}^T\mathbf{Q}^C + \delta\mathbf{q}^T\mathbf{\Phi_q}^T\boldsymbol{\lambda} = 0$. The fact that $\delta\mathbf{q}^T\mathbf{Q}^C = 0$ is a consequence of the workless condition. If forces $\mathbf{Q}^C$ do no work, thus the admissible displacements $\delta\mathbf{q}$ are perpendicular to $\mathbf{Q}^C$. A complete proof of the theorem of Lagrange multipliers can be found in Haug (1992). Proceeding this way,

$$\mathbf{Q}^C + \mathbf{\Phi_q}^T\boldsymbol{\lambda} = 0 \tag{10}$$

Substituting Eq. (10) into Eq. (9), and rearranging the terms we obtain the Differential Algebraic Equation form of a multibody system equations of motion .

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi_q}^T\boldsymbol{\lambda} = \mathbf{Q}^A \tag{11a}$$
$$\mathbf{\Phi}(\mathbf{q}, t) = \mathbf{0} \tag{11b}$$

Equations (11a) and (11b) are said to be *Differential Algebraic Equations* (DAE) of index 3. DAE is system of differential equations that describes the behavior of a dynamical system which is coupled to a set of algebraic equations that should be always satisfied.A DAE can be written as an ODE differentiating Eq. (11a) twice. In doing so, it is convenient to isolate the term $\mathbf{\Phi_q}\ddot{\mathbf{q}}$ in Eq.(3):

$$\boldsymbol{\gamma} \equiv -(\mathbf{\Phi_q}\cdot\dot{\mathbf{q}})_{\mathbf{q}}\cdot\dot{\mathbf{q}} - 2\mathbf{\Phi}_{t\mathbf{q}}\dot{\mathbf{q}} - \mathbf{\Phi}_{tt} \tag{12}$$

Thus, Eqs. (11a) and (11b) can be written in an unified form:

$$\begin{bmatrix} \mathbf{M} & \mathbf{\Phi_q}^T \\ \mathbf{\Phi_q} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}^A \\ \boldsymbol{\gamma} \end{bmatrix} \tag{13}$$

2.3 Inverse dynamics of a regularly actuated mechanism

Rewriting Eq. (13) we have

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi_q}^T\boldsymbol{\lambda} = \mathbf{Q}^A \tag{14a}$$
$$\mathbf{\Phi_q}\ddot{\mathbf{q}} = \boldsymbol{\gamma} \tag{14b}$$

It can be shown that Lagrange multipliers uniquely determines the constraint forces and torques that act in the system. Thus, inverse dynamics can be calculated solving sequentially Eqs. (12) and (15b). In the example studied the last term of vector $\boldsymbol{\lambda}$ will be the desired actuator torque at joint A (Fig. 1), because the driving constraint was inserted at the last element of vector $\mathbf{\Phi}$ (see Eq. (21)).

$$\ddot{\mathbf{q}} = \mathbf{\Phi_q}^{-1}\boldsymbol{\gamma} \tag{15a}$$
$$\boldsymbol{\lambda} = (\mathbf{\Phi_q}^T)^{-1}(\mathbf{Q}^A - \mathbf{M}\ddot{\mathbf{q}}) \tag{15b}$$

2.4 Inverse dynamics of a redundantly actuated mechanism

Valasek and Sika (2001) presents a modified version of Eq. (14a), introducing the term $\mathbf{Tn}$ at the right side of Eq. (14a), where $\mathbf{T}$ is the *Transmission matrix* and $\mathbf{n}$ is vector of external actuators. The Transmission matrix is defined as $\mathbf{T} = \partial\mathbf{s}/\partial\mathbf{q}^T$, where $\mathbf{s}$ is the vector of coordinates over which the actuators directly act.

