DETERMINATION OF THE DISTRIBUTION OF SITES AND BONDS OF PERCOLATION NETWORKS USING THE METHOD OF THE OPENING-DILATION

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Abstract. *This paper shows a proposal of master's degree dissertation, whose main objective is to esteem permeability values through computational models. The proposed model is of percolation network that considers the flow through a hypothetical porous media, a three-dimensional representation formed by sites and/or bonds. Some morphologics characteristics (distribution of pores size and throats) and topologicals (interconnectivity of porous structure) of the way is gotten from technical analysis of images. The innovative character consists in using the methods of Opening-Dilation, in digitalized images, for the obtaining of parameters of pores and throats distribution, that will be used in the generation of the percolation network.*

Keywords*: permeability, percolation, revervoir rocks.*

1. INTRODUCTION

The characterization and understanding of the properties of the porous constituted in activity required that is essential for understanding the behavior of production of hydrocarbons reservoir, and definition of recovery methods to be used. Among the properties to be characterized, the permeability has been receiving great attention by geologists and petroleum engineers.

Conventionally values of permeability are determined by testing petrographics in laboratory experiments involving injection of fluid in a sample of rock. Such experiments require the existence of samples of testimony of good quality and a significant spent time in the laboratory and operating costs. Therefore demand through new techniques to optimize the obtaining of these values.

Among these new techniques, the image analysis comes out in the scientific literature due to its applicability in areas of knowledge. By using image analysis is possible to estimate values of properties, such as porosity, and, to develop methods able to characterise the geometry of the system and rebuild the porous (Arns et.al., 2004; Bueno, 2001; Fernandes et.al., 1994).

Several authors are getting good results in determining petrophysical property of reservoir rock with the use of models used to rebuild in three dimensions the porous. Between these Santos et.al. (2002), developed the model of reconstruction overlapping spheres , in Zhirong (1997) is introducing model of Gaussian truncated and Bueno (2001) displays a reconstruction of the porous called Gaussian truncated 2. From these models the permeability of porous using three dimensional representations can be estimate by reconstruction 3-D and numeric models for calculations of permeability.

The main disability practice presented in the patterns of reconstruction developed by Bueno (2001) and Zhirong (1997) is in the ability to generate representations 3D image 2D with great length correlation parameter (which leaves a sense of order of size of objects that make up the image). In these models the reconstructed images would have to have dimensions above 1000^3 , which requires super workstations with a lot of memory RAM (16GB) or use of parallel processing in a cluster. An alternative is to use patterns percolation networks.

The study of physical phenomenom of behavior of a fluid in a real porous structure is hampered due to the great complexity of the geometry of these systems. It is possible to get an approximation of the geometric model of porous with the representation of porous structure through a geometric network of interconnected capilar elements. On this network the nodes represent the pore, that is, the empty spaces with the fluid capacity storage; and bonds correspond to the canyons, that connect a pore to the other, and track properties of fluid transmission.

This article contains a proposal for a dissertation of masters, whose main goal is to estimate values of permeability through computer models. The model proposed is the percolation network that considers the flow through a hypothetical porous media, a three-dimensional representation made up of sites and / or bonds. Models, algorithms and programs simulation to estimate values of permeability will be developed, all using object oriented modeling (UML) and programming language c++. The innovative character is to use method of Opening – Dilation, in digital images, to obtain parameters of distribution the sites and bonds, to be used in the generation of percolation networks.

This work is organized as follows: The section 2 presents a brief bibliographic review about image analysis (section 2.1) and percolation networks (section 2.2). In section 2.3 appears the methodology for the development of the proposal. In section 3 is approached the construction of the percolation networks based on distribution of size pores. And in section 4 appears the method Opening – Dilatation to obtain values of distribution of size pores and bond.

2. BIBLIOGRAPHICAL REVIEW

2.1. Analysis of images

The processing and analysis of images to get properties about middle porous have received great interest by researchers in recent decades. There are several jobs in the literature where the authors were able to determine good accuracy, properties of the porous as permeability, porosity, curves of capillary pressure, mineralogical composition and electrical properties (Arns et. al., 2004; Monteagudo et.al., 2001; Gasperi, 1999).

