DESCRIPTION OF HELICAL BROACH GEOMETRY USING MATHEMATICAL MODELS ASSOCIATED WITH CAD 3D MODELS

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Abstract: *In order to supply the constant demand for quality and low price, companies continually need to upgrade their technologies, knowledge on their products and production processes. In reaching these goals, the rationalization of the production process is one of the main factors affecting the cost and quality of products. In this context, the broaching process offers a solution for certain types of operations. The helical broaching process uses a single tool that performs roughing, semi-finishing and finishing in a machining cycle, making the process highly productive and attractive in terms of mass production. Helical broach is a tool of high value (and high cost), but if it is well designed, properly manufactured and used within the specifications, it provides a great return on investment. This tool presents high geometrical complexity, which directly affects the efficiency of the process. In order to gain a better understanding of the geometry of the broach and to correctly design the tool, a study is proposed herein based on a mathematical model which describes the tool geometry and compares it to CAD 3D models. These mathematical model algorithms are then implemented in computational language. With the inputting of some parameters, like rake angle and helical spline angle, it is possible to obtain some important information on the tool geometry, for example, the cross sectional area, the moment of inertia and the tool pitch. To obtain these characteristics through other methods, like direct measurement or using a CAD 3D software, for example, more time consuming and expensive methods would be necessary, and in some cases it would even be impossible. Thus, these models are used to gain information on many geometrical characteristics of the broach used in the design in a quicker and cheaper way.*

Keywords*: broaching, modeling, simulation.*

1. INTRODUCTION

Broaching is a process of machining where the cutting movement is predominantly linear, characterized by the use of a tool with multiple teeth with increasing pitches, arranged in series (Stemmer, 2005). The tool used is called broach, and it can be forced by traction or compression inside or outside the piece. The operation consists of removing material from a workpiece in order to build straight and flat surfaces or a specific internal or external cross section. Almost any cross section can be broached as long as all machined surfaces remain parallel to the direction of the broach movement. The exception to this rule is uniform rotation sections such as spiral gear teeth, produced by twisting the broach tool during the machining cycle (SME, 1983).

In the broaching process, the cutting speed is the only machining parameter there can be changed during execution of the process. In helical broaching, the cutting speed is a combination of a linear motion and a rotating motion. Other parameters such as feed and depth of cutting are defined by the geometry of the tool.

Historically, broaches have been designed based on trial and error or on the designer`s experience (Sutherland *et al*., 1996). The greater the precision of the component required, the less the design should be based on experience. To design a broach in the early life cycle of a product, a model of how the broach will perform during the operation would be extremely advantageous (Sutherland *et al*., 1996). The broach, a tool of high cost, lacks key information required for its design and manufacture, such as the behavior of the cutting forces during the machining cycle, stress distribution in the tool, machining angles, and so on.

2. THEORY REVIEW

2.1. Helical broach

The helical broach differs from the traditional traction broach since it or the workpiece is rotated, depending on whether or not the teeth of the broach produce a helix with an angle corresponding to the required piece. For some applications, when the helix angle is smaller than 15°, the rotating broach does not need to be held by an actuator, because the workpiece has the ability to auto-rotate due to the small helix angle (Stemmer, 2005; SME, 1983). However, in cases of large-scale production or large helix angles, there must be a tool system of rotation synchronized with the linear movement. An example of a helical broach is shown in Fig. 1.

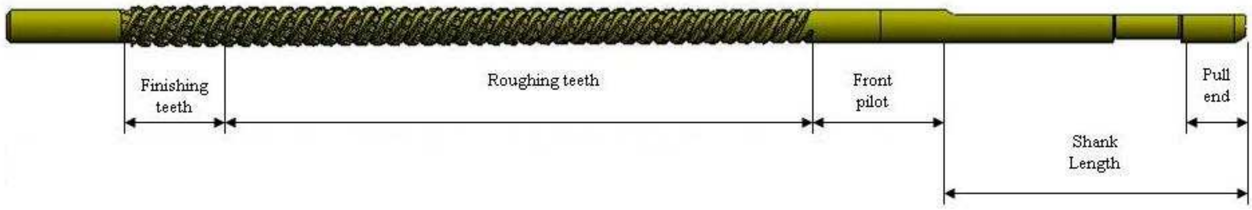

Figure 1. Typical internal helical broach with standard nomenclature used

2.2. Geometry of helical broach teeth

One of the key features of the geometry of the broach teeth is the space between them, i.e., the distance between one teeth and the next in the same row, since this determines the number of teeth working on the piece simultaneously. This is an important design factor because the tool tension values can be calculated. With a helical broach the pitch can be mathematically determined according to the number of tooth gullets, the number of broach grooves, and the helix angle, as seen in Fig. 2.

