THERMAL SYSTEM CONTROL SIZING AIDED BY GENERALIZED INTEGRAL TRANSFORM TECHNIQUE

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***Abstract**. This paper provides an hybrid numerical-analytical solution to a transient problem of heat transfer determining the temperature field inside a cylinder full of water with imposed boundary and initial conditions. The Generalized Integral Transform Technique (GITT) is applied to solve the heat conduction differential equation considering temperature time and radial dependence. For each set of eigenvalues a temperature field is obtained and analyzed, taking into account the number of eigenvalues used. Further, the results consistence is verified. The influence of the cylinder diameter in time response of the system was studied. To optimize sensors localization the sensibility of the temperature with the radial position was investigated. Analysis for cylinders with different radios is alsoheld. In this way, important data to feed experimental control apparatus could be taken.*

***Keywords**: GITT, heat transfer, thermal system sizing*

1. INTRODUCTION

Temperature measurement and control has been a typical needing associated to several science areas, industrial processes, agricultural, environment, etc. Thermal systems projects presuppose prior knowledge of relevant heat exchanges mechanisms occurring during the process. There are many works about thermal control as Belo, 2003 and 2004, Nascimento, 2004, especially inside enclosures.

Mathematical models and computer simulation are very import tools to design experimental procedure and predict its conditions benefiting the dimension of the experiment and making it less expensive. Therefore, it is possible to compare the system performance just changing a few parameters and it is no so hard taking an overview in time response, global and local temperature behavior accordingly the system sizing in computational model. This paper presents thermal analysis of a cavity full with fluid initially at a uniform temperature (prescribed initial condition) and that is submitted to a boundary condition (imposed temperature at the surface) which puts all the system in disequilibrium. Then, dynamic system response is evaluated.

Transport phenomena (heat and/or mass transfer) involve partial differential equations solution, frequently, boundary and initial conditions problems. Analytical and numeric methods have been always studied for solving the proposed system dealing with accuracy or computational time. Here, we applied the Generalized Integral Transform Technique (GITT) to solve the problem, which is a well-known hybrid numerical–analytical approach that can efficiently handle diffusion and convection–diffusion partial differential formulations. It is based on expansions of the original potentials in terms of eigenfunctions and the solution is obtained through integral transformation in all but one of the independent variables, thus reducing the partial differential formulations to an ordinary differential system for the expansion coefficients, which can be then solved using numerical techniques or in some special cases, analytical procedures (Almeida *et al*., 2008).

The GITT has quite recently appeared in the literature as an alternative to conventional discrete numerical methods, for various partial differential formulations (Venezuela *et al*., 2009, Naveira *et al*., 2007, Barros *et al*., 2006, Macêdo *et al*., 1999). Its hybrid numerical - analytical structure permits the automatic control of the global error in the simulation, which avoids the need for several computer program runs to inspect for the convergence on the final results, and therefore yields codes that automatically work towards user prescribed accuracy targets. Symbolic computational languages have been a point of emphasis in the GITT applications due to the use of direct mathematical expressions during programation. However, the application of this technique in problems of thermal systems control is still rare. Therefore, the present work addresses the solution via GITT of a transient one-dimensional radial diffusion formulation describing the dynamic behavior of temperature inside a cylindrical enclosure.

2. PHYSICAL SYSTEM AND MATHEMATICAL MODELLING

2.1. Model assumptions

System consists in an enclosure thermally insulated that is filled of a liquid. The cavity is made from metal material and has tiny walls. Electronic apparatus make temperature control. Specific equipment (heater or cooler) is employed to fix the wall temperature at a prescribed value. Sensors are disposed in some points to determine temperature internal field. As change sensors localization or cylinder diameter, time response varies.

The following hypotheses were considered aiming problem solution:

- Heat conduction equation in cylindrical coordinates;
- Initially, thermal equilibrium;
- System is insulated, analysis neglects losses effects to environment;
- Cylinder's length is considered much longer than its radio;
- Temperature gradients in longitudinal and angular directions weren´t took into account;
- Constant properties.