Through optical or eletronic microscopy is possible to get images of two-dimensional porous. These images when subjected to processes of segmentation, characterization and reconstruction of image analysis, result in an important tool for obtaining properties about porous media. Images can be obtained in different extensions (depending on the technique used), considering also the resolution, since the fundamental criterion for the resolution is to preserve the images, objects related to property that you want to find (Gasperi, 1999).

After the acquisition, the images must pass through a preprocessor to correct or minimize problems in the acquisition. Then the process of segmentation that consists in transforming a image in grayscale or color levels in a binary image (Fig. 1), which separates the solid phase from the porous phase (Rego and Bueno, 2008). From a two-dimensional image duly segmented to the characterization where parameters as porosity, connectivity function, autocorrelation function, among others are determinated. In this work is used as a parameter the distribution of sizes pores in order to obtain informations that will be used in the construction of percolation network.

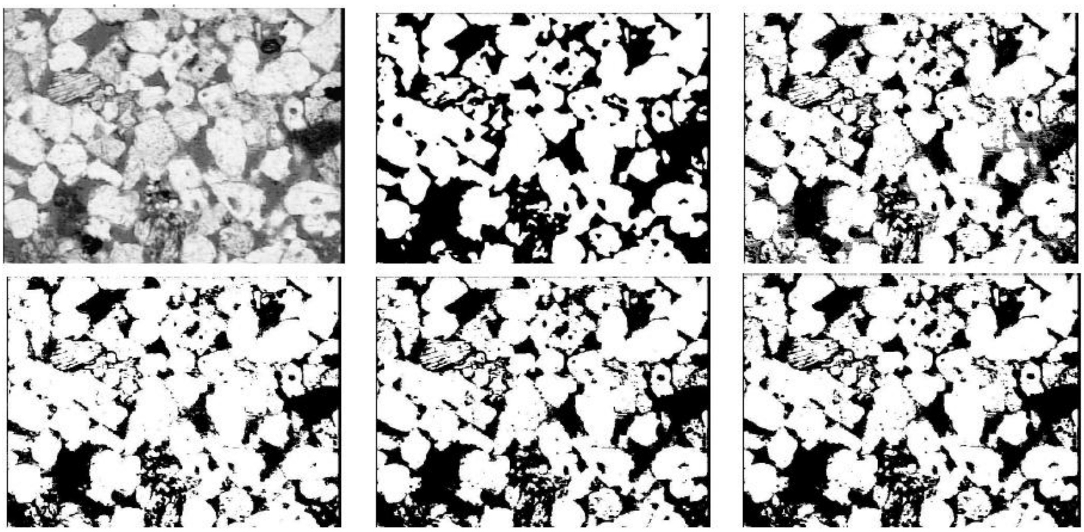

Figure 1. Gray image Berea 500, as a binary number using different algorithms described in Parker (1997). Rego and Bueno, 2008.

Considering X a point of coordinated (x, y) where *x*, *y*, are integer values in a discreet two-dimensional space, you can define the function phase *Z(X)* and the porosity (ε) by:

$$\text{(a)}\quad Z(x) = \begin{cases} 1 \text{ if x belongs to the pore space,} \\ 0 \text{ otherwise.} \end{cases} \qquad \text{(b)}\quad \varepsilon = \overline{Z(x)} \tag{1}$$

The result of porosity obtained by equation (1b), depends on the correct image segmentation and it is associated with the pores that have been invaded by the resin in the impregnation process. A study on methods of images segmentation can be found at (Rego and Bueno, 2008).