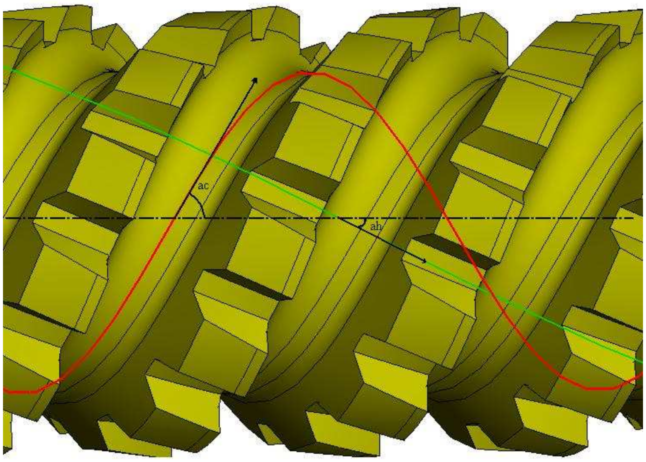

Figure 2. Tooth gullets, grooves and helix angles

The rake angle γ plays an important role in the development of broaching tools due to its influence on the mechanisms of chip formation and its consequences in terms of the strength of machining, cutting temperature, chip adherence to the tooth, etc. For materials such as medium to low carbon steel, the use of rake angles of 15-20°, and for aluminum the use of rake angles of 10-15°, is recommended (Stemmer, 2005; Walsh and Cormier, 2006; ASM, 1997).

The rise per tooth a_{sf} is also one of the main features, because it permits the calculation of the cutting force, together with the tooth width and the material constant specific cutting pressure. With the value of the cutting force it is then possible to ascertain the tension on the tool and, therefore, the tool resistance can also be calculated. For steel with medium to low carbon the use of cutting depths of 0.03 to 0.05mm is recommended. For aluminum a cutting depth of 0.1 to 0.2 mm is recommended (Stemmer, 2005; Walsh and Cormier, 2006; ASM, 1997).

Besides these main features considered in the design of a broach, there are also other parameters, for example : chip bags radius r_1 and r_2, which are needed to receive the chips during machining; the clearance angle α, which affects the tool life, because the smaller the angle the greater the friction on the flank and also the less the tool needs to be sharpen. In Fig. 3 the main elements of broach tooth geometry can be seen.

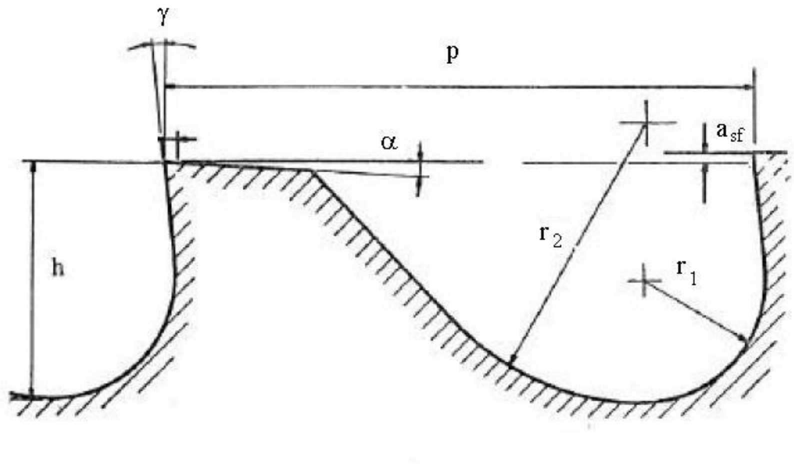

Figure 3. Main tooth angles

3. GEOMETRIC MODEL DESIGNED IN CAD 3D

The model, designed in CAD 3D software, used in this study refers to a broach used by a company in its production line, and it was designed to produce helical gears. The original design, drawn in CAD 2D, was converted into a CAD 3D model of the tool, providing key features for the modeling.

3.1. Broach modeling with CAD 3D

The model designed in CAD 3D provides important information on the tool geometry, showing the model space. This method, however, when compared to purely mathematical models, has the disadvantages of being laborious and time consuming. In Fig. 4 some steps for modeling in CAD 3D can be observed.