2.2. Mathematical Modeling

Starting from heat conduction transient equation in cylindrical coordinates and considering the prior assumptions, the problem in question can be represented by:

$$\frac{\partial T(r,t)}{\partial t} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T(r,t)}{\partial r}\right) \quad \text{at } 0 < r < r_0 \tag{1a}$$

Initial and boundary conditions are listed below to complete the numerical formulation of the problem:

- Boundary conditions

$$\frac{\partial T}{\partial r}\Big|_{r=0} = 0 \quad \text{at } r = 0 \text{ and } t > 0 \tag{1b}$$

$$T(r_0,t) = T_s \quad \text{at } r = r_0 \text{ and } t > 0 \tag{1c}$$

- Initial condition

Uniform initial temperature:

$$T(r,0) = T_{ini} \quad \text{at } 0 \le r \le r_0 \text{ and } t = 0 \tag{1d}$$

where α is the thermal diffusivity of the water (m^2s^{-1}), T is the temperature (K), T_s is the imposed temperature at the surface, T_{ini} is the initial temperature, r is the radial position (m), r_0 is the maximum radius and t is time (s).

In a dimensionless form Eqs. (1) become:

$$\frac{\partial \theta(R,\tau)}{\partial \tau} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \theta(R,\tau)}{\partial R}\right) \quad \text{at } 0 < R < 1 \text{ and } \tau > 0 \tag{2a}$$

with boundary and initial conditions:

$$\frac{\theta(0,\tau)}{\partial R} = 0 \quad \text{at } R = 0 \text{ and } \tau > 0 \tag{2b}$$

$$\theta(1,\tau) = 0 \quad \text{at } R = 1 \text{ and } \tau > 0 \tag{2c}$$

$$\theta(R,0) = 1 \quad \text{at } 0 \le R \le 1 \text{ and } \tau = 0 \tag{2d}$$

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where the dimensionless groups are:

$$\tau = \frac{\alpha t}{r_0^2}; \qquad R = \frac{r}{r_0}; \qquad \theta(R,\tau) = \frac{T(r,t) - T_s}{T_{ini} - T_s}$$

Mathematical modeling allows the temperature field determination. We just have to solve the proposed equations set by now. The solution method applied here is the Generalized Integral Transform Technique (GITT) as described below.

3. SOLUTION METHOD

The Generalized Integral Transform Technique (GITT) is a powerful hybrid numerical-analytical tool used in several heat diffusion problems. Such a technique, as applied to time dependent problems, includes the basics steps below (Cotta *et al*, 1997):

(i) Selection of an associated auxiliary eigenvalue problem, that retains the highest capacity of information of the original problem;
(ii) Development of the appropriate transform/inverse formulae pair;
(iii) Integral transformation of the original problem by substituting the inverse formula into non-transformable terms or by using the integral balance approach;
(iv) Solving the resulting coupled system of ordinary differential equations in the time variable;
(v) Applying the inverse formula to the transformed field in order to obtain the solution for the original problem.

System (2) is solved using the Generalized Integral Transform Technique (GITT) (Özisik, 1980). The first step is to choose the so-called auxiliary problem. If we consider the differential Eq. (2) with its boundary conditions, this can be expressed as (Özisik, 1980)

$$\frac{1}{R}\frac{d}{dR}\left(R\frac{d\psi_i}{dR}\right) + \mu_i^2\psi_i = 0 \tag{3a}$$

$$\frac{d\psi_i}{dR}\Big|_{R=0} = 0 \tag{3b}$$

$$\psi_i(1) = 0 \tag{3c}$$

Eqs. (3) represent a classical Sturm–Liouville problem. Its solution is obtained in the form of eigenfunctions:

$$\psi_i(R) = J_0(\mu_i R)$$

where J_0 are zero order first kind Bessel functions, μ_i are the functions' roots (eigenvalues) and i = 1, 2, 3.