The autocorrelation function *(C(u))* is defined by,

$$C(u) = \langle [Z(x)][Z(x+u)] \rangle \tag{2}$$

where *u* is the displacement vector. This function provides information about distribution of pores in the porous media, in other words, it measures the degree of spatial correlation between points representing this space. The autocorrelation curve (Fig 2) assumes a value for maximum $|u| = 0$, and decreases as it $|u|$ grows. For a particular value of $|u|$, C(u) takes a minimum named correlation length (λ). The correlation length represents the distance from which the middle is no longer correlated.

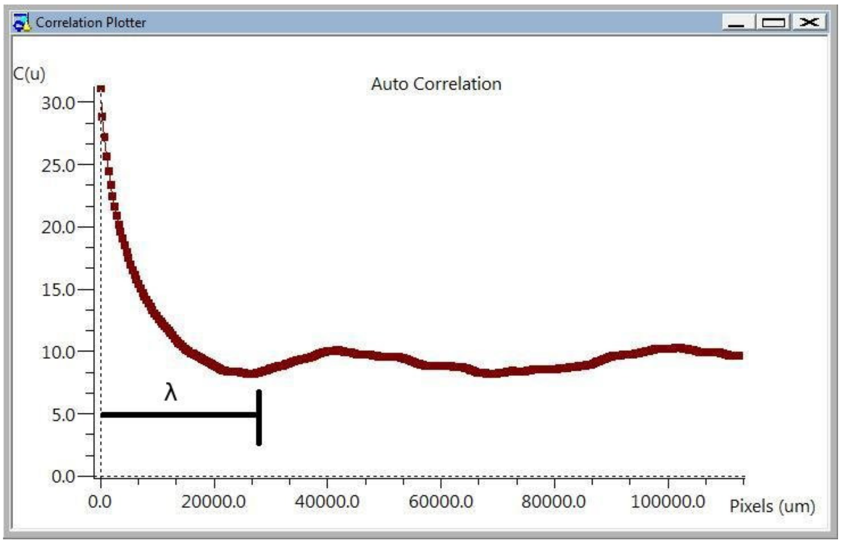

Figure 2. Autocorrelation curve.

The autocorrelation curve illustrated in Fig. 2 is used by the algorithm Liang (1997) to generate the 3D representation of the porous. When the correlation length (λ)[μm] is large, the method of Liang (1997) can only be used in super workstations or clusters of computers.

The domain correlation or integral scale (I_s) represents the total intensity of spatial correlation of a phenomenon, providing information about the distribution of size of pores. This parameter can be obtained from the area below the curve of autocorrelation function:

$$I_s = \int C(u)du \tag{3}$$

A strong correlation domain found in an image represents a high spatial continuity of the porous, and can be therefore associated with a sample with larger pore sizes.

2.2. Percolations Networks

It has been observed in the literature the use of percolation networks to represent the microscopic structure system of pourus and simulating the flow of fluids for the understanding of the properties transmissibility. In these works the distribution of pore size and/or bonds is an important data to build representative network of the porous media.

According to Souza (1993), the term percolation is associated with the concept of diffusion of an object or property through a random interconnected partially, similar to porous media. The theory of percolation was first developed by Flory (1941) and Stockmayer (1943) in order to describe the process of polymerization of macromolecules, from the increase of branches of minors molecules, forming a molecules network connected by chemical bonds, which is the central concept of the theory of percolation.

In mathematics literature, the theory of percolation was introduced by Hammersley (1957) in order to describe the connectivity and conductivity of porous media geometrically complex, which is seen as a network of random canals where a fluid seeps. The application of models percolation has been relevant to many scientific and industrial problems, such as the problem of statistical mechanical systems ferromagnetic diluted (Chayes, 1994), spread fire across forests (Henley, 1993) prediction properties of porosity during the morphological evolution (Monteagudo et al., 2001), problems of prospect of oil (Sahimi, 1993a), estimating parameters performance in fractured reservoirs (Masihi et al., 2005) determination of hydraulic conductivity using simulation processes of imbibition and drainage (Souza, 1993).