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi_q}^T\boldsymbol{\lambda} = \mathbf{Q}^A + \mathbf{Tn} \tag{16}$$

Equation (16) can be re-written in the form of Eq. (17) in order to put all unknown quantities ($\boldsymbol{\lambda}$ and $\mathbf{n}$) in the same column vector.

$$\left[\mathbf{\Phi_q}^T, -\mathbf{T}\right] \begin{bmatrix} \boldsymbol{\lambda} \\ \mathbf{n} \end{bmatrix} = \mathbf{Q}^A - \mathbf{M}\ddot{\mathbf{q}} \tag{17}$$

Proceeding this way one can define the matrix $[\mathbf{P}] = [\mathbf{\Phi_q}^T, -\mathbf{T}]$ by juxtaposing matrices $[\mathbf{\Phi_q}]^T$ and $[-\mathbf{T}]$. Additionally, defining $[\mathbf{G}] = \mathbf{Q^A} - \mathbf{M}\ddot{\mathbf{q}}$ it is possible to find internal forces and redundant actuated torques solving Eq. (18) where $\mathbf{P}^{+1} = \mathbf{P}^T \left(\mathbf{P} \cdot \mathbf{P}^T\right)^{-1}$ is the Moore-Penrose pseudo inverse matrix (Falco, 2005).

$$\mathbf{X} = \mathbf{P}^{+}\mathbf{G} \tag{18}$$

It should be noted that $\mathbf{P}$ has more columns than rows and therefore it cannot be solved by conventional methods. An important property of pseudo inverse matrix is that a solution of the system of equations (18) is equivalent to the solution of optimization problem defined at Eqs. (19a) and (19b). Therefore, the solution obtained is optimal, in the sense of the least squares of vector $\mathbf{X}$.

$$\textit{minimize}: \quad g \equiv ||\mathbf{X}||_2^2 \tag{19a}$$
$$\textit{subject to}: \quad \mathbf{P} \cdot \mathbf{X} = \mathbf{G} \tag{19b}$$

3. COMPUTATIONAL IMPLEMENTATION

The reference frames of four-bar mechanism are depicted in Fig. 1. The lengths of the bars are $L_1 = 0,5$ m, $L_2 = 0,9$ m, $L_3 = 0,7$ m e $L_4 = 1,0$ m and their masses $m_1 = 6,590$ kg, $m_2 = 11,550$ kg and $m_3 = 9,070$ kg. The center of mass are in the middle of all bars and moment of inertia were calculated by $J = \frac{1}{12} m L^2$. The crank is assumed to turn with a constant velocity (ω) of 60 rpm around point A. The study will comprise a complete rotation of the crank which takes 1 second to be performed. Thus, the vector of coordinates $\mathbf{q}$ is defined bellow.

$$\mathbf{q} = [x_1,\ y_1,\ \phi_1,\ x_2,\ y_2,\ \phi_2,\ x_3,\ y_3,\ \phi_3]^T \tag{20}$$

The constraints of vector $\mathbf{\Phi}(\mathbf{q}, t)$ are shown at Eq. (21). The eight first lines are the kinematic constraints, and can be interpreted graphically as the vector sums indicated in Fig. 2. The last term of Eq. (21) is the driving constraint, where $\phi_{1\circ} = 60°$ is the initial crank angle and $\omega = 60$ RPM is the constant angular velocity.

$$\mathbf{\Phi} = \begin{bmatrix} x_1 - \frac{1}{2}L_1\cos(\phi_1) - X_A \\ y_1 - \frac{1}{2}L_1\sin(\phi_1) - Y_A \\ x_1 + \frac{1}{2}L_1\cos(\phi_1) - x_2 + \frac{1}{2}L_2\cos(\phi_2) \\ y_1 + \frac{1}{2}L_1\sin(\phi_1) - y_2 + \frac{1}{2}L_2\sin(\phi_2) \\ x_2 + \frac{1}{2}L_2\cos(\phi_2) - x_3 + \frac{1}{2}L_3\cos(\phi_3) \\ y_2 + \frac{1}{2}L_2\sin(\phi_2) - y_3 + \frac{1}{2}L_3\sin(\phi_3) \\ x_3 + \frac{1}{2}L_3\cos(\phi_3) - X_D \\ y_3 + \frac{1}{2}L_3\sin(\phi_3) - Y_D \\ \phi_1 - \phi_{1o} - \omega t \end{bmatrix} = \mathbf{0} \tag{21}$$

Differentiating vector $\mathbf{\Phi}(\mathbf{q}, t)$ with respect to time and to $\mathbf{q}$ we have Eq. (22) and the Jacobian matrix Eq. (23), respectively.