Second step is to define the Transformed/Inverse pair

$$\overline{\theta}_i(\tau) = \frac{1}{\sqrt{N_i}}\int_0^1 R\psi_i\theta(R,\tau)dR \qquad \text{Transformed} \tag{4a}$$

$$\theta(R,\tau) = \sum_{i=1}^{\infty}\frac{\psi_i\overline{\theta}_i(\tau)}{\sqrt{N_i}} \qquad \text{Inverse} \tag{4b}$$

where $\overline{\theta}_i(\tau)$ is the transformed dependent variable and the norm N_i is given by:

$$N_i = \int_0^1 RJ_0^2(\mu_i R)dR \tag{4c}$$

The normalized eigenfunctions are defined by:

$$\tilde{\psi}_i(R) = \frac{\psi_i(R)}{\sqrt{N_i}} \tag{4d}$$

Moreover, the eigenfunctions have the following orthogonality property:

$$\int \tilde{\psi}_i(R)\tilde{\psi}_j(R)dR = \begin{cases} 0, i \neq j \\ 1, i = j \end{cases} \tag{4e}$$

At this point, the integral transformation can be held applying the operator below in the Eq. (2)

$$\int_0^1 R\tilde{\psi}_i(R)dR$$

Using orthogonality property (4e), Integral Transform (4a) and Inverse Formula (4b), from Eqs. (2) we get

$$\frac{d\overline{\theta}_i(\tau)}{d\tau} + \mu_i^2\overline{\theta}_i(\tau) = 0 \tag{5a}$$

The integral transform of the entry condition (Eq. (4a)) produces the following transformed initial condition:

$$\overline{\theta}_i(0) = -\frac{1}{\mu_i} J_1(\mu_i) \tag{5b}$$

where J_1 is an one order first kind Bessel function.

Ordinary differential equations system presented in Eqs. (5) was solved in the Matlab. Obviously, for computational purposes this system is truncated to a sufficiently large finite order, N, for the required convergence control.

3. RESULTS AND DISCUSSION

3.1. Convergence analysis

It is necessary to observe the temperature field profile at the initial instants for convergence analysis purposes. Therefore, temperature was evaluated in two distinct moments (dimensionless time) varying dimensionless positions. Convergence development can be seen through the increase in the system truncation order, denoted by the letter N, see Table 1. A maximum of 16 terms in the temperature series were considered for this demonstration with excellent convergence characteristics.

In the beginning of the process much series terms are required to get converged temperature values. Considering τ = 0.005, stabilized values occurred with a maximum of 15 terms in any position. It is possible to realize that at R = 0.1 the worst case happened, physically, this position has always been affected by symmetry boundary condition and it is far from perturbation imposed at surface, so there is a delay in response to excitation.

For τ = 0.010, a time step later, we can notice that convergence occurs faster, only 11 terms are necessary to obtain converged temperature values. Same behavior was observed at R = 0.1, in this point, a larger number of terms were used to series convergence. It is remarkable that the number of series terms decreases with observation time increasing, of course, if the convergence analysis is made for posterior moments less terms will be required.

To visualize the system behavior, one 3D curve was generated. Figure (1) represents temperature distribution inside the studied body. In this case, we considered a cylinder with radio equal 0.010m and the plot was obtained from converged values. Starting from 300K curve indicates that the temperature variation occurs quickly near surface, in the first 50s it reached 330K and for positions closers to the center it changes slowly, it took almost 350s to hit the same 330K. Clearly, the imposed surface boundary condition makes the most external points reach higher temperature values taking less time than middle ones due to water thermal diffusivity. As the water has low thermal diffusivity it takes a large time period to transfer the energy trough the fluid mass. At the center there is null flux boundary condition, so temperature doesn't change with position in the surroundings.