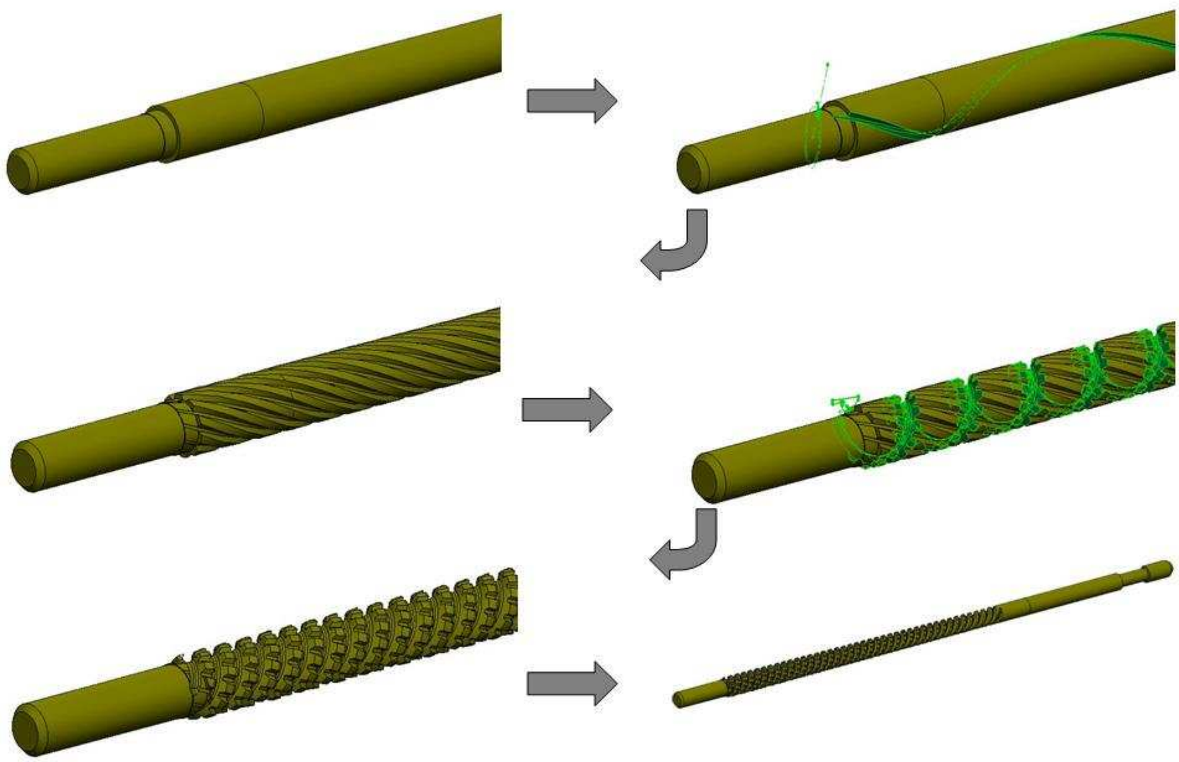

Figure 4. Helical broach modeling processes

3.2. Broach mathematical modeling

Compared to models designed in CAD 3D, a drawback of mathematical models is the need to develop equations which represent the tool. This can be very time consuming and often it is even impossible to obtain an analytical solution. In these cases, it is only possible to obtain a result through numerical methods or by simplifying the real situation.

In this study, the first step was to determine the edge position of each broach tooth in space using parametric equations, which represent the paths followed by a row of teeth and the tooth gullets. At the intersection of these two equations the cutting edges are found. To find these points a parameter represented by Eq. (1) is used:

$$t = \frac{p_2}{p_1 + p_2} \cdot \left(-\theta + \frac{2 \cdot k \cdot \pi}{nc} \right) \tag{1}$$

where t is the parameter used to calculate the position of the tooth, p_2 is the helix pitch of the tooth gullets, p_1 is the helix pitch of the groove (Fig. 5), θ is the initial angle of the groove, which is dependent on the number of grooves in the broach, nc is the number of tooth gullets and k, where $k \in Z$, is a variable. This variable is used to determine the positions of all teeth in a row. The values of interest for this variable vary for each row of teeth and will be discussed later.