$$\mathbf{\Phi}_t = [0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\omega]^T \tag{22}$$

$$\mathbf{\Phi_q} = \begin{bmatrix} 1 & 0 & \frac{1}{2}L_1\sin(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{2}L_1\cos(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -\frac{1}{2}L_1\sin(\phi_1) & -1 & 0 & -\frac{1}{2}L_2\sin(\phi_2) & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2}L_1\cos(\phi_1) & 0 & -1 & \frac{1}{2}L_2\cos(\phi_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{1}{2}L_2\sin(\phi_2) & -1 & 0 & -\frac{1}{2}L_3\sin(\phi_3) \\ 0 & 0 & 0 & 0 & 1 & \frac{1}{2}L_2\cos(\phi_2) & 0 & -1 & \frac{1}{2}L_3\cos(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\frac{1}{2}L_3\sin(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{2}L_3\cos(\phi_3) \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{23}$$

Once these terms are defined, it is possible to calculate $\boldsymbol{\gamma}$ with Eq. (12).

$$\boldsymbol{\gamma} = \begin{bmatrix} -\frac{1}{2}L_1\cos(\phi_1)\,\dot{\phi}_1^2 \\ -\frac{1}{2}L_1\sin(\phi_1)\,\dot{\phi}_1^2 \\ \frac{1}{2}L_1\cos(\phi_1)\,\dot{\phi}_1^2 + \frac{1}{2}L_2\cos(\phi_2)\,\dot{\phi}_2^2 \\ \frac{1}{2}L_1\sin(\phi_1)\,\dot{\phi}_1^2 + \frac{1}{2}L_2\sin(\phi_2)\,\dot{\phi}_2^2 \\ \frac{1}{2}L_2\cos(\phi_2)\,\dot{\phi}_2^2 + \frac{1}{2}L_3\cos(\phi_3)\,\dot{\phi}_3^2 \\ \frac{1}{2}L_2\sin(\phi_2)\,\dot{\phi}_2^2 + \frac{1}{2}L_3\sin(\phi_3)\,\dot{\phi}_3^2 \\ \frac{1}{2}L_3\cos(\phi_3)\,\dot{\phi}_3^2 \\ \frac{1}{2}L_3\sin(\phi_3)\,\dot{\phi}_3^2 \\ 0 \end{bmatrix} \tag{24}$$

Finally, $\mathbf{M}$ is a $[9\times 9]$ diagonal matrix and $\mathbf{Q}^A$ is the generalized force vector whose non-zero elements are the weights of the bars, as presented in Eqs. (25) and (26), respectively.

$$\mathbf{M} = \mathrm{diag}(m_1,\ m_1,\ J_1,\ m_2,\ m_2,\ J_2,\ m_3,\ m_3,\ J_3) \tag{25}$$

$$\mathbf{Q}^A = [0,\ -m_1 g,\ 0,\ 0,\ -m_2 g,\ 0,\ 0,\ -m_3 g,\ 0]^T \tag{26}$$

With this information, after performing the kinematic analysis, the only unknown in Eq. (13) is $\boldsymbol{\lambda}$ and the inverse dynamics of the regularly actuated case can be calculated with Eqs. (14a) and (14b). This result is shown in Fig. 3(a) and represent the last element of $\boldsymbol{\lambda}$ calculated for all positions from $\phi_1 = 60°$ to $\phi_1 = 420°$.

For the redundantly actuated case, we consider two actuators at joints A and B, three actuators at joints A, B and C and four actuators at all joints. The explicit form of Transmission matrices for the two, three and four actuators case are presented in Eq. (27). As the directions of actuation $\mathbf{s}$ coincide with some elements of $\mathbf{q}$, the non-zero elements of $\mathbf{T}_2$, $\mathbf{T}_3$ and $\mathbf{T}_4$ are simply 1 and -1 and represent pairs of action-reaction.

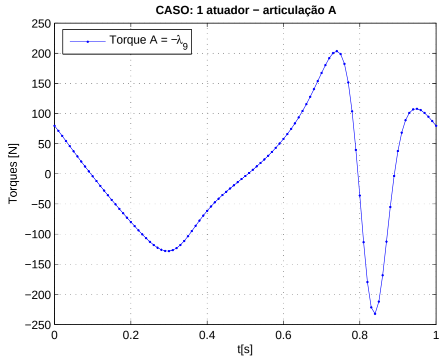
$$\mathbf{T_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{T_3} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \mathbf{T_4} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \tag{27}$$

The form of vector $\mathbf{X}_i$ from Eq. (18), where $i = \{2, 3, 4\}$ represents the number of actuation, is as follows:

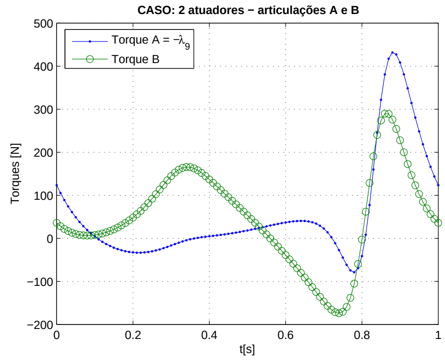
$$\mathbf{X}_i = [\lambda_1,\ \ldots,\ \lambda_9,\ n_1,\ \ldots,\ n_4]^T \tag{28}$$

4. RESULTS AND DISCUSSION

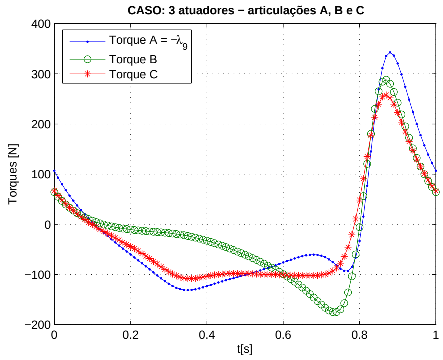
Applying the methodology described above to the four-bar mechanism with 1, 2, 3 and 4 actuators, the torque curves shown in Fig. 3 (from a to d) are obtained. In Fig. 3(a), the torque for the one actuator case is shown for both solutions, with Eq. (14) and Eq. (19). It can be observed that both curves are equal, as should be expected. Comparing Figs. 3(b) to 3(d) with Fig. 3(a), the smaller peak torque was still that obtained with one single actuator (Table 1). The torques shown in Fig. 1 were re-introduced in the model of the mechanism, and the forward dynamical simulation produced the same kinematics used to generate the inverse dynamics solution (Silva, 2007).



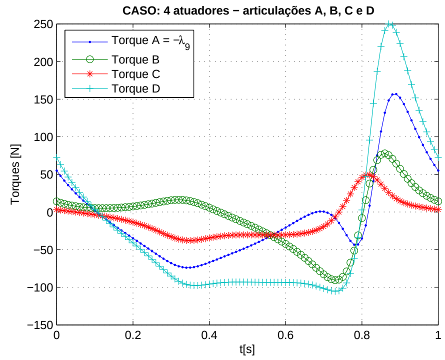
(a) One actuador at jointA



(b) Two actuadors at joints A and B



(c) Three actuadors at joints A, B and C



(d) Four actuadors at joints A, B, C and D

Figure 3. Comparison of torques calculated via inverse dynamics analysis of four-bar mechanism actuated from 1 to 4 actuators.

Table 1. Maximum and minimum torque actuators for a complete cycle of the four-bar mechanism. Values expressed in Nm.

	n_1		n_2		n_3		n_4	
	max	*min*	*max*	*min*	*max*	*min*	*max*	*min*
1 actuator	203	-232	-	-	-	-	-	-
2 actuators	431	-78	289	-173	-	-	-	-
3 actuators	342	-131	288	-174	257	-108	-	-
4 actuators	156	-73	77	-90	49	-37	249	-105

Table 1 shows that the inclusion of actuation redundancy led to an increase of the peak torques. This effect can be attributed to a sort of "competition" among the actuators. On the other hand, this result may be related to an increase of the stiffness of the mechanism, what may be advantageous in some applications. The smaller peak torque, among the redundant cases, was observed when the mechanism is driven by four actuators (Fig. 3(d)).

5. CONCLUSIONS

In Silva (2007) we have shown that it is possible to drastically reduce the peak torque in the redundant actuation cases, by using an optimal control formulation. However, the optimal control numerical problem is much harder to solve, and could not be implemented in real time applications. The presented DAE formulation actually provides an optimal solution, but the entire vector $\mathbf{X}$ is minimized, including both torques and Lagrange Multipliers $\boldsymbol{\lambda}$, i. e., internal mechanism forces that does not produce work.

Therefore, in the specific mechanism studied, it can be concluded that, by using the DAE approach (namely, using the $\mathbf{T}$ matrix from Valasek's method) to formulate the redundant inverse dynamics problem, a solution is found without special numerical difficulties but solving a pseudo-inverse matrix.

6. ACKNOWLEDGMENTS

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8. Responsibility notice

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