Table 1. Temperature field profile convergence behavior

τ = 0.005									
N	Temperature (K)								
	R = 0.1	R = 0.2	R = 0.3	R = 0.4	R = 0.5	R = 0.6	R = 0.7	R = 0.8	R = 0.9
1	281.6416	284.5691	288.1102	292.8525	298.5935	305.0894	312.0673	319.2380	326.3093
5	298.9966	300.9644	300.3764	299.1010	299.9457	300.9656	299.6946	300.7946	312.1874
10	299.9875	300.0102	299.9906	300.0091	299.9912	300.0112	300.0990	301.6868	311.0428
11	300.0046	299.9965	300.0024	299.9989	299.9997	300.0044	300.1041	301.6834	311.0444
12	299.9986	300.0007	300.0000	299.9994	300.0008	300.0023	300.1064	301.6815	311.0455
13	300.0003	300.0000	299.9998	300.0002	300.0001	300.0025	300.1068	301.6809	311.0460
14	299.9999	300.0000	300.0000	300.0000	300.0000	300.0027	300.1067	301.6808	311.0461
15	300.0000	300.0000	300.0000	300.0000	300.0000	300.0027	300.1067	301.6808	311.0461
16	300.0000	300.0000	300.0000	300.0000	300.0000	300.0027	300.1067	301.6808	311.0461
τ = 0.010									
N	Temperature (K)								
	R = 0.1	R = 0.2	R = 0.3	R = 0.4	R = 0.5	R = 0.6	R = 0.7	R = 0.8	R = 0.9
1	283.8242	285.9494	289.3896	293.9968	299.5741	305.8849	312.6639	319.6303	326.5000
5	299.7811	300.2275	300.0606	299.8010	300.0451	300.3777	301.2269	305.7014	316.8796
6	300.0305	299.9356	300.0355	300.0189	299.9716	300.2314	301.3530	305.7725	316.7368
7	299,9997	300,0095	299,9883	300,0097	300,017	300,1954	301,3495	305,8070	316,7058
8	299.9990	299.9998	300.0011	299.9997	300.0208	300.1984	301.3422	305.8150	316.7008
9	300.0003	299.9998	300.0002	300.0011	300.0191	300.2000	301.3408	305.8160	316.7003
10	299.9999	300.0000	300.0000	300.0012	300.0191	300.2000	301.3408	305.8160	316.7003
11	300.0000	300.0000	300.0000	300.0012	300.0191	300.2000	301.3408	305.8160	316.7003
12	300.0000	300.0000	300.0000	300.0012	300.0191	300.2000	301.3408	305.8160	316.7003

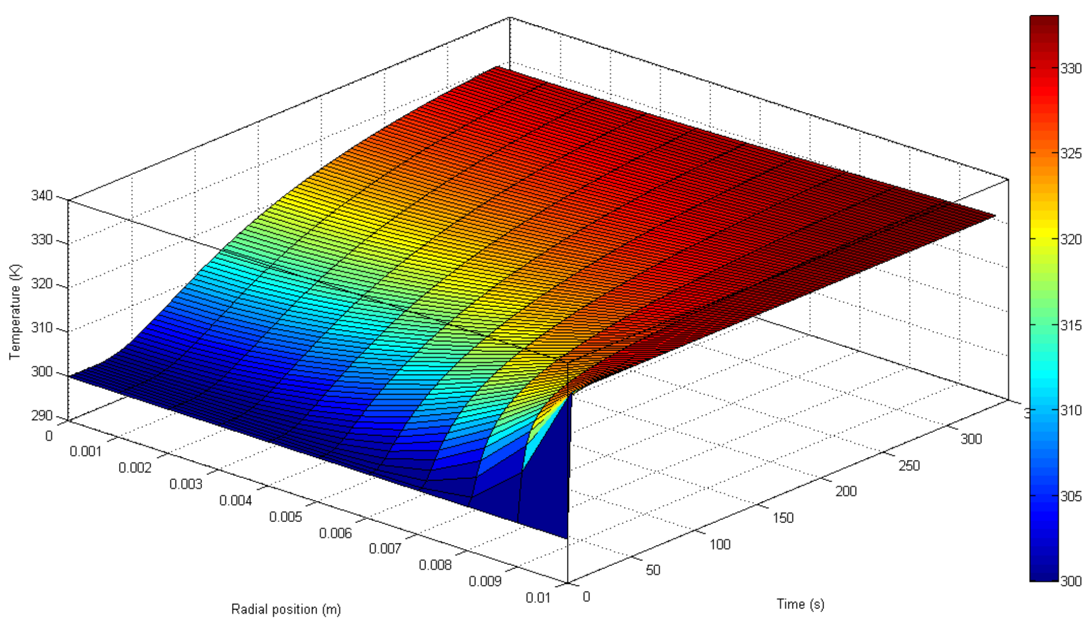

Figure 1: Visualization in 3D of the temperature variation inside the cylinder, using Matlab, for N = 16