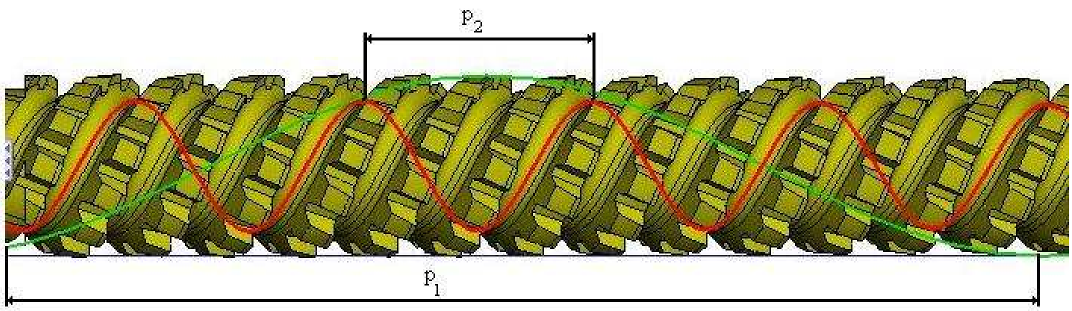

Figure 5. Section of a helical broach showing the pitch of the tooth gullets and the grooves helix.

The initial angle of the groove, θ, is determined through its angular symmetry distributions in the broach, taking one as a reference. It can be calculated through Eq. (2):

$$\theta = \frac{n}{nr} 2.\pi \text{ ; } n \in Z, 0 \le n < nr \tag{2}$$

To determine the radius of the cross section in terms of a parameter, Eq. (3) is used:

$$r(t) = \frac{D_o}{2} + \frac{p_1}{4.\pi.L}.(D_f - D_o).t \tag{3}$$

where D_o is the initial diameter, D_f is the final diameter, L is the length of roughing teeth (Fig. 6).

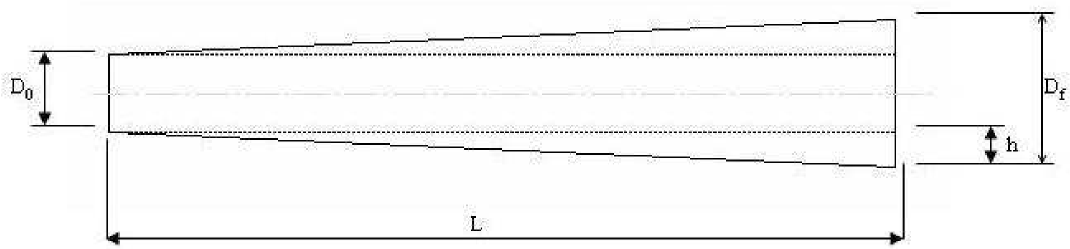

Figure 6. Design of a broach showing the initial and final diameters as well as the length

To obtain the position of each edge, Eqs. (4), (5) and (6) are used:

$$x = r(t) \cdot \cos(t + \theta) \tag{4}$$

$$y = r(t).\sin(t + \theta) \tag{5}$$

$$z = \frac{p_1}{2.\pi}.t \tag{6}$$

where *x, y* and *z* represent the Cartesian coordinates of a point in space and θ is the initial angle of the teeth row. The points of interest for the model are those which are located within the range:

$$0 \le z \le L$$

Therefore:

$$\frac{nc.\theta}{2.\pi} \le k \le \frac{nc}{2.\pi}.\left(2.\pi.L.\frac{p_1 + p_2}{p_1.p_2} + \theta\right) \tag{7}$$

In other words, the k values which are of interest for the model are all of the values which lie within these limits.

For the value of the cross sectional area of this tool the actual area represented in Fig. 7 can be approximated. This figure shows a division into 3 parts. Each part is identical to the other two and is formed by the sum of two areas: area 1 and area 2 (A_1 e A_2, respectively). Area 1 has a constant radius from 0° to θ_1, and area 2 has a variable radius and ranges from 0° to θ_2.

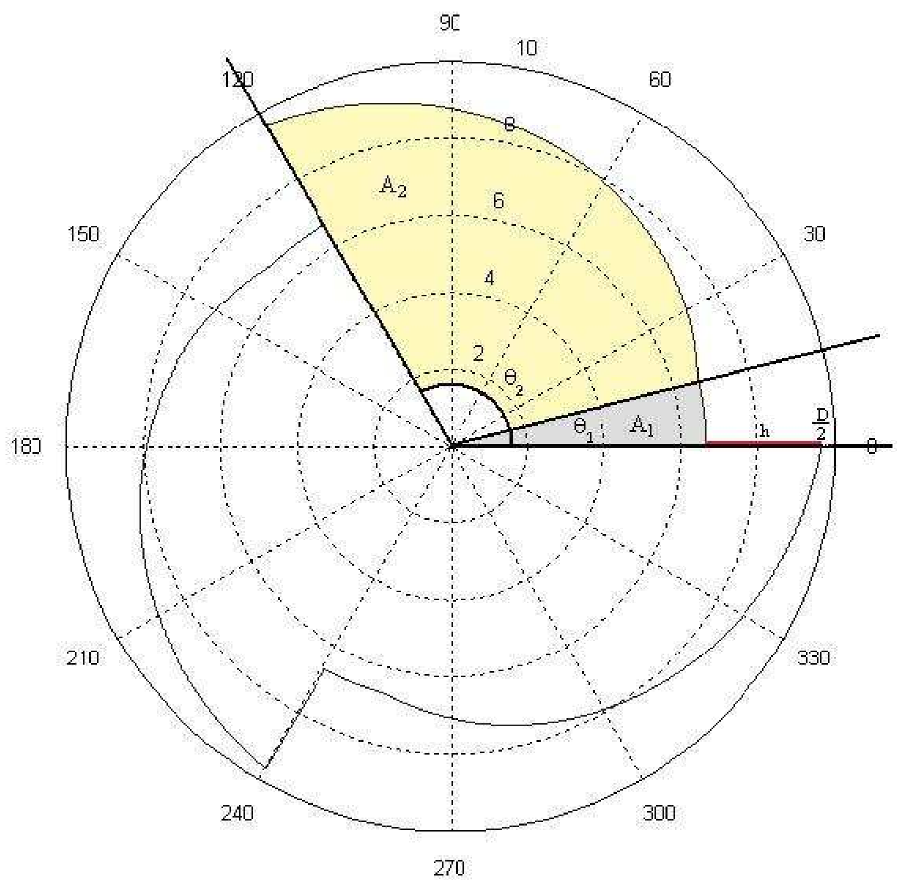

Figure 7. Mathematical model of the cross sectional area

For area 2, the radius is determined by Eq. (8):

$$r_2 = \left(\frac{D}{2} - h\right) + h.\frac{\theta}{\theta_2} \tag{8}$$

where h is the gullet depth and D is the diameter of the section.
The equations representing the model of the cross sectional area shown above are:

$$A_1 = \int_0^{\theta_1} \int_0^{\frac{D}{2}-h} r dr d\theta_1 = \frac{1}{2}\left(\frac{D}{2} - h\right)^2 .\theta_1 \tag{9}$$

$$A2 = \int_0^{\theta_2} \int_0^{r_2} r dr d\theta_2 = \frac{1}{2}\int_0^{\theta_2} r_2^2 dr \tag{10}$$

since:

$$d\theta = \frac{\theta_2}{h}.dr \tag{11}$$

Substituting (11) in (10) we find that:

$$A_2 = \frac{\theta_2}{6.h}\left[\left(\frac{D}{2}\right)^3 - \left(\frac{D}{2} - h\right)^3\right] \tag{12}$$

Finally, the total area is:

$$A = 3.(A_1 + A_2) \tag{13}$$

Another important factor in terms of the future tool design is the polar moment of inertia, used in the calculation of stresses. Based on Fig. 7 the following equations can be drawn:

$$J_1 = \int_0^{\theta_1} \int_0^{\frac{D}{2}-h} r^3 dr d\theta = \frac{\left(\frac{D}{2} - h\right)^4}{4}.\theta_1 \tag{14}$$

For area 2:

$$J_2 = \int_0^{\theta_2} \int_0^{r_2} r^3 dr d\theta = \frac{1}{4}\int_0^{\theta_2} r_2^4 dr \tag{15}$$

since:

$$d\theta = \frac{\theta_2}{h}.dr \tag{16}$$

Substituting (16) for (15) we find that:

$$J_2 = \frac{\theta_2}{20.h}\left[\left(\frac{D}{2}\right)^5 - \left(\frac{D}{2} - h\right)^5\right] \tag{17}$$

Finally:

$$J = 3.(J_1 + J_2) \quad (18)$$

To obtain a better adjustment of these values for the area and polar moment of inertia to the values found in the CAD 3D model, the angles θ_1 and θ_2 change according the position on the broach and have following relations:

$$\theta_1 = \frac{\pi}{4.nc.L}.z \quad (19)$$

$$\theta_2 = \left(1 - \frac{z}{8.L}\right).\frac{2.\pi}{nc} \quad (20)$$

where z is the position on the broach, L is the length of roughing teeth and nc is the number of tooth gullets.

Taking into account the trapezoidal shape of the broach teeth, the size of the tooth edge can be inferred based on the design data related to the crest length and the tooth base. In this case, the variation of the tooth width will be linear with the diameter and the coefficients of the equation can be determined using the following conditions:

1 – When $d=D_0$, $b=b_0$;

2 – When $d=D_f$, $b=b_f$;

where D_0 and D_f are the initial and the final diameters and b_0 and b_f are the initial and final length of the edge.

Thus, the following linear system is found:

$$\begin{cases} b_0 = A.D_0 + B \\ b_f = A.D_f + B \end{cases} \quad (21)$$

Solving the linear system, the equation for b is the following:

$$b = \frac{d - D_0}{D_f - D_0}.(b_f - b_0) + b_0 \quad (22)$$

4. COMPARISON BETWEEN CAD 3D AND MATHEMATICAL MODELS

In order to validate the mathematical model, the results can be compared to those of the model designed in CAD 3D.

For the size of the edge, the values obtained for the first teeth in CAD 3D were around 3.65mm and using the mathematical model they were 3.70mm, which represents a percentage difference of around 1.3%. The CAD 3D last teeth has the value of 2.42mm and using the mathematical model the value was 2.70mm, which represents a percentage difference of around 11.5%. The comparison between these two models can be seen in Fig. 8.

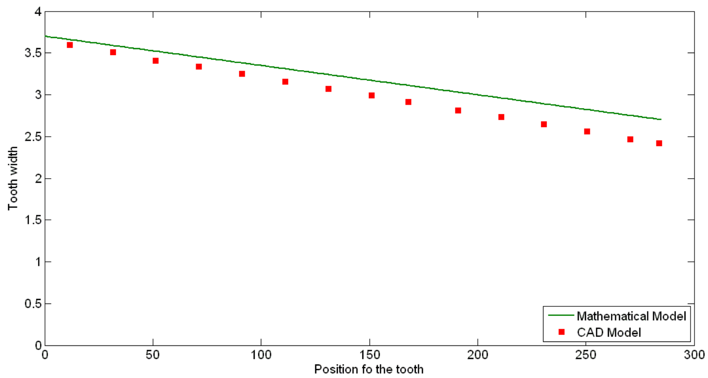

Figure 8. Comparison between the tooth widths obtained from the mathematical and CAD models.

For the cross sectional area in the CAD 3D model, the value at the beginning of the broach was 151.3mm^2 and in the mathematical model it was 140.3mm^2, a difference of 6.6%. At the end of the broach the model in CAD 3D had a value of 206.9mm^2 and in the mathematical model it was 198.2mm^2, a difference of 4.2%. A comparison between these two models can be seen in Fig 9.

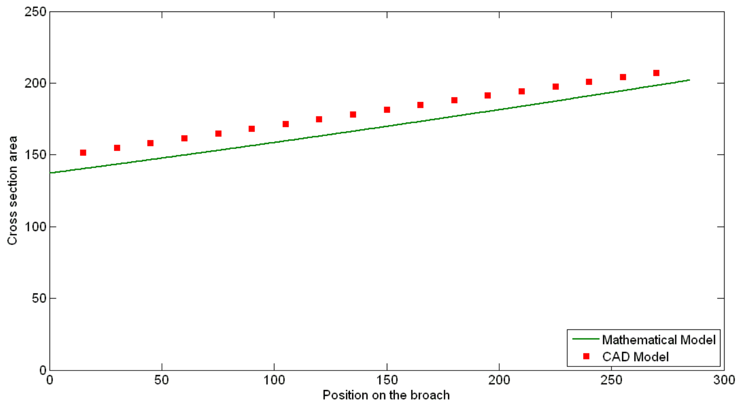

Figure 9. Comparison between the cross sectional areas obtained with the mathematical and CAD models.

5. CONCLUSIONS

Both methods used in this study showed advantages and disadvantages related to their usage. The advantages of the graphical method using CAD 3D in terms of visualization of the final product and reliability of the result were evident. However, when there is a need to predict the cutting forces or obtain the size and position of each edge related to the piece to be machined, the drawbacks of this method become clear, since the cutting force of each edge would need to be calculated manually. In contrast, the mathematical method is the best alternative to study the cutting force since, once the model equations are obtained, it is easy to implement these equations in an algorithm.

The differences between the values found using the two methods are mainly due to the broach geometry simplifications in the mathematical model. However, for application in predicting the machining forces, this variation is negligible, since the variation in the cutting force due to the formation of chips generates a greater variation.

5. ACKNOWLEDGEMENTS

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