Applied to area of porous media, the theory of percolation demands to model the porous structure in which will occur the flow through a network model. In these models, the sites or network`s nodes correspond to the pores, while bonds represent the canyons of pores. Hammersley (1957) shows two situations for networks of percolation: first, called the percolation of sites, refers to networks in which percolation is considered the level of sites, that only the sites contain properties to be studied. When percolation is considered the level of bonds, in other words, only bonds contain properties to be studied, this is called bond percolation. There are cases where both sites and bonds represent properties of the problem to be studied, in these cases the process is called percolation of sites and bonds. Figure 3 illustrates some examples of networks of percolation.

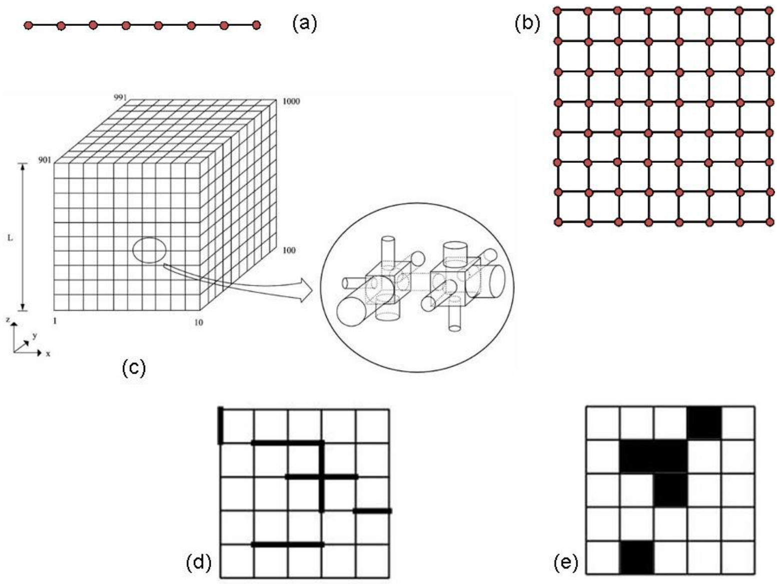

Figure 3. Models of percolation networks. In (a) one-dimensional network; in (b) two-dimensional network; in (c) three-dimensional network; in (d) bond percolation; in (e) site percolation.

During the process of percolation the bonds and/or sites are busy or assets randomly, and independently. A site (or bond) is active when its diameter meets the condition of invasion and it can be attacked or occupied by a liquid wetting or not, depending on the transport process to be simulated. The active bonds may be understanding as representations of the regions of high permeability of the porous, in other words, where occurs effectively the flow. The inactive bonds may represent the areas of low permeability system porous. A group of actives sites linked to each other through bonds is also called grouping.

A fundamental property of connected networks is the critical probability (or limit of percolation), *pc*, below which there is no path connecting with the faces of network, preventing the flow. Above the *pc* , there is at least one infinite grouping of active sites crossing the network.

In percolation network are used algorithms of labelling for the identification of percolate groups. The purpose of these algorithms is to identify the nodes belonging to the same class, giving them a label in common. The algorithm Hoshen and Kopelman (1976) has been extensively used in studies which involves such concepts. However, in this work it is used the algorithm proposed by (Herrero and Fernández, 2000). The main difference of this algorithm in relation to the previous one is the use of recurrent mechanisms that brings in a simplicity of code and computational efficiency. A study on the computational efficiency of the two algorithms can be found at (Michel and Zara, 2003).

As introduced previously the porous and connections can be represented by patterns of percolation of sites and patterns of percolation of bonds, for these ones it is also set a limit of percolation pcs e pc_b, respectively. The values of *pcs* e *pcb* , may be determined analytically only for networks regular 2-D. For other cases, these are obtained through simulations (Monteagudo et al., 2001). Other important topologicals properties that characterize the connectivity of groups and their properties are:

- ***the coordination number, z:*** that indicates the number of connections that are connected to each site of the network.
- ***the probability of percolation, P(p):*** what it is the probability of a site belongs to a infinity grouping when the network connections are occupied with the probability *p*.
- ***the fraction accessible, $X^A(p)$:*** what it is the fraction of active sites or bonds belonging to an infinity grouping.
- ***the average number of a grouping size s,* $\mathbf{n_s(p)}$:** defines the probability of a any grouping site contain exactly *s* active sites isolated.
- ***the connectivity function, g(r, p):*** it is the probability that an active object (a site) meet another active object (another site) as a displacement r units with the requirement that the two objects belong to the same grouping.
- ***the length of correlation, ξ:*** links the connectivity function with the displacement *r*. For values *p* next to limit of percolation, it is inversely proportional to the difference between the limit of percolation and the probability of activation of an object of the network.

$$\xi \approx \frac{1}{p_c - p} = (p_c - p)^{-1} \tag{4}$$

- ***the effective diffusivity, D_e:*** or hydrodynamic permeability *k*, which it is the fraction *p* that is active for the broadcast or to the flow of fluid.

Values of Z or the geometry of the elements vary for each percolation network and depend on the microscopes system details. But it is possible to reproduce the system by a set of universal parameters using laws of scale to infinite systems and critical exponents [Souza, 1993, Sahimi, 1993a].

2.3. Methodology

The methodology for the development of the proposal essay includes the following steps:

- Bibliographic Review: in this step a review of concepts of image analysis was made, models of determination of reservoir rock permeability, model of the serial connection graph and fundamentals and applications of the percolation theory and graph theory. Articles, thesis, dissertations and books, all with relevant importance in the scientific community were used as references.
- Studies of methods to be used: concepts of image analysis, model of serial connection graph for the determination of intrinsic permeability, percolation theory, and the methodology for the construction of percolation networks models based on information obtained by the parameter of the distribution of size pores (section 3) and the study of Imago software, used to assist in the processing.
- Development of models and algorithms: development of models and algorithms for the construction of percolation networks, graph, development of Opening-Dilatation model (section 4) and simulation of values of intrinsic permeability..
- Development of programs: design and implementation of a computer program with the following functions: generate networks percolation representing the porous structure of the porous media; simulate values of intrinsic permeability.
- Analysis and interpretation of results.

3. CONSTRUCTION OF NETWORK BASED ON THE DISTRIBUTION OF PORE SIZE

The first step consists in loading images, which can be images in grayscale or color images. In Imago software files images must be in TIFF and BMP, and may have variable size until the maximum of 1024 x 1024 pixels. Then it is done the segmentation of images, which consist in transforming a image in grayscale or color levels in a binary image; that separates the solid phase from the porous phase.

In the characterization of the images phase the relevant parameters as porosity, the autocorrelation function (Fig. 2), the connectivity function and distribution of size pores function, described in section 2.1, are determined.

3.1. Input Data

As cited formerly, to contruct the percolation network parameters as the distribution of size pores are used. The Table 1 shows na exemple of data of distribution of size pores obtained through the Imago software.

In this step of the work carried out using a percolation network only of sites. According to (Souza, 1993), a network only of sites can be understood as a network where connections (throat of pores) are considered negligible volume. Sites are divided into classes representing each diameter pore. In the literature multiple geometric forms are used to represent the pores, Souza (1993) uses the spherical, (Gasperi, 1999), and (Siqueira, 2000) describe the pores of cubic form with bonds in prisms form. The way in which the pore is represented does not interfere in the results, being enough that the distribution of pores evaluated in the middle is reproduced in the network (Siqueira, 2000).

Table1. Distribution of size and cumulative frequency of image i310 Berea 500 rock.