3.2. Sensibility analysis

Sensibility is another important feature to investigate. How temperature varies with the position is estimated, the Fig. 2 shows dT / dr (temperature variation with radial position) as a time function. Temperature sensibility with position increases as distance from the center increases, it is expected because the perturbation occurs at the surface. For analyzed points, the highest value to dT / dr happens at r = 0.009m (more than 160 K/m) and the lowest at r = 0.001m (values near 1 K/m). It can be seen that sensibility is greater in the beginning of the process, all peaks happens before

80s and temperature stops to change with position approximately in 300s, meaning that a equilibrium temperature has been found. But temperature variation with position is greater and faster closer to the surface (taller peaks in minor time), there is a point of maximum in 3,4s at r = 0.009m. As we move away from the surface temperature variation with position is smaller and slowly (shorter peaks in major time), there is a point of maximum in 6,3s at r = 0.001m. A remarkable fact is that around 95s sensibility becomes greater at r = 0.007m instead of 0.009m, once nearest surface temperature has reached almost the steady state value and for other positions it still has a considerable variation.

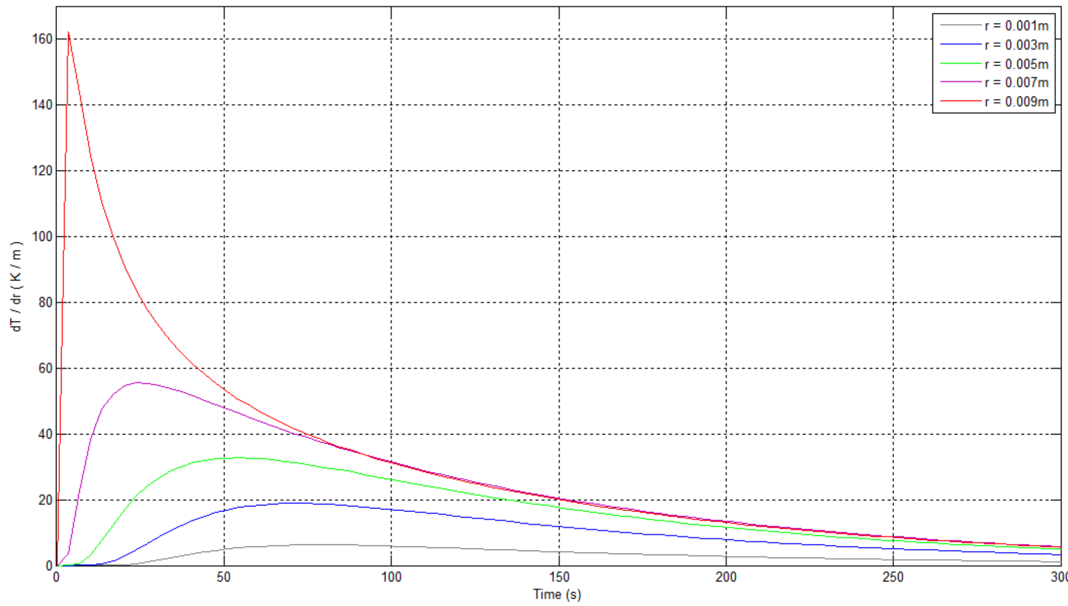

Figure 2: Temperature variation with radial position versus time, r_0 = 0.010m.

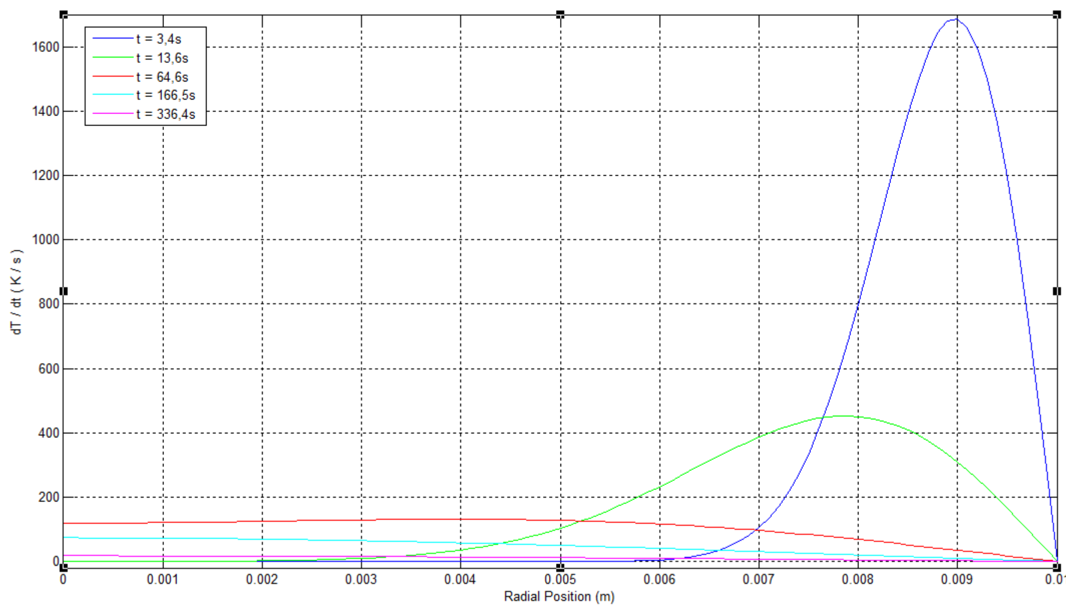

Figure 3: Temperature variation rate versus radial position, r_0 = 0.010m.

Another point of view in sensibility analysis is temperature variation with time (dT/dt). Accordingly Fig.3, there is a clear difference in time response for each position within cylinder. It is possible to separate temperature variation rate behavior in ranges. For a localization range from the center of the cylinder to 0.0033m dT / dt follow the decreasing order: 64,6s, 166,5s, 336,4s, 13,6s and 3,4s. Thus, in these positions we can't say that the variation is greater neither it is smaller for the first time steps. This region suffers flux null effect (symmetry condition) and is quite distant from the disturbance). Between 0.0033 and 0.0044m temperature variation with time obeys the sequence 64,4s, 166,5s, 13,6s, 336,4s and 3,4s. In the range 0.0044m – 0.0052m the higher variation values occurs in 64,6s, 13,6s, 166,5s, 336,4s and 3,4s. Others ranges are noticed: 0.0052m – 0.0061m, 0.0061m – 0.0066m, 0.0066m – 0.007m, 0.0070m – 0.0077m and 0.0077m – 0.010m were, finally, variation is more intense for the first time steps, dT / dt assume higher values in the beginning of the process (in decreasing order: 3,4s, 13,6s, 64,6s, 166,5s and 336,4s), what indicates strong influence of the boundary condition in this zone. Considering t = 3,4s, dT / dt hits almost 1700 K/s at r = 0.009m, greatest detected variation rate. For cylinder center surroundings very smaller rates are realized, in any moment. These results lead us to realize a sensibility displacement as time and radial position function.