Pore Radius (*pixel*)	Cumulative Frequency	Pore Radius (*pixel*)	Cumulative Frequency
1	0.0127391	25	0.676519
2	0.0260494	26	0.721741
3	0.039959	27	0.728773
4	0.0561909	28	0.746374
5	0.0707566	29	0.752802
6	0.0860965	30	0.766268

7	0.10221	31	0.766268
8	0.11654	32	0.798076
9	0.136869	33	0.832404
10	0.156513	34	0.850161
11	0.178258	35	0.876446
12	0.204841	36	0.910425
13	0.23157	37	0.961665
14	0.262179	38	0.962113
15	0.287737	39	0.963029
16	0.307868	40	0.963548
17	0.332883	41	0.963756
18	0.0127391	42	0.964416
19	0.0260494	43	0.965087
20	0.039959	44	0.965582
21	0.0561909	45	0.966342
22	0.0707566	46	0.966673
23	0.0860965	47	0.967168
24	0.66117	48	1

3.2. Generation of network percolation

The network model used consists of a regular cubic mesh in which the pores are represented by the nodes and these can be connected to a maximum of six neighbors nodes through bonds (troaths) associated with them.

The first step to generate the model network is to define its size. According to Siqueira (2000), to determine the size of network it is important to consider the representativeness of the model, the processing time and memory computer available. To 3D network the size used in (Gasperi, 1999) will be the same adopted in this step of the work.

The second step is in the construction of the network itself. A routine of randomics numbers is used to generate a random number between 0 and 1. To obtain a network of sites randomly distributed, the random number with the cumulative proportion of each type of diameter is compared, sites are then placed from the upper left corner of the front face ending with the lower right corner of the opposite face (in case of 3D networks).

The last step is the proportion of the active and inactive elements of the network, for both are used concepts of the theory of percolation. Initially, all nodes are considered inactive and are labeled, randomly, with indices 1 until the number of classes of nodes obtained by distributition of pore size. Then from the index 1, it is added step-by-step the index reference and certain nodes of the network are being activated, as a node with label less than or equal to the index reference is encountered. The increase in the index reference continues until the point where the percolation limit is reached, in other words, when, at least, one node of the face of output is opened, forming a continuous way of opened nodes between the injection face and output face. From this moment the number of average coordination of open pores can be calculated for comparison with experimental data.

With the network model appropriate to the characteristics of simulated porous media, estimates of the values of permeability are result of the simulation of flow in the model (Ioannids et al., 1995).

The model developed in this work for the determination of distribution sites and bonds is following.

4. OPENING-DILATION METHOD

The determination of distribution the sites and bonds in two-dimensional images of porous media is important in studies of characterization of the material correlated in areas such as the theory of percolation.

Fernandes (1994) shows that it is possible to determine the distribution of porous and the number of connections in images, using the operation of the opening (erosion followed dilation) of mathematical morphology. However, the method used by Fernandes cannot determine the area of connections, as necessary in some methods of reconstruction.

This section provides an alternative method, called Opening - Dilatation. This method has as advantage, determining the distribution of porous and bonds.

The method used by Fernandes (1994) is simple, given a binary image (black / white), where white represents the solid matrix and black the cavities (pores and bonds). It consists in applying the opening operation with certain structural element (usually the circle); each increase in the element structure determines the number of objects in the image through operation of labelling of objects. From the difference of objects in the image, it is possible to determine the number of connections, with the same diameter of the structural element. Evaluating the area of black objects before and after the implementation of the opening operation, it is possible to determine the distribution of pores of the image. Figure 4 illustrates the process. Note that the whole area eroded and not dilated is accounted as pores area, and that it is not possible the determination of the area of connections, only the number of these ones.