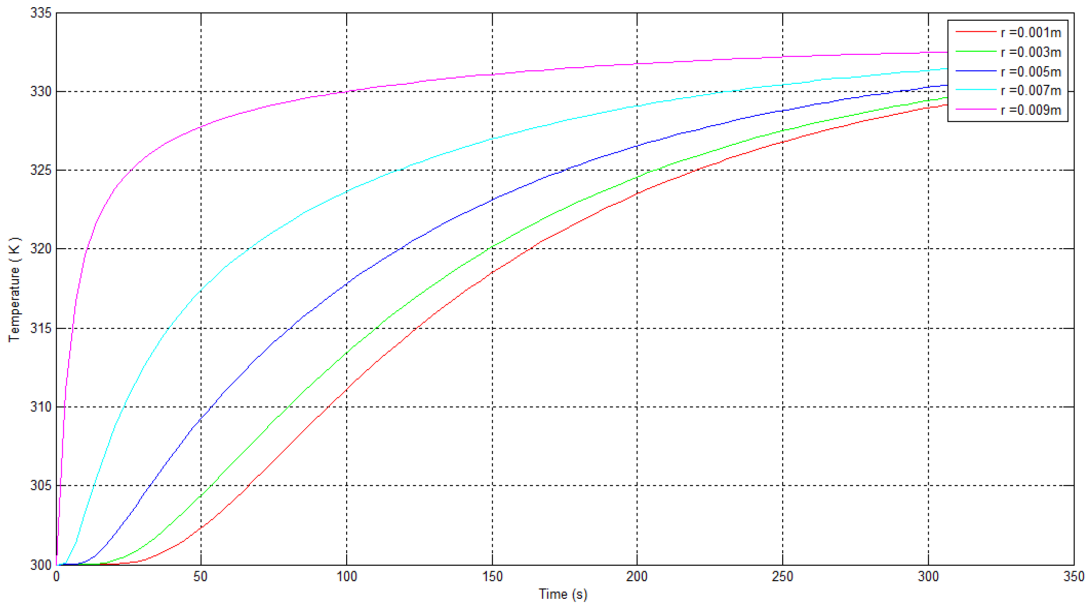

Figure 4: Temperature profile at some radial position, $r_0 = 0.010m$.

Figure 4 illustrates temperature profile for several radial positions. As expected, at r = 0.001m temperature changes with a big delay if compared with r = 0.009, we had just already justified this behavior taking fluid low thermal diffusivity into account. Temperature shows exponential ongoing and dT / dr plots confirm that temperature varies with higher speed at the process starting and lower at the end.

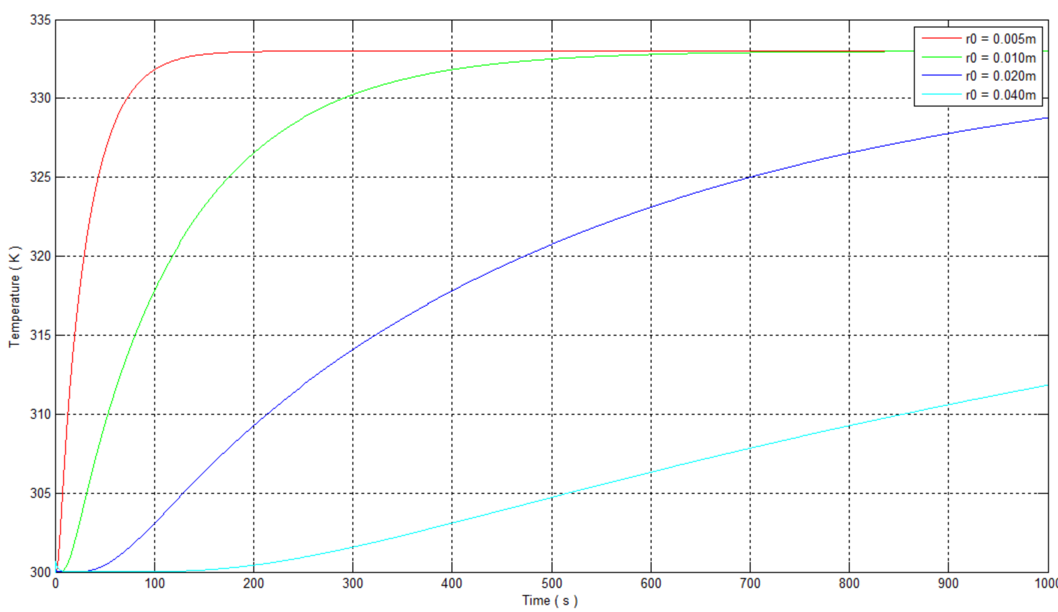

Figure 5: Temperature profile for cylinders with different radios.

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Comparing cylinders with distinct radios, in Fig.5, time response decreases when radios decreases showing larger inertia for bigger systems. Clearly, more mass implicates in greater thermal inertia. Thus, finer cylinders promote faster equilibrium temperature reaching.

4. CONCLUSIONS

In this work, the Generalized Integral Transform Technique (GITT) was successfully applied in a heat transfer problem generating reliable values. This modeling allows illustrating temperature distribution within a cylinder full of water, using converged series. Evaluating sensibility is relevant for sensor positioning, because of the identification of more sensibly places into enclosure as system time response too. Simple parameter changes, as thermal diffusivity or cavity radio in the mathematical model make possible others studies aiming concrete experiment sizing. A posterior research, already in progress, includes the development of high accuracy thermal control apparatus for cylindrical enclosures filled with a fluid applied to industrial processes.

5. ACKNOWLEDGEMENTS

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