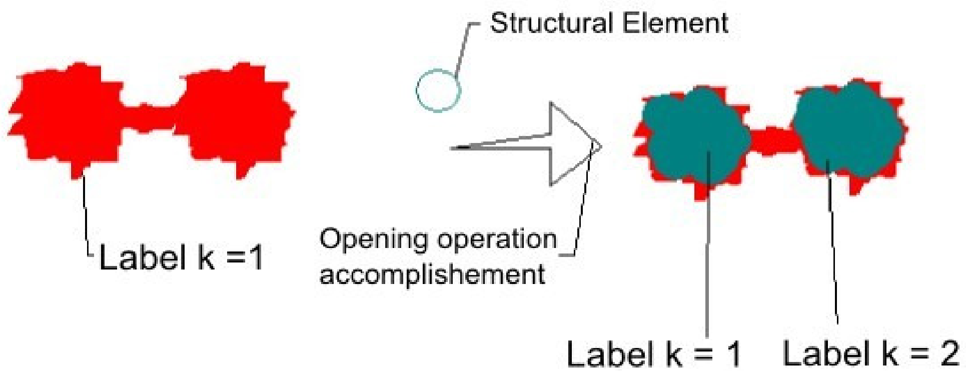

Figure 4. Procedure for determining the distribution and the number of bonds. The red area is pores area and there is a connection, Fernandes (1994).

It is possible to observe that the structural element has divided the object label k=1 in two objects, label k=1 and k=2, the area in red after erosion operationis the area of pores to diameter equal to the structural element. As the object was split into two, it has a diameter equal to the structural element, but the area of the connection is not determinated.

4.1. Opening-Dilatation method

The first alteration in relation to the method of Fernandes (1994) is the storage of information of the images in three matrix. The matrix M1 is the original matrix, the matrix M2 is used for a second expansion, and the matrix M3 is used for storage of pixels bonds (optional, only preview). It is kept of the label matrix (Fig. 5).

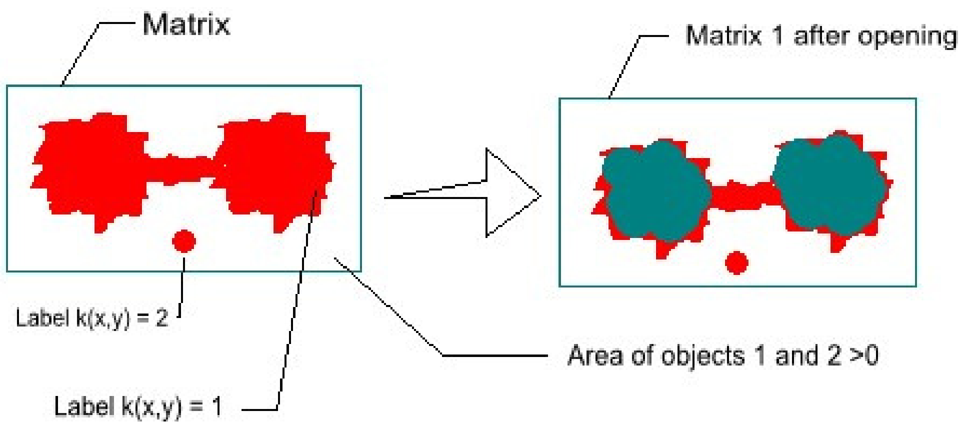

Figure 5. Operation of opening on the matrix M1.

In Figure 5, the arrow indicates the execution of the opening operation. The area in red after the opening operation represents the area of pores and bonds. The second difference of this method, the area in red is no longer the area of pores, it is the area of pores and bonds. It is determined the area of pores and bonds by expression:

$$A_{porous} + A_{bonds} = A_{red,M1\ after\ opennig} \quad (5)$$

The area in red in the matrix M1 is determined by the difference porosity of M1 and M1 after opening. After opening operation, the matrix M1 has as active pixels the pixels in green in Fig. 5.

It is created a second matrix, M2, equal M1 (M2 is formed by the green area in the matrix M1), and it is proceeded the dilatation operation on M2. The result is similar to that shown in Fig. 6, where the white circles represent the dilated pixels.

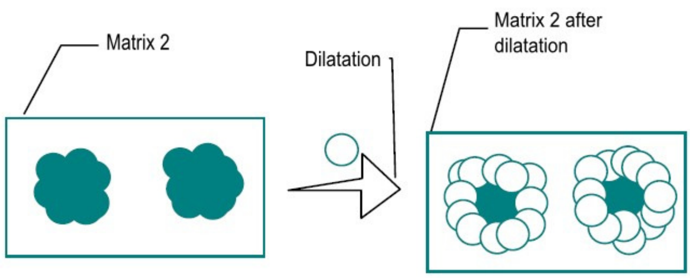

Figure 6. Operation of dilation on matrix 2

The third difference about the previous method is the achievement of one more dilatation operation on a auxiliary matrix, so called method of Opening - Dilatation.

It is achieved the mapping from the matrix M1 to label matrix. The mapping process is simple, it consists in verifying that a given pixel is active in M1, and if it is active, assign it the index object of label matrix. The matrix M1 is the matrix with the objects after the achievement of the opening operation, which now has the labels of the label matrix.

It is performed the determination of the area of the objects of the matrix M1. Note that the objects that have been completely eliminated will have area = 0, and the objects that had only their bonds eliminated will have area > 0. The images of M1 and M2 are then compared, as Fig. 7.

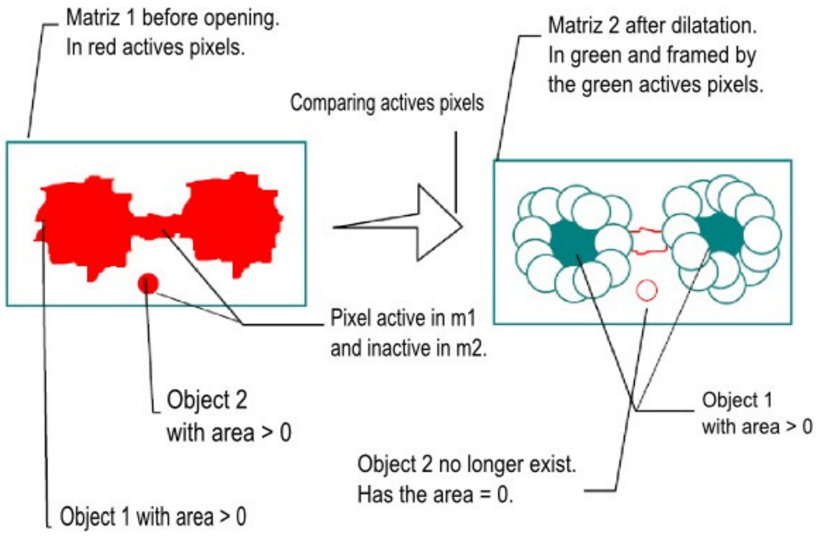

Figure 7. Comparison of M1 and M2, and area objects M1 to determe the pixels of connection.

Note that the area surrounded in red in the matrix M2, or it is fully eroded pore eroded or it is area of connections.

The area of connections is determined as follows: speeding the whole image and compare the matrixs M1 and M2, if the pixel is active in the matrix M1 (> 1), and inactive in the matrix M2 (=0) then it is a connection pixel or pore fully eroded. If the area of the object (based on the matrix M1 mapped) is greater than zero, then it is a connection pixel. Optionally a third matrix is created, M3, which it is used to store the bond pixels, for later viewing.

It can be determined the area of bonds, by establishing the area of active pixels in M3. It is obtained then relation:

$$A_{bonds} = A_{red,M3} \qquad (6)$$

From the equations 4 and 5 the area of pores, is determined as:

$$A_{porous} = A_{red,M1\ after\ openning} - A_{red,M3} \qquad (7)$$

The process follows iteratively, increasing the structural element and repeating the calculations until the entire distribution of pores and connections is obtained.

5. CONCLUSIONS

The application of models of percolation networks to represent a complex microstructure porous of reservoir rocks, associated to techniques of analysis of images, has the potential to become a powerful tool in petrophysical evalutions.

Using analysis of images it is possible to determine important parameters for the construction of the percolation network as porosity, autocorrelation function and distribution of size of pores function. This article showed the methodology for the construction of the percolation network and it presents the method of the Opening-Dilation for determination of the distribution of pores and connections in 2D images. The method of the Opening-Dilation differs of the other methods previously published by its capacity to determine not only the distribution of pores, but also the distribution of connections, necessary information in some methods of reconstruction of images